TYPICAL QUESTIONS & ANSWERS

PART - I

OBJECTIVE TYPE QUESTIONS

Each Question carries 2 marks.

Choose correct or the best alternative in the following:

Q.1 To calculate Thevenin's equivalent value in a circuit

(A) all independent voltage sources are opened and all independent current sources are short circuited.
(B) both voltage and current sources are open circuited
(C) all voltage and current sources are shorted.
(D) all voltage sources are shorted while current sources are opened.

Ans: **D**
To calculate Thevenin's equivalent impedance value in a circuit, all independent voltage sources are shorted while all independent current sources are opened.

Q.2 A 26 dBm output in watts equals to

(A) 2.4W. **(B)** 0.26W.
(C) 0.156W. **(D)** 0.4W.

Ans: **A**
A 26dBm output in watts equals to 0.4 W because

$$10\times\log_{10}\left(\frac{400\text{mW}}{1\text{mW}}\right)=10\times2.6=26\,\text{dB}$$

Q.3 The Characteristic Impedance of a low pass filter in attenuation Band is

(A) Purely imaginary. **(B)** Zero.
(C) Complex quantity. **(D)** Real value.

Ans: **A**
The characteristic impedance of a low pass filter in attenuation band is purely imaginary.

Q.4 The real part of the propagation constant shows:

(A) Variation of voltage and current on basic unit.
(B) Variation of phase shift/position of voltage.
(C) Reduction in voltage, current values of signal amplitude.
(D) Reduction of only voltage amplitude.

Ans: **C**
The real part of the propagation constant shows reduction in voltage, current values of signal amplitude.

Q.5 The purpose of an Attenuator is to:

(A) increase signal strength. **(B)** provide impedance matching.
(C) decrease reflections. **(D)** decrease value of signal strength.

Ans: **D**
The purpose of an Attenuator is to decrease value of signal strength.

Q.6 In Parallel Resonance of:
R – L – C circuit having a R – L as series branch and 'C' forming parallel branch. Tick the correct answer only.

(A) Max Impedance and current is at the frequency that of resonance.
(B) Value of max Impedance = L / (CR).
(C) ranch currents are 180 Degree phase shifted with each other.
(D) $f_r = \frac{1}{2}\pi\left[\frac{1}{LC} - \frac{R^2}{L^2}\right]$.

Ans: **D**
In parallel resonance of R-L-C circuit having a R-L branch and 'C' forming parallel branch,

$$fr = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Q.7 In a transmission line terminated by characteristic impedance, Z_o

(A) There is no reflection of the incident wave.
(B) The reflection is maximum due to termination.
(C) There are a large number of maximum and minimum on the line.
(D) The incident current is zero for any applied signal.

Ans: **A**
In a transmission line terminated by characteristic impedance, $Z_{o,}$ there is no reflection of the incident wave.

Q.8 For a coil with inductance L and resistance R in series with a capacitor C has

(A) Resonance impedance as zero.
(B) Resonance impedance R.
(C) Resonance impedance L/CR.
(D) Resonance impedance as infinity.

Ans: **B**
For a coil with inductance L and resistance R in series with a capacitor C has a resonance impedance R.

Q.9 Laplace transform of a unit Impulse function is

(A)s. **(B)** 0.
(C) e^{-s}. **(D)** 1.

Ans: **D**

Laplace transform of a unit Impulse function is **1**

Q.10 Millman's theorem is applicable during determination of

(A) Load current in a network of generators and impedances with two output terminals.
(B) Load conditions for maximum power transfer.
(C) Dual of a network.
(D) Load current in a network with more than one voltage source.

Ans: D
Millman's theorem is applicable during determination of **Load current in a network with more than one voltage source.**

Q.11 Asymmetrical two port networks have

(A) $Z_{sc_1} = Z_{oc_2}$ **(B)** $Z_{sc_1} = Z_{sc_2}$
(C) $Z_{oc_1} \neq Z_{oc_2}$ **(D)** $Z_{oc_1} \neq Z_{oc_2}$ and $Z_{sc_1} \neq Z_{sc_2}$

Ans: D
Asymmetrical two port networks have $\mathbf{Z_{OC1} \neq Z_{OC2}}$ **and** $\mathbf{Z_{SC1} \neq Z_{SC2}}$

Q.12 An attenuator is a

(A) R's network. **(B)** RL network.
(C) RC network. **(D)** LC network.

Ans: A
An attenuator is a **R's network**.

Q.13 A pure resistance, R_L when connected at the load end of a loss-less $100\ \Omega$ line produces a VSWR of 2. Then R_L is

(A) $50\ \Omega$ only. **(B)** $200\ \Omega$ only.
(C) $50\ \Omega$ or $200\ \Omega$. **(D)** $400\ \Omega$.

Ans: C
A pure resistance, R_L when connected at the load end of a loss-less $100\ \Omega$ line produces a VSWR of 2. Then R_L is **50 Ω or 200 Ω,** as follows:

$$VSWR = \frac{R_O}{R_L} = \frac{100}{R_L} = 2 \qquad \Rightarrow R_L = 50\Omega$$

$$VSWR = \frac{R_L}{R_O} = \frac{R_L}{100} = 2 \qquad \Rightarrow R_L = 200\Omega$$

Q.14 The reflection coefficient of a transmission line with a short-circuited load is

(A) 0. **(B)** ∞.
(C) $1.0\angle 0°$. **(D)** $1.0\angle 180°$.

Ans: A
The reflection coefficient of a transmission line with a short-circuited load is **0**.

Q.15 All pass filter

(A) passes whole of the audio band.
(B) passes whole of the radio band.
(C) passes all frequencies with very low attenuation.
(D) passes all frequencies without attenuation but phase is changed.

Ans: D
All pass filters, **passes all frequencies without attenuation but phase change.**

Q.16 A series resonant circuit is inductive at f = 1000 Hz. The circuit will be capacitive some where at

(A) f > 1000 Hz.
(B) f < 1000 Hz.
(C) f equal to 1000 Hz and by adding a resistance in series.
(D) f = 1000+ f_o (resonance frequency)

Ans: B
A series resonant circuit is inductive at f = 1000 Hz. The circuit will be capacitive some where at **f < 1000 Hz.**

Q.17 Compensation theorem is applicable to

(A) non-linear networks. **(B)** linear networks.
(C) linear and non-linear networks. **(D)** None of the above.

Ans: C
Compensation theorem is applicable to **linear and non-linear networks.**

Q.18 Laplace transform of a damped sine wave $e^{-\alpha t}\sin(\theta t).u(t)$ is

(A) $\frac{1}{(s+\alpha)^2+\theta^2}$. **(B)** $\frac{s}{(s+\alpha)^2+\theta^2}$.

(C) $\frac{\theta}{(s+\alpha)^2+\theta^2}$. **(D)** $\frac{\theta^2}{(s+\alpha)^2+\theta^2}$.

Ans: C
Laplace transform of a damped sine wave $e^{-\alpha t}\sin(\theta t)\,u(t)$ is

$$\frac{\theta}{(s+\alpha)^2+\theta^2}$$

Q.19 A network function is said to have simple pole or simple zero if

(A) the poles and zeroes are on the real axis.
(B) the poles and zeroes are repetitive.
(C) the poles and zeroes are complex conjugate to each other.
(D) the poles and zeroes are not repeated.

Ans: D
A network function is said to have simple pole or simple zero if **the poles and zeroes are not repeated.**

Q.20 Symmetrical attenuators have attenuation $'\alpha'$ given by

(A) $20\log_{10}\left(\frac{I_R}{I_S}\right)$ **(B)** $20\log_{10}\left(\frac{I_R\,R_R}{I_S\,R_S}\right)$.

(C) $10\log_{10}\left(\frac{I_R}{I_S}\right)$. **(D)** $20\log_{10}\left(\frac{I_S}{I_R}\right)$.

Ans: D
Symmetrical attenuators have attenuation 'α' given by

$$\alpha = 20\log_{10}\left[\frac{I_S}{I_R}\right]$$

Q.21 The velocity factor of a transmission line
(A) is governed by the relative permittivity of the dielectric.
(B) is governed by the skin effect.
(C) is governed by the temperature.
(D) All of the above.

Ans: A
The velocity factor of a transmission line **is governed by the relative permittivity of the dielectric.**

Q.22 If $'\alpha'$ is attenuation in nepers then
(A) attenuation in dB = α / 0.8686. **(B)** attenuation in dB = 8.686 α.
(C) attenuation in dB = 0.1 α. **(D)** attenuation in dB = 0.01 α.

Ans: B
If 'α'̣ is attenuation in nepers then **attenuation in dB = 8.686 α.**

Q.23 For a constant K high pass π-filter, characteristic impedance Z_0 for $f < f_c$ is
(A) resistive. **(B)** inductive.
(C) capacitive. **(D)** inductive or capacitive.

Ans: D
For a constant K high pass π-filter, characteristic impedance Z_o for $f < f_c$ is **inductive or capacitive.**

Q.24 A delta connection contains three impedances of $60\,\Omega$ each. The impedances of equivalent star connection will be
(A) 15 Ω each. **(B)** 20 Ω each.
(C) 30 Ω each. **(D)** 40 Ω each.

Ans: B
A delta connection contains three impedances of 60Ω each. The impedances of equivalent star connection will be **20Ω each.**

Q.25 Which one of the following is a passive element?
(A) A BJT. **(B)** An Inductor.
(C) A FET. **(D)** An Op-amp.

Ans: B
Which one of the following is a passive element? **An Inductor**

Q.26 Millman theorem yields
(A) equivalent resistance of the circuit.
(B) equivalent voltage source.
(C) equivalent voltage OR current source.
(D) value of current in milli amperes input to a circuit from a voltage source.

Ans: C
Millman's theorem yields equivalent **voltage or current source.**

Q.27 The z-parameters of the shown T-network at Fig.1 are given by
(A) 5, 8, 12, 0
(B) 13, 8, 8, 20
(C) 8, 20, 13, 12
(D) 5, 8, 8, 12

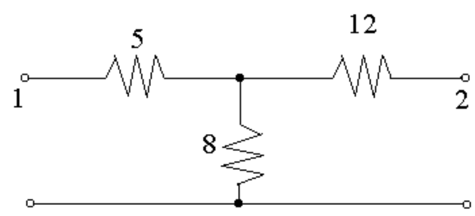


Fig. 1

Ans: B
The Z parameters of the T - network at Fig 1.1 are given by **13, 8, 8, 20**
$\mathbf{Z_{11} = Z_1 + Z_3 = 5 + 8 = 13, Z_{12} = Z_3 = 8, Z_{21} = Z_3 = 8, Z_{22} = Z_2 + Z_3 = 12 + 8 = 20}$

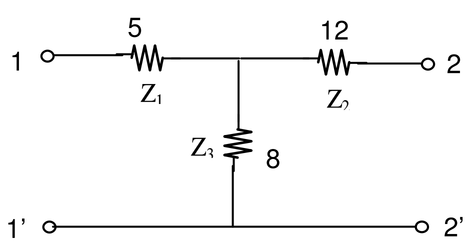


Fig 1.1

Q.28 To a highly inductive circuit, a small capacitance is added in series. The angle between voltage and current will
(A) decrease. **(B)** increase.
(C) remain nearly the same. **(D)** become indeterminant.

Ans: C
To a highly inductive circuit, a small capacitance is added in series. The angle between voltage and current will **remain nearly the same**.

Q.29 The equivalent inductance of Fig.2 at terminals 1 1' is equal to

(A) $L_1 + L_2 + 2M$
(B) $L_1 + L_2 - 2M$
(C) $L_1 + L_2$
(D) $L_1 - L_2 + 2M$

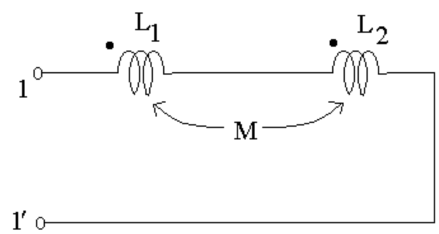


Fig. 2

Ans: A
The equivalent inductance of Fig 1.2 at terminals 11' is equal to

$L_1 + L_2 + 2M$

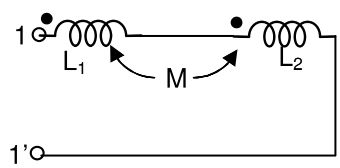


Fig 1.2

Q.30 The characteristic impedances z_0 of a transmission line is given by, (where R, L, G, C are the unit length parameters)

(A) $(R + j\omega L)/(G + j\omega C)$ **(B)** $(R + j\omega L)(G + j\omega C)$
(C) $(R + j\omega L)^2/(G + j\omega C)$ **(D)** $[(R + j\omega L)/(G + j\omega C)]^{1/2}$

Ans: D
The characteristic impedance Z_o of a transmission line given by, (where R, L, G, C are the unit length parameters

$$Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

Q.31 The relation between R_1 and R_2 for the given symmetrical lattice attenuator shown in Fig.3 is

(A) $R_1 = R_2 = R_0$
(B) $R_1 = R_0^2 / R_2$
(C) $R_1 = R_2^2 / R_0$
(D) $R_2 = R_1^2 / R_0$

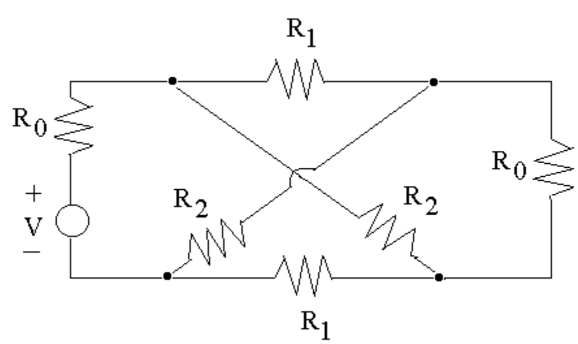


Fig. 3

Ans: B

The relation between R_1 and R_2 for the given symmetrical lattice attenuator shown in Fig 1.3 is

$$R_1 = \frac{R_o^2}{R_2}$$

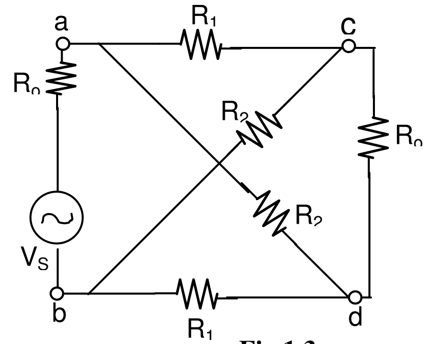


Fig 1.3

Q.32 If Laplace transform of x(t) = X(s), then Laplace transform of $x(t-t_0)$ is given by

(A) $(-t_0)X(s)$ **(B)** $X(s-t_0)$

(C) $e^{t_0 s}X(s)$ **(D)** $e^{-t_0 s}X(s)$

Ans: D

If Laplace transform of x(t) = X(s), then laplace transform of $x(t - t_0)$ is given by

$$e^{-t_0 s} X(s)$$

Q.33 The following constitutes a bilateral element

(A) A resistor. **(B)** FET.

(C) Vacuum tube. **(D)** metal rectifier.

Ans: A

Q.34 Voltages v_1 and v_2 in the given circuit are

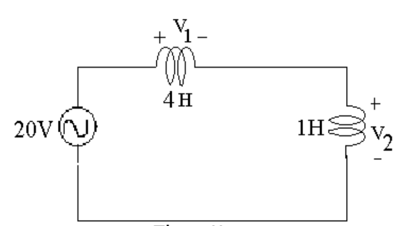


Figure 1b

(A) 20 volts each.

(B) 10 volts each.

(C) 16 volts, 4 volts.

(D) 4 volts, 16 volts.

Ans: B

Voltages v_1 and v_2 in the given circuit are

Q.35 Step response of series RC circuit with applied voltage V is of the form

(A) $i(t) = \frac{V}{R}e^{-t/RC}$ **(B)** $i(t) = \frac{V}{R}\left(1 - e^{-t/RC}\right)$

(C) $i(t) = -\frac{V}{R}e^{-t/RC}$ **(D)** $i(t) = -\frac{V}{R}\left(1 - e^{-t/RC}\right)$

Ans: Step response of series RC circuit with applied voltage V is of the form

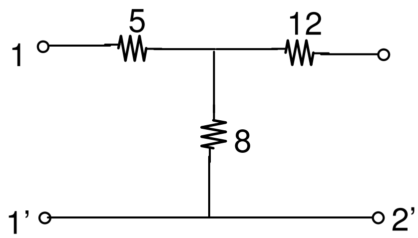


Fig 1.1

Q.36 In the given circuit switch S is opened at time t=0, then $\frac{dv}{dt}\left(0^{+}\right)$ is

(A) 10^{6} volt / sec.
(B) 100 volt / sec.
(C) 10^{5} volt / sec.
(D) 10 volt / sec.

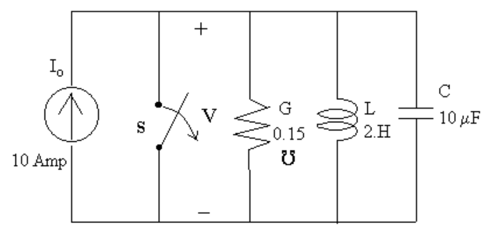
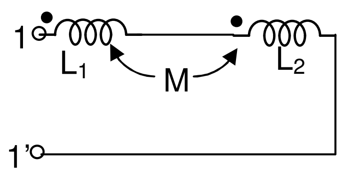


Fig. 1 d

Ans:
In a given circuit, switch S is opened at time t = 0, then



$L_1 + L_2 + 2M$

Fig 1.2

Q.37 In the circuit shown, maximum power will be transferred when

(A) $Z_L = (4.5 + j6.5)\Omega$
(B) $Z_L = (4.5 - j6.5)\Omega$
(C) $Z_L = (6.5 + j4.5)\Omega$
(D) $Z_L = (6.5 - j4.5)\Omega$

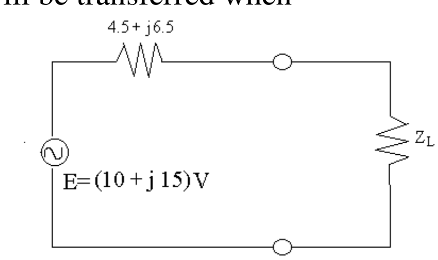


Fig. 1 e

Ans: B
In the circuit shown, maximum power will be transferred when Z_L = **(4.5 – j 6.5)Ω**

Q.38 Voltage Standing Wave Ratio (VSWR) in terms of reflection coefficient ρ is given by

(A) $\frac{1-\rho}{1+\rho}$. **(B)** $\frac{\rho-1}{\rho+1}$.

(C) $\frac{1+\rho}{1-\rho}$. **(D)** $\frac{\rho}{1+\rho}$.

Ans: C

$$VSWR = \frac{1+\rho}{1-\rho}$$

Q.39 For a 2-port network, the output short circuit current was measured with a 1V source at the input. The value of the current gives

(A) h_{12} **(B)** y_{12}

(C) h_{21} **(D)** y_{21}

Ans:

$$R_1 = \frac{R_o^{\ 2}}{R_2}$$

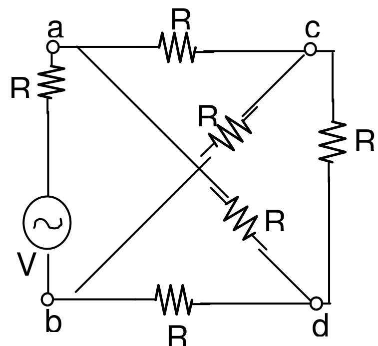


Fig 1.3

Q.40 $H(s) = \frac{V(s)}{I(s)} = \frac{s+3}{(s+2)^2}$ When i(t) is the unit step function, the value of v (t) in the steady state is given by

(A) $\frac{3}{2}$. **(B)** 1.

(C) 0. **(D)** $\frac{3}{4}$.

Ans:

Q.41 An RLC series circuit is said to be inductive if

(A) $\omega L > 1/\omega C$ **(B)** $\omega L = 1/\omega C$

(C) $\omega L < 1/\omega C$ **(D)** $\omega L = \omega C$

Ans: A

A RLC series circuit is said to be inductive if **$\omega L > 1/\omega C$.**

Q.42 Laplace transform of an unit impulse function is given by

(A) 1 **(B)** -1

(C) $1/s$ **(D)** $1/s^2$

Ans: A
Laplace transform of an unit impulse function is given by **1**.

Q.43 A function $H(s) = 2s/(s^2 + 8)$ will have a zero at

(A) $s = \pm j4$ **(B)** Anywhere on the s-plane.
(C) On the imaginary axis. **(D)** On the origin.

Ans: D
A function $H(s) = 2s/(s^2 + 8)$ will have a zero at **the origin**.

Q.44 For a two port reciprocal network, the three transmission parameters are given by A = 4, B = 7 and C = 5. The value of D is equal to

(A) 8.5 **(B)** 9
(C) 9.5 **(D)** 8

Ans: B
For a two port reciprocal network, the three transmission parameters are given by A = 4, B = 7 and C = 5. The value of D is equal to **9**.

$AD - BC = 1 \Rightarrow 4D = 1 + 35 = 36 \Rightarrow D = 36/4 = 9$

Q.45 Higher the value of Q of a series circuit

(A) Sharper is its resonance. **(B)** Greater is its bandwidth.
(C) Broader is its resonant curve. **(D)** Narrower is its bandwidth.

Ans: D
Higher the value of Q of a series circuit, **narrower is its pass band**.

Q.46 An ideal filter should have

(A) Zero attenuation in the pass band.
(B) Zero attenuation in the attenuation band.
(C) Infinite attenuation in the pass band.
(D) Infinite attenuation in the attenuation band.

Ans: A
An ideal filter should have **Zero attenuation in the pass band**.

Q.47 For an m-derived high pass filter, the cut off frequency is 4KHz and the filter has an infinite attenuation at 3.6 KHz, the value of m is

(A) 0.436 **(B)** 4.36
(C) 0.34 **(D)** 0.6

Ans: A
For an m-derived high pass filter, the cut off frequency is 4KHz and the filter has an infinite attenuation at 3.6KHz, the value of m is **0.436**

$$m = \sqrt{1 - \frac{f_\infty^{\,2}}{f_c^{\,2}}} = \sqrt{1 - \frac{(3.6 \times 1000)^2}{(4 \times 1000)^2}} = 0.436$$

Q.48 If $Z_{oc} = 120\Omega$ and $Z_{sc} = 30\Omega$, the characteristic impedance is

(A) 30Ω **(B)** 60Ω
(C) 120Ω **(D)** 150Ω

Ans: B
If $Z_{oc} = 120\Omega$ and $Z_{sc} = 30\ \Omega$, the characteristic impedance is 60Ω.

$$Z_o = \sqrt{Z_{oc} Z_{sc}} = \sqrt{120 \times 30} = 60\,\Omega$$

Q.49 The reflection coefficient of a line is -1. The line is

(A) Open circuited. **(B)** Short circuited.
(C) Terminated in Z_o. **(D)** Of infinite length.

Ans: A
The reflection coefficient of a line is –1. The line is **open circuited.**

Q.50 If a transmission line of length less than $\lambda/4$ is short circuited, it behaves as

(A) Pure capacitive reactance. **(B)** Series resonant circuit.
(C) Parallel resonant circuit. **(D)** Pure inductive reactance.

Ans: D
If a transmission line of length less than λ/4 is short circuited, it behaves **as pure inductive reactance**.

Q.51 A line becomes distortion less if

(A) It is properly matched **(B)** It is terminated into Zo
(C) LG = CR **(D)** LR = GC

Ans: C
A line becomes distortion less if **LG = CR**

Q.52 Double stub matching eliminates standing waves on the

(A) Source side of the left stub **(B)** Load side of the right stub
(C) Both sides of the stub **(D)** In between the two stubs

Ans: A
Double stub matching eliminates standing waves on the **Source side of the left stub.**

Q.53 If $Z_{OC} = 100\Omega$ and $Z_{SC} = 64\Omega$, the characteristic impedance is

(A) $400\,\Omega$ **(B)** $60\,\Omega$
(C) $80\ \Omega$ **(D)** $170\,\Omega$

Ans: (C)
If $Z_{oc} = 100\,\Omega$ and $Z_{sc} = 64\,\Omega$ the characteristic impedance is **80Ω**

Q.54 The final value of f (t) for a given $F(s) = \frac{s}{(s+4)(s+2)}$

(A) Zero **(B)** 1/15

(C) 1/8 **(D)** 1/6

Ans: (A)

The final value of f(t) for a given $f(s) = \frac{s}{(s+4)(s+2)}$ is **Zero.**

Q.55 If the given network is reciprocal, then according to the reciprocity theorem

(A) $y_{21} = y_{12}$ **(B)** $y_{22} = y_{12}$

(C) $y_{11} = y_{12}$ **(D)** $y_{11} = y_{22}$

Ans: A

If the given network is reciprocal, then according to the reciprocity theorem $\mathbf{y_{21} = y_{12}}$

Q.56 The frequency of infinite attenuation (f_∞) of a low pass m-derived section is

(A) Equal to cut off frequency (f_c) of the filter.

(B) $f_\infty = \infty$.

(C) Close to but greater than the f_c of the filter.

(D) Close to but less than the f_c of the filter.

Ans: C

The frequency of infinite attenuation (f_∞) of a low pass m-derived section is **Close to but greater then the f_c of the filter.**

Q.57 The dynamic impedance of a parallel RLC circuit at resonance is

(A) C/LR **(B)** R/LC

(C) L/CR **(D)** LC/R

Ans: C

The dynamic impedance of a parallel RLC circuit at resonance is $\frac{L}{CR}$

Q.58 Laplace transform of the function e^{-2t} is

(A) 1/2s **(B)** $(s+2)$

(C) $1/(s+2)$ **(D)** 2s.

Ans: (C)

Laplace transform of the function e^{-2t} is $\frac{1}{s+2}$

Q.59 A (3 + 4j) voltage source delivers a current of (4 + j5) A. The power delivered by the source is

(A) 12 W **(B)** 15 W
(C) 20 W **(D)** 32 W

Ans: A
A (3 + 4j) voltage source delivers a current of (4+ j5) A. The power delivered by the source is **12 W**

Q.60 In a variable bridged T-attenuator, with $R_A = R_o$, zero dB attenuation can be obtained if bridge arm R_B and shunt arm R_C are set as

(A) $R_B = 0, R_C = \infty$ **(B)** $R_B = \infty, R_C = 0$
(C) $R_B = R, R_C = \infty$ **(D)** $R_B = 0, R_C = R$

Ans: A
In a variable bridged T-attenuator, with RA = RO, zero dB attenuation can be obtained if bridge arm RB and shunt arm RC are set as RB = 0, RC = ∞.

Q.61 Consider a lossless line with characteristic impedance Ro and VSWR = S. Then the impedance at **the** point of a voltage maxima equals

(A) SR_0 **(B)** R_0/S
(C) S^2R_0 **(D)** R_0

Ans: A
Consider a lossless line with characteristic impedance Ro and VSWR = S. Then the impedance at the point of a voltage maxima equals SRo

Q.62 If f_1 and f_2 are half power frequencies and f_o is the resonance frequency, the selectivity of RLC circuit is given by

(A) $\frac{f_2 - f_1}{f_0}$ **(B)** $\frac{f_2 - f_1}{2f_0}$
(C) $\frac{f_2 - f_1}{f_1 - f_0}$ **(D)** $\frac{f_2 - f_0}{f_1 - f_0}$

Ans: A
If f1 and f2 are half power frequencies and f_0 be resonant frequency, the selectivity of RLC circuit is given by $\frac{f_2 - f_1}{f_0}$

Q.63 A symmetrical T network has characteristic impedance Z_o and propagation constant γ. Then the series element Z_1 and shunt element Z_2 are given by

(A) $Z_1 = Z_o \sinh\gamma$ and $Z_2 = 2Z_o/\tanh\gamma/2$
(B) $Z_1 = Z_o/\sinh\gamma$ and $Z_2 = 2Z_o \tanh\gamma/2$
(C) $Z_1 = 2Z_o \tan\gamma/2$ and $Z_2 = Z_o/\sinh\gamma$

(D) $Z_1 = Z_o \tanh \gamma/2$ and $Z_2 = 2Z_o/\sinh \gamma$

Ans: C
A symmetrical T network has characteristic impedance Zo and propagation constant γ. Then the series element Z1 and shunt element Z2 are given by
Z1 = 2 Zo tan γ/2 and Z2 = Zo / sinh γ

Q.64 A function is given by $F(s) = \frac{2s}{(s^2+8)}$. It will have a finite zero at

(A) Infinity **(B)** Anywhere on the s-plane
(C) On the imaginary axis **(D)** On the origin

Ans: D
A function is given by. It $F(s) = \frac{2s}{(s^2+8)}$ will have a zero on the origin.

Q.65 For a linear passive bilateral network

(A) $h_{21} = h_{12}$ **(B)** $h_{21} = -h_{12}$
(C) $h_{12} = g_{12}$ **(D)** $h_{12} = -g_{12}$

Ans: B
For a linear passive bilateral network $\mathbf{h_{21} = - h_{12}}$

Q.66 A constant K band-pass filter has pass-band from 1000 to 4000 Hz. The resonance frequency of shunt and series arm is a

(A) 2500 Hz. **(B)** 500 Hz.
(C) 2000 Hz. **(D)** 3000 Hz.

Ans: C
A constant k band pass filter has pass band from 1000 to 4000 Hz. The resonant frequency of shunt and series arm is 2000Hz

Q.67 A constant voltage source with 10V and series internal resistance of 100 ohm is equivalent to a current source of

(A) 100mA in parallel with 100 ohm.
(B) 1000mA in parallel with 100 ohm.
(C) 100V in parallel with 10-ohms.
(D) 100mA in parallel with 1000 ohm.

Ans: A
A constant voltage source with 10V and series internal resistance of 100 ohm is equivalent to a current source of 100mA in parallel with 100 ohm.

Q.68 Input impedance of a short-circuited lossless line with length $\lambda/4$ is

(A) Z_o **(B)** zero
(C) infinity **(D)** z_0^2

Ans: C
Input impedance of a short-circuited loss less line with length □/4 is ∞

Q.69 Laplace transform of unit impulse is
(A) u(s) **(B)** 1
(C) s **(D)** 1/s

Ans: B
Laplace transform of unit impulse is 1

Q.70 In a two terminal network, the open circuit voltage at the given terminal is 100V and the short circuit at the same terminal gives 5A current. If a load of 80 Ω resistance is connected at the terminal, the load current is given by
(A) 1Amp **(B)** 1.25 Amp
(C) 6 Amp **(D)** 6.25 Amp

Ans: A
Ina two terminal network, the open circuit voltage at the given terminal is 100V and the short circuit at the same terminal 5A. If a load of 80 Ω resistance is connected at the terminal, the load current is given by **1 Amp.**

Q.71 Given V_{TH} = 20V and R_{TH} = 5 Ω, the current in the load resistance of a network,
(A) is 4A **(B)** is more than 4A.
(C) is 4A or less **(D)** is less than 4A.

Ans: D
Given V_{TH} = 20V and R_{TH} = 5 Ω, the current in the load resistance of a network, **is less than 4A.**

Q.72 The Laplace transform of a function is 1/s x Ee^{-as}. The function is
(A) E sin ωt **(B)** Ee^{at}
(C) E u(t–a) **(D)** E cos ωt

Ans: C
The Laplace transform of a function is 1/s × Ee-as. The function is E u(t - α).

Q.73 For a symmetrical network
(A) $Z_{11} = Z_{22}$ **(B)** $Z_{12} = Z_{21}$
(C) $Z_{11} = Z_{22}$ and $Z_{12} = Z_{21}$ **(D)** $Z_{11} \times Z_{22} - Z_{12}^2 = 0$

Ans: C

Q.74 A constant k low pass T-section filter has Z_0 = 600Ω at zero frequency. At f = f_c the characteristic impedance is
(A) 600Ω **(B)** 0
(C) ∞ **(D)** More than 600Ω

Ans: B

A constant k low pass T-section filter has Zo = 600Ω at zero frequency. At f = fc, the characteristic impedance is 0.

Q.75 In m-derived terminating half sections, m =

(A) 0.1 **(B)** 0.3
(C) 0.6 **(D)** 0.95

Ans: C
In m-derived terminating half sections, **m = 0.6**.

Q.76 In a symmetrical T attenuator with attenuation N and characteristic impedance R_0, the resistance of each series arm is equal to

(A) R_0 **(B)** $(N-1)R_0$
(C) $\frac{2N}{N^2-1}R_0$ **(D)** $\frac{N}{N^2-1}R_0$

Ans: C
In a symmetrical T attenuator with attenuation N and characteristic impedance R_o, the resistance of each series arm is equal to $\frac{2N}{N^2-1}R_o$

Q.77 For a transmission line, open circuit and short circuit impedances are 20Ω and 5Ω. The characteristic impedance of the line is

(A) 100 Ω **(B)** 50 Ω
(C) 25 Ω **(D)** 10 Ω.

Ans: D
For a transmission line, open circuit and short circuit impedances are 20 Ω and 5. Ω The characteristic impedance of the line is 10 Ω

Q.78 If K is the reflection coefficient and S is the Voltage standing wave ratio, then

(A) $k = \frac{VSWR-1}{VSWR+1}$ **(B)** $|k| = \frac{VSWR-1}{VSWR+1}$
(C) $k = \frac{VSWR+1}{VSWR-1}$ **(D)** $|k| = \frac{VSWR+1}{VSWR-1}$

Ans: B
If K is the reflection coefficient and S is the Voltage standing wave ratio, then

$$|k| = \frac{VSWR-1}{VSWR+1}$$

Q.79 A parallel RLC network has R=4Ω, L =4H, and C=0.25F, then at resonance Q=

(A) 1 **(B)** 10
(C) 20. **(D)** 40

Ans: A

A parallel RLC network has R = 4Ω, L = 4H, and C = 0.125F, then at resonance Q = 1.

Q.80 A delta connection contains three impedances of 60 Ω each. The impedances of the equivalent star connection will be

(A) 15Ω each. **(B)** 20Ω each.
(C) 30Ω each. **(D)** 40Ω each.

Ans: B
A delta connection contains three impedances of 60 Ω each. The impedances of the equivalent star connection will be 20 Ω each.

Q.81 If V_{TH} and R_{TH} are the Thevenin's voltage and resistance and R_L is the load resistance, then Thevenin's equivalent circuit consists of

(A) series combination of $R_{TH,}V_{TH}$ and R_L.
(B) series combination of R_{TH} and V_{TH}.
(C) parallel combination of $R_{TH,}V_{TH}$ and R_L.
(D) parallel combination of R_{TH} and V_{TH}.

Ans: B
If VTH and RTH are the Thevenin's voltage and resistance and RL is the load resistance, then Thevenin's equivalent circuit consists of series combination of RTH and VTH.

Q.82 If $f(t) = r(t-\alpha), F(s) =$

(A) $\frac{e^{-\alpha s}}{s^2}$ **(B)** $\frac{\alpha}{s+\alpha}$
(C) $\frac{1}{s+\alpha}$ **(D)** $\frac{e^{-\alpha s}}{s}$

Ans: A
If f(t) = r(t -α), $F(s) = \frac{e^{-\alpha s}}{s^2}$

Q. 83 The integral of a step function is

(A) A ramp function. **(B)** An impulse function.
(C) Modified ramp function. **(D)** A sinusoid function.

Ans: A
The integral of a step function is **a ramp function**.

Q.84 For a prototype low pass filter, the phase constant β in the attenuation band is

(A) ∞ **(B)** 0
(C) π **(D)** $\pi/2$

Ans: C
For a prototype low pass filter, the phase constant π in the attenuation band is β

Q.85 In the m-derived HPF, the resonant frequency is to be chosen so that it is
(A) above the cut-off frequency. **(B)** Below the cut-off frequency.
(C) equal to the cut-off frequency. **(D)** None of these.

Ans: B
In the m-derived HPF, the resonant frequency is to be chosen so that it is **below the cut off frequency.**

Q.86 In a symmetrical π attenuator with attenuation N and characteristic impedance R_o, the resistance of each shunt arm is equal to
(A) R_O **(B)** $(N-1)R_O$
(C) $\frac{N-1}{N+1}R_o$ **(D)** $\frac{N+1}{N-1}R_o$

Ans: D
In a symmetrical π attenuator with attenuation N and $\left(\frac{N+1}{N-1}\right)R_o$ characteristic impedance $R_{o,}$ the resistance of each shunt arm is equal to

Q.87 In terms of R,L,G and C the propagation constant of a transmission line is
(A) $\sqrt{R+j\omega L}$ **(B)** $\sqrt{(R+j\omega L)(G+j\omega C)}$
(C) $\sqrt{G+j\omega C}$ **(D)** $\sqrt{\frac{R+j\omega L}{G+j\omega C}}$

Ans: B
In terms of R, L, G and C, the propagation constant of a transmission line is

$$\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}$$

Q. 88 A line has $Z_o = 300\angle 0\Omega$. If $Z_L = 150\angle 0\Omega$, Voltage standing wave ratio, S =
(A) 1 **(B)** 0.5
(C) 2 **(D)** ∞

Ans: C
A line has $Z_o = 300\angle 0\,\Omega$. If $Z_L = 150\angle 0\,\Omega$, Voltage standing wave ratio, since $Z_O > Z_L$ **, S = 2**

$$\frac{Z_o}{Z_L}=\frac{300\angle 0°}{150\angle 0°}$$

Q.89 In a series resonant circuit, the resonant frequency will be
(A) Geometric mean of half power frequencies.
(B) Arithmetic mean of half power frequencies.
(C) Difference of half power frequencies.

(D) Sum of half power frequencies

Ans: A
In a series resonant circuit, the resonant frequency is the **geometric mean of half power frequencies.**

Q.90 A function is given by $F(S) = \frac{1}{s+3}$. It would have a zero at

(A) real axis of s-plane. **(B)** imaginary axis of s-plane.
(C) at infinity. **(D)** at the origin.

Ans: C
A function is given by $F(S) = \frac{1}{s+3}$. It would have a zero **at infinity.**

Q.91 In a series parallel circuit, any two resistances in the same current path must be in-:

(A) Parallel with each other **(B)** Series with each other
(C) Parallel with the voltage source **(D)** Series with the voltage source

Ans: B
In a series parallel circuit, any two resistances in the same current path must be in **Series with each other**

Q. 92 Superposition theorem is not applicable in:

(A) Voltage responses **(B)** Power responses
(C) Current responses **(D)** All the three

Ans: B
Superposition theorem is not applicable in **Power responses.**

Q.93 Kirchoff's first law is used in the formation of:

(A) Loop equations **(B)** Nodal equations
(C) Both **(D)** None of the above

Ans: B
Kirchoff's first law is used in the formation of **Nodal equations**.

Q.94 Bridged T network can be used as:

(A) Attenuator **(B)** Low pass filter
(C) High pass filter **(D)** Band pass filter

Ans: A
Bridged T network can be used as **Attenuator.**

Q.95 One neper is equal to

(A) 0.8686 dB **(B)** 8.686 dB
(C) 118.686 dB **(D)** 86.86 dB

Ans:
One neper is equal to **0.1151 x attenuation in dB.**

Q.96 Total reflection can take place if the load is:

(A) 0 **(B)** ∞
(C) 0 and ∞ **(D)** Zo

Ans: C
Total reflection can take place if the load is **0 and ∞.**

Q.97 The characteristic impedance of a distortion less line is:

(A) Real **(B)** Inductive
(C) Capacitive **(D)** Complex

Ans: A
The characteristic impedance of a distortion less line is **Real.**

Q.98 Terminating half sections used in composite filters are built with the following value of m:

(A) m = 0.6 **(B)** m = 0.8
(C) m = 0.3 **(D)** m = 1

Ans: A
Terminating half sections used in composite filters are built with the following value of **m = 0.6.**

Q.99 A transmission line works as an

(A) Attenuator **(B)** LPF
(C) HPF **(D)** Neither of the above

Ans: B
A transmission line works as an **LPF (Low Pass Filter).**

Q.100 In a loss free RLC circuit the transient current is:

(A) Sinusoidal **(B)** Square wave
(C) Oscillating **(D)** Non-oscillating

Ans: A
In a loss free RLC circuit the transient current is **Sinusoidal.**

PART – II

NUMERICALS

Q.1. Open and short circuit impedances of a transmission line at 1.6 KHz are $900\angle{-30^0}\,\Omega$ and $400\angle{-10^0}\,\Omega$. Calculate the characteristic impedance of the Line. **(7)**

Ans:
Given $Z_{oc} = 900\angle{-30^0}$
$Z_{sc} = 400\angle{-10^0}$
The characteristic impedance of the line is given by

$$Z_o = \sqrt{Z_{oc} \times Z_{sc}} = \sqrt{900 \times 400}\left[\frac{1}{2}\angle\left(-30^0 - 10^0\right)\right]$$

$$= \sqrt{360000}\left[\frac{1}{2}\angle -40^0\right]$$

$$= 600\angle -20^0\,\Omega$$

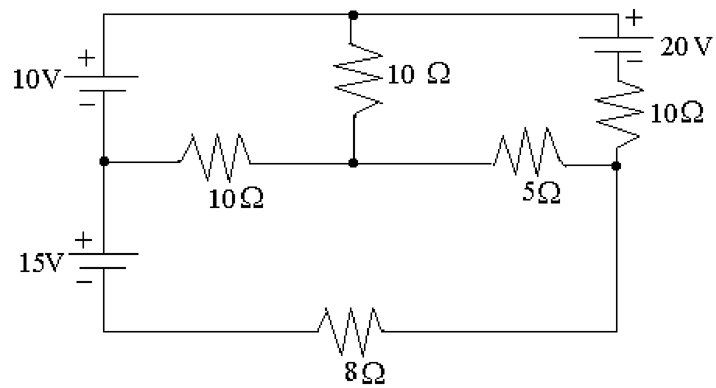
Q.2. Define Laplace transform of a function f(t). Find the Laplace transforms for the functions
$f_1(t) = e^{-at}\sin\omega t \,.\, u(t)$

Ans:
$F_1(s) = L(e^{-at}\sin\omega t)$

$$F_1(s) = \int_0^\infty e^{-at}\sin\omega t \cdot e^{-st}\,dt = \frac{1}{2j}\int_0^\infty \sin\omega t \cdot e^{-(s+a)t}\,dt$$

$$= \left[\frac{-(s+a)\sin\omega t e^{-(s+a)t} + \omega\cos\omega t e^{-(s+a)t}}{(s+a)^2+\omega^2}\right]_0^\infty = \frac{\omega}{(s+a)^2+\omega^2}$$

Q.3. Find the power dissipated in 8 Ω resistors in the circuit shown below using Thevenin's theorem.



(6)

Ans:

To find R_{TH}, open circuiting the 8Ω resistor and short-circuiting the voltage sources

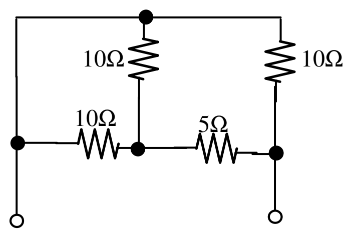


Fig 2.b.2

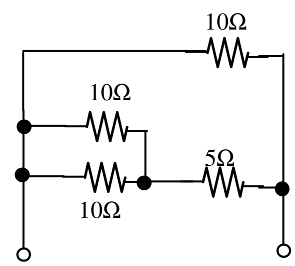


Fig 2.b.3

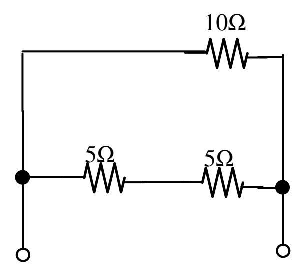
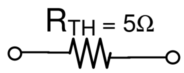


Fig 2.b.4

$R_{TH} = 5\Omega$



To find V_{OC},

Let the potential at x be V_1.

On applying kirchoff's current law at point x

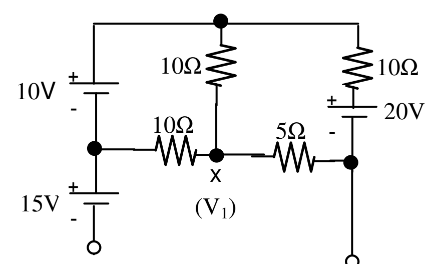


Fig 2.b.5

$$\frac{V_1+20}{10+5}+\frac{V_1}{10}+\frac{V_1+10}{10}=0$$

$$\frac{V_1+20}{15}+\frac{V_1}{10}+\frac{V_1+10}{10}=0$$

$$\frac{2V_1+40+3V_1+3V_1+30}{30}=0$$

$\Rightarrow 8V_1 + 70 = 0$

$\Rightarrow V_1 = -70/8 = -8.75V$

$\therefore$ Current through 5Ω resistor is

$$\frac{V_1+20}{15}=\frac{20-8.75}{15}=\frac{11.25}{15}\text{A}$$

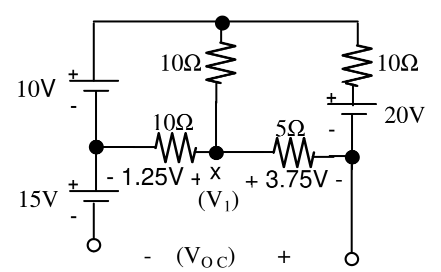


Fig 2.b.6

$\therefore$ Drop across 5Ω resistor is

$$5\times\frac{11.25}{15}=3.75\text{ V}$$

Current through 10Ω resistor left to point x is

$$\frac{V_1+10}{10}=\frac{10-8.75}{10}=\frac{1.25}{10}=0.125\text{A}$$

Drop across 10Ω resistor is

$$10\times\frac{1.25}{10}=1.25\text{V}$$

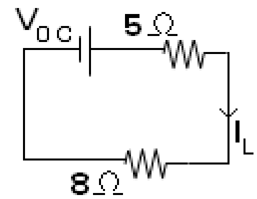


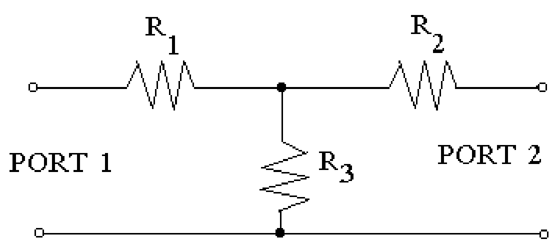
Fig 2.b.7

$\therefore V_{OC}$ = - 15 - 1.25 + 3.75 = - 12.5 V

$$I_L=\frac{V_{OC}}{R_L+R_{TH}}=\frac{12.5}{8+5}=\frac{12.5}{13}=0.96\,\text{A}$$

Power loss in 8Ω resistor = $(0.96)^2$ x 8 = 7.37 Watts

Q.4. Design an asymmetrical T-network shown below having $Z_{oc_1}=1000\,\Omega$, $Z_{oc_2}=1200\,\Omega$ and $Z_{sc_1}=700\,\Omega$.



(6)

Ans:

Given Z_{OC1} = 1000Ω, Z_{OC2} = 1200 Ω and Z_{SC1} = 700 Ω.

From the Fig 3.b $Z_{OC1} = R_1 + R_3 = 1000\ \Omega$,

$R_1 = 1000 - R_3$

$Z_{OC2} = R_2 + R_3 = 1200\ \Omega$,

$R_2 = 1200 - R_3$

$$Z_{SC1}=R_1+\frac{R_2R_3}{R_2+R_3}=700\Omega$$

$$\Rightarrow Z_{SC1}=(1000-R_3)+\frac{(1200-R_3)R_3}{1200}=700\Omega$$

$$\therefore 300=R_3-\frac{(1200-R_3)R_3}{1200}=\frac{R_3^{\,2}}{1200}$$

$$\Rightarrow R_3^2=360000$$

$$\therefore R_3=600\Omega$$

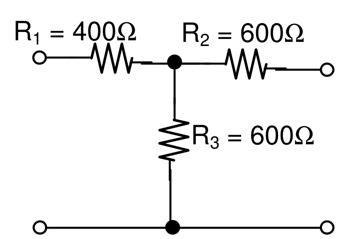


Fig 3.b

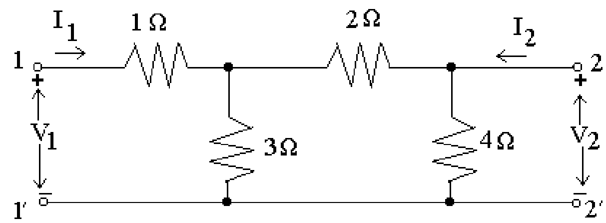
we know that $R_1 + R_3 = 1000\ \Omega$

$\therefore R_1 = 400\Omega$

we know that $R_2 + R_3 = 1200\ \Omega$

$\therefore\ R_2 = 600\ \Omega$

Q.5. Calculate the transmission parameters of the network shown below. Also verify the reciprocity & symmetricity of the network. **(8)**



Ans:

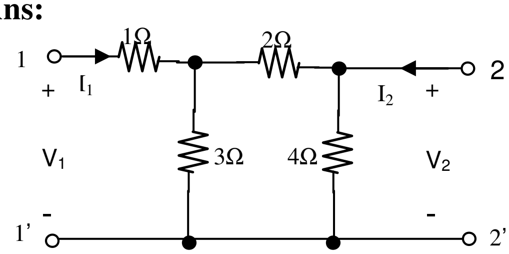


Fig 4.b.1

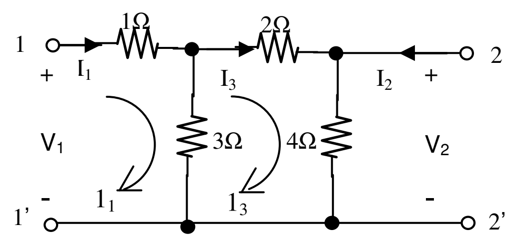


Fig 4.b.2

On open circuiting the terminals 2-2' as in Fig 4.b.2
Applying Kirchoff's voltage law (KVL) for the first loop
$V_1 = I_1 + 3(I_1 - I_3) = 4I_1 - 3I_3$ --- (1)
Applying KVL for the second loop

$$0 = 9\,I_3 - 3\,I_1$$

$$\therefore I_1 = 3\,I_3$$

$$7I_3 = I_1$$

$$I_3 = \frac{I_1}{3} \qquad \text{---(2)}$$

From (1) and (2)

$$V_1 = 4I_1 - \frac{3}{3}I_1 = 3I_1 \qquad \cdots(3)$$

$$V_2 = 4I_3 = \frac{4}{3}I_1 \qquad \cdots(4)$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{(4/3)I_1} = \frac{3}{4}\ \text{mhos}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{3I_1}{\frac{4}{3}I_1} = \frac{9}{4}$$

On short circuiting the terminals 2-2' as in Fig 4.b.3

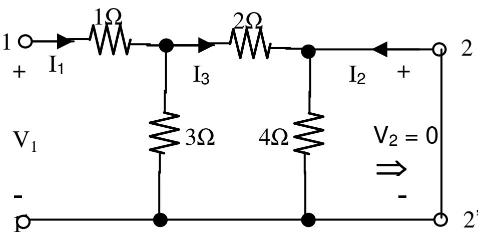


Fig 4.b.3

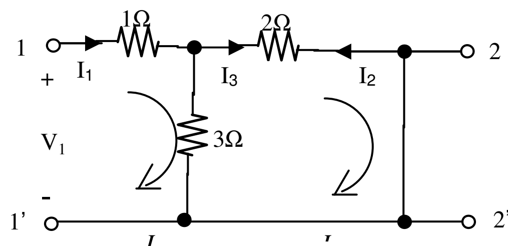


Fig 4.b.4

Applying Kirchoff's voltage law (KVL) for the first loop of Fig 4.b.4

$V_1 = I_1 + 3(I_1 - I_3) = 4I_1 - 3I_2$ --- (5)

$I_2 = -I_3$

Applying KVL for the second loop

$0 = 3(I_3 - I_1) + 2I_3$

$5I_3 = 3I_1$ and also $I_2 = -I_3$. Hence are get

$$I_1 = \frac{5}{3} I_3 \quad and \quad I_1 = -\frac{5}{3} I_2 \qquad \ldots(6)$$

From (5) and (6)

$$V_1 = -4 \times \frac{5}{3} I_2 - 3I_2 = -\frac{29}{3} I_2 \qquad \cdots(7)$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{29}{3} \text{ ohms}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{5}{3} \qquad A = \frac{9}{4}, \quad B = \frac{29}{3} \text{ ohms}, \; C = \frac{3}{4} \text{ mhos}, \quad D = \frac{5}{3}$$

$\Rightarrow A \neq D$

$$AD - BC = \frac{9}{4} \times \frac{5}{3} - \frac{29}{3} \times \frac{3}{4} = \frac{15}{4} - \frac{29}{4} = -\frac{7}{2} = 4 \neq 1$$

$\therefore AD - BC \neq 1$

$\therefore$ The circuit is neither reciprocal nor symmetrical.

Q.6. In a symmetrical T-network, if the ratio of input and output power is 6.76. Calculate the attenuation in Neper & dB. Also design this attenuator operating between source and load resistances of 1000 Ω. **(8)**

Ans:

Let P_{in} be the input power, P_{out} be the output power and N be the attenuation in Nepers.

$$\frac{P_{in}}{P_{out}} = 6.76 \qquad (given)$$

$$N = \sqrt{\frac{P_{in}}{P_{out}}} = \sqrt{6.76} = 2.6$$

Attenuation D = 20 $\log_{10}$(N)

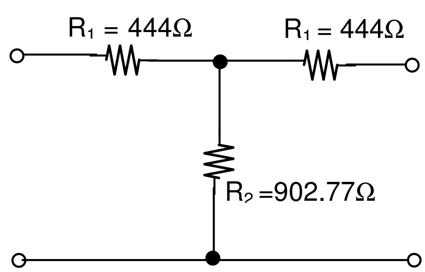


Fig 5.b

$= 20 \log_{10}(2.6)$
$= 8.299\text{dB}$

Load Resistance, $R_O = 1000\ \Omega$ (given)

$$\text{Series arm resistance } (R_1) = R_O \frac{(N-1)}{(N+1)} = 1000 \frac{(2.6-1)}{(2.6+1)} = 1000 \times 0.44 = 444\Omega$$

$$\text{Shunt arm resistance } (R_2) = R_O \frac{2N}{(N^2-1)} = 1000\left(\frac{2 \times 2.6}{(2.6)^2-1}\right) = 1000\left(\frac{5.2}{6.76-1}\right)$$

$$= 1000 \times 0.90277 = 902.77\ \Omega$$

Q.7. Determine the Laplace transform of the function $f(t) = \left(1 - e^{-\alpha t}\right)\sin \alpha t$, where α is a constant. **(6)**

Ans:

$f(t) = (1 - e^{-\alpha t}) \sin\alpha t$

$(f(t))= L((1-e^{-\alpha t}) \sin\alpha t)$

i.e. $F(s) = L(\sin\alpha t) - L(e^{-\alpha t} \sin\alpha t)$

$F(s) = F_1(s) - F_2(s)$

$F_1(s) = L(\sin\alpha t)$

$$F_1(s) = \frac{1}{2j}\int_0^\infty (e^{j\alpha t} - e^{-j\alpha t})e^{-st}\,dt = \frac{1}{2j}\int_0^\infty (e^{-(s-j\alpha)t} - e^{-(s+j\alpha)t})dt$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\alpha} - \frac{1}{s+j\alpha}\right]_0^\infty = \frac{s}{s^2+\alpha^2}$$

$F_2(s) = L(e \sin t)$

$$F_2(s) = \int_0^\infty e^{-\alpha t} \sin \alpha t e^{-st}\,dt = \frac{1}{2j}\int_0^\infty \sin \alpha t e^{-(s+\alpha)t}\,dt$$

$$= \left[\frac{-(s+\alpha)\sin \alpha t e^{-(s+\alpha)t} + \alpha \cos \alpha t e^{-(s+\alpha)t}}{(s+\alpha)^2+\alpha^2}\right]_0^\infty = \frac{\alpha}{(s+\alpha)^2+\alpha^2}$$

$$F(s) = F_1(s) - F_2(s) = \frac{s}{s^2+\alpha^2} - \frac{\alpha}{(s+\alpha)^2+\alpha^2}$$

Q.8. A low-loss coaxial cable of characteristic impedance of 100 Ω is terminated in a resistive load of $150\,\Omega$. The peak voltage across the load is found to be 30 volts. Calculate,

(i) The reflection coefficient of the load,
(ii) The amplitude of the forward and reflected voltage waves and current waves.
(iii) and V.S.W.R. **(8)**

Ans:

Given $Z_O = 100\Omega$ and $Z_R = 150\Omega$

i) $$K = \frac{Z_O - Z_R}{Z_O + Z_R} = \frac{100-150}{100+150} = -\frac{50}{250} = -0.2$$

ii) Let the amplitude of the forward voltage wave and the reflected voltage wave be V_R and V_i respectively.

$$K = \frac{V_R}{V_i} \qquad \Rightarrow -0.2 = \frac{V_R}{V_i}$$

$\Rightarrow V_R = -0.2\ V_i$
$V_R + V_i = 30$ (given)
$V_i - 0.2\ V_i = 30$

$$V_i = \frac{30}{0.8} = 37.5 \text{ Volts}$$

$$V_R = -0.2\ V_i = -0.2 \times 37.5 = -7.5 \text{ Volts}$$

$$\therefore \left|V_i\right| = 37.5 \text{ Volts}, \left|V_R\right| = 7.5 \text{ Volts}$$

Let I_R and I_i be the reflected and forward current respectively

$$\text{The peak value of the current at the terminated end} = \frac{\text{Total voltage}}{\text{Load resistance}} = \frac{30}{150} = 0.2 \text{ Amp}$$

$$-K = \frac{I_R}{I_i}$$

$\Rightarrow I_R = 0.2\ I_i$

But $I_R + I_i = 0.2$

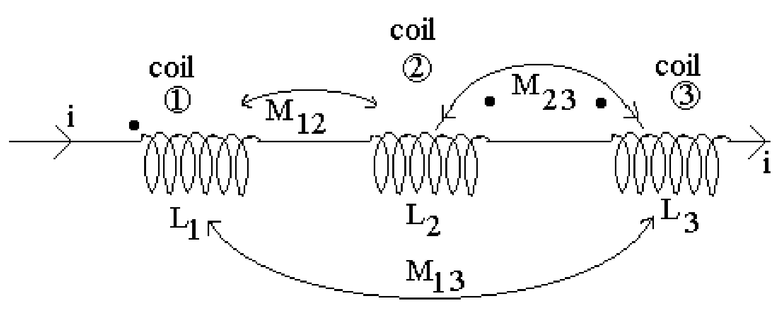
$\Rightarrow \quad 0.2\ I_i + I_i = 0.2$

$$\Rightarrow I_i = \frac{0.2}{1.2} = 0.167 \text{ Amp}$$

$$I_R = 0.2\ I_i = 0.2 \times 0.167 = 0.0334 \text{ Amp}$$

iii) $$\text{VSWR} = \frac{1+K}{1-K} = \frac{1-0.2}{1+0.2} = -\frac{0.8}{1.2} = 0.66$$

Q.9. Three series connected coupled coils are shown below in Fig.



Calculate

(i) The total inductance of these coils.

(ii) The coefficient of coupling between coils ① and ②, coils ② & ③ and coils ③ and ①.

It is given that $L_1 = 1.0\text{H}, L_2 = 3\text{ H}, L_3 = 7\text{ H}$,

$M_{12} = 0.5H, M_{23} = 1H, M_{13} = 1H$ **(7)**

Ans:

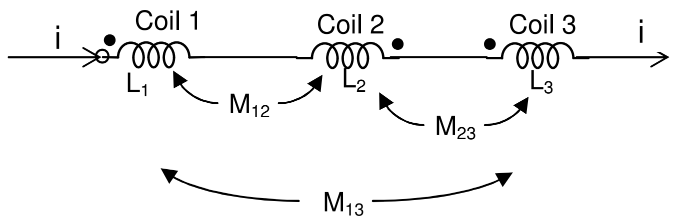


Fig 10.a

The inductance of the coils is given by
For coil 1 = $L_1 - M_{12} + M_{13} = 1.0 - 0.5 + 1.0 = 1.5$ H
For coil 2 = $L_2 - M_{23} - M_{12} = 3.0 - 1.0 - 0.5 = 1.5$ H
For coil 3 = $L_3 + M_{13} - M_{32} = 7.0 - 1.0 + 1.0 = 7$ H
The total inductance of the coils = 1.5 + 1.5 + 7 = 10 H
The coefficient of coupling between coils (1) and (2) is

$$K_{12} = \frac{M_{12}}{\sqrt{L_1 L_2}} = \frac{0.5}{\sqrt{3}} = \frac{0.5}{1.732} = 0.289$$

The coefficient of coupling between coils (2) and (3) is

$$K_{23} = \frac{M_{23}}{\sqrt{L_2 L_3}} = \frac{1}{\sqrt{21}} = \frac{1}{4.58} = 0.22$$

The coefficient of coupling between coils (3) and (1) is

$$K_{31} = \frac{M_{31}}{\sqrt{L_3 L_1}} = \frac{1}{\sqrt{7}} = \frac{1}{2.65} = 0.38$$

Q.10. It is required to match 300 Ω load to a 400 Ω transmission line, to reduce the VSWR along the line to 1.0. Design a quarter-wave transformer at 100 MHz. **(7)**

Ans:

When the line is made λ/4 long, the input impedance becomes,

$$Z_{in}(\lambda/4) = \frac{Z_O^{\ 2}}{Z_R} \quad \text{At any value of s}$$

But when standing wave ratio s is equal to 1, then $Z_O = Z_R$

$$\therefore Z_S(\lambda/4) = \frac{Z_O^{\ 2}}{Z_O} = Z_O$$

$$\therefore Z_S(\lambda/4) = 400\,\Omega = Z_{in}$$

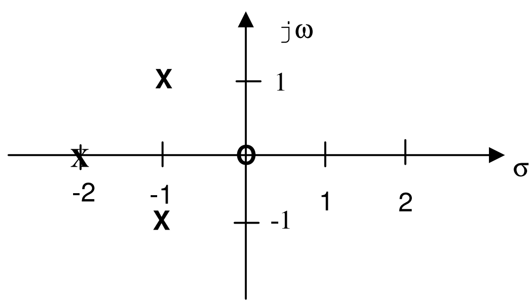
Hence, the characteristic impedance of a quarter-wave transformer should be 400Ω.

Q.11. A network function is given below

$$P(s)=\frac{2s}{(s+2)\left(s^2+2s+2\right)}$$

Obtain the pole-zero diagram (use graph paper). **(6)**

Ans:
The Scale factor H = 2.
On factorization of the denominator
we get $(s + 2)(2s + s^2 + 2) = (s + 2)(s + 1 – j)(s + 1 + j)$
The poles are situated at s = - 2, s = (-1 + j), s = (-1 – j)
The zeroes are situated at s = 0.
The pole-zero diagram is shown below.



Q.12. For the network of Figure 1, replace the parallel combination of impedances with the compensation source. **(6)**

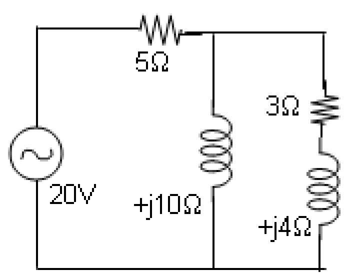


Fig.2.b.1

Z_{eq}

Ans:
The equivalent impedance of the parallel combination is given by

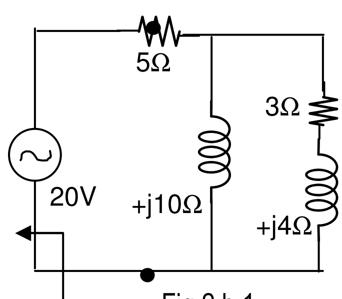


Fig.2.b.1

Z_{eq}

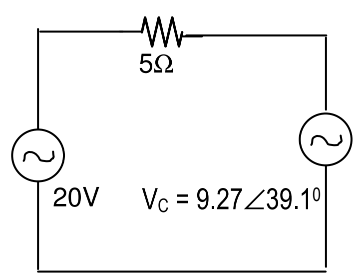


Fig.2.b.2

$$Z_{eq} = \frac{j10(3+j4)}{j10+3+j4} = \frac{-40+j30}{3+j14} = 1.46 + j3.17 = 3.5\angle 65.3° ohms$$

The total impedance of the circuit,

$$\sum Z = 5 + (1.46 + j3.17) = 6.46 + j3.17 = 7.18\angle 26.2° ohms$$

The current I,

$$I = \frac{V}{\sum Z} = \frac{20}{7.18\angle 26.2°} = 2.79\angle -26.2° \; amp$$

The compensation source,

$$V_C = I \times Z_{eq} = (2.79\angle -26.2°)\times(3.5\angle 65.3°) = 9.77\angle 39.1° \quad Volts$$

The replacement of the parallel combination of impedances with a compensation source V_C is shown in the Fig 2.b.2

Q.13. Find by convolution integral of the Laplace inverse of $\frac{1}{(s+2)(s+3)}$ taking $\frac{1}{(s+2)}$ as first function and $\frac{1}{(s+3)}$ as the second function. **(6)**

Ans:
It is given that

$$F_1(s) = \frac{1}{(s+1)} \quad and \quad F_2(s) = \frac{1}{(s+2)}$$

Hence

$$F_1(t) = e^{-t} \quad and \quad F_2(t) = e^{-2t}$$

$$L^{-1}\left[\frac{1}{(s+1)(s+2)}\right] = L^{-1}[F_1(s)\cdot F_2(s)] = f_1(t) * f_2(t)$$

$$= \int_0^t e^{-(t-\tau)} e^{-2\tau} d\tau = e^{-t}\int_0^t e^{-\tau} d\tau = e^{-t}.\left[-e^{-t}\right]$$

$$= e^{-t}[-e^{-\tau} + 1] = -e^{-2t} + e^{-t} = e^{t} - e^{-2t}$$

Q.14. Find the sinusoidal steady state solution (i_{ss}) for a series RL circuit. **(8)**

Ans:
The driving voltage is given by

$$v(t) = V\cos\omega t = \frac{V}{2}[e^{j\omega t} + e^{-j\omega t}] \text{ --- Eq - 1}$$

Considering voltage source $Ve^{j\omega t}/2$ and applying Kirchoff's voltage law

$$L\frac{di}{dt} + Ri = V\frac{e^{j\omega t}}{2} \qquad \text{--- Eq – 2}$$

The steady state current is given by $i_{ss1} = A\ e^{j\omega t}$ where A is the undetermined coefficient.

From Eq - 1 and Eq - 2

$$j\omega LA + RA = \frac{V}{2}$$

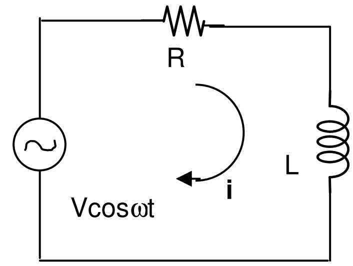


Fig.5.a.

$$A = \frac{V/2}{R + \mathrm{j}\omega L}$$

Considering voltage source $Ve^{-j\omega t} / 2$ and applying Kirchoff's voltage law

$$L\frac{di}{dt} + Ri = V\frac{\mathrm{e}^{-j\omega t}}{2} \qquad \text{--- Eq - 3}$$

The steady state current is given by $i_{ss2} = B\ e^{-j\omega t}$ where B is the undetermined coefficient.

From Eq -1 and Eq - 3

$$-j\omega LB + RB = \frac{V}{2} \qquad B = \frac{V/2}{R - \mathrm{j}\omega L}$$

On applying superposition principle, the total steady state current i_{ss} is the summation of the currents i_{ss1} and i_{ss2}.

$\therefore i_{ss} = i_{ss1} + i_{ss2}$

$= i_{ss1} + i_{ss2} = A\ e^{j\omega t} + B\ e^{-j\omega t}$

$$i_{ss} = \frac{V}{2}\left[\frac{e^{j\omega t}}{R + \mathrm{j}\omega L} + \frac{e^{-j\omega t}}{R - \mathrm{j}\omega L}\right] = \frac{V}{R^2 + \omega^2 L^2}\left[R\cos\omega t + \omega L\sin\omega t\right]$$

$$i_{ss} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

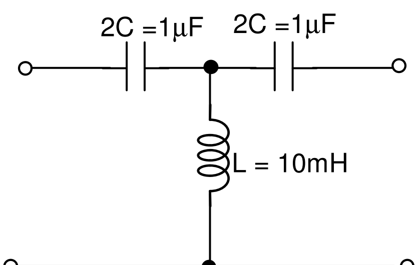
Q.15. Given two capacitors of 1 μ F each and coil L of 10mH, Compute the following:

(i) Cut-off frequency and characteristic impedance at infinity frequency for a HPF.

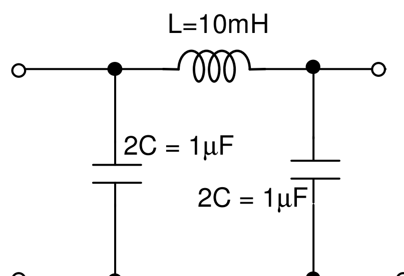
(ii) Cut-off frequency and characteristic impedance at zero frequency for an LPF.

Draw the constructed sections of filters from these elements **(6)**

Ans:



T-section HPF
Fig 5.b.i



Π-section LPF
Fig 5.b.ii

(i) For a HPF, $Z_2 = j\omega L$ and $Z_1 = 1/j\omega C$ (Fig 5.b.i)
At f = ∝, $Z_{OT} = R_O$

$$R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{10\times 2\times 10^{-3}}{1\times 10^{-6}}} = \sqrt{10\times 2\times 10^{3}} = 1.414\times 10^{2}\,\Omega$$

The cut off-frequency f_c

$$f_c = \frac{1}{4\pi\sqrt{LC}} = \frac{1}{4\pi\sqrt{5\times 10^{-3}\times 1\times 10^{-6}}} = 1.12\ KHz$$

(ii) For a LPF, $Z_1 = j\omega L$ and $Z_2 = 1/j\omega C$ (Fig 5.b.ii)
At f = 0, $Z_{OT} = R_O$

$$R_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{5\times 10^{-3}}{1\times 10^{-6}}} = 707\,\Omega$$

The cut off-frequency f_c

$$f_c = \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi\sqrt{5\times 10^{-3}\times 1\times 10^{-6}}} = 4.5\,KHz$$

Q.16. In a transmission line the VSWR is given as 2.5. The characteristic impedance is $50\,\Omega$ and the line is to transmit a power of 25 Watts. Compute the magnitudes of the maximum and minimum voltage and current. Also determine the magnitude of the receiving end voltage when load is $(100 - j80)\Omega$. **(8)**

Ans:
Given the standing wave ratio, S = 2.5
Characteristic impedance, $Z_o = 50\Omega$
Power, P = 25 Watts.
We know that

$$P = \frac{|V_{\max}|^2}{Z_{\max}} = \frac{|V_{\max}|^2}{SZ_o}$$

$$25 = \frac{|V_{\max}|^2}{2.5\times 50} \qquad \Rightarrow |V_{\max}| = 56\ \text{Volts}$$

$$|V_{\min}| = \frac{|V_{\max}|}{S} = \frac{56}{2.5} = 22.36\ \text{Volts}$$

$$|I_{\max}| = \frac{|V_{\max}|}{Z_o} = \frac{56}{50} = 1.12\ \text{Amp} \quad \text{as} \quad |V_{\max}| = |I_{\max}|\times Z_o$$

$$|I_{\min}| = \frac{|V_{\min}|}{Z_o} = \frac{22.36}{50} = 0.45\ \text{Amp} \quad \text{as} \quad |V_{\min}| = |I_{\min}|\times Z_o$$

$$P = |I_R|^2 R_R = 25$$

$$\left(\because Z_R = 100 - j80 \ and \ \therefore R_R = 100\Omega\right)$$

Since the powers at the sending end and receiving end are the same

$$\Rightarrow 25 = \left|I_R\right|^2 \times 100$$

$$\therefore \left|I_R\right| = \sqrt{\frac{25}{100}} = 0.5 \ \text{Amp}$$

$$\left|V_R\right| = \left|I_R\right|\left|Z_R\right| = 0.5\times\sqrt{100^2 + 80^2} = 0.5\times\sqrt{10000+6400}$$

$$= 0.5\times\sqrt{16400} = 0.5\times 128.06 = 64.03 \ \text{Volts}$$

Q.17. Compute the values of resistance, inductance and capacitance of the series and shunt elements of a 'T' network of 10 Km line having a characteristic impedance of $280\angle-30^\circ$ and propagation constant of $0.08\angle 40^\circ$ per loop Km at a frequency of $\frac{5000}{2\pi}$ Hz. Draw the 'T' network from the calculated values. **(14)**

Ans:

Given l= 10km, $Z_o = 280\angle\text{-}30^0$, $\gamma = 0.08\angle 40^0$, f = 2500/π,ω, 2πf = 5000rad/sec

$$\gamma l = 10\times 0.08\angle 40^\circ = 0.8\angle 40^\circ = 0.8(\cos 40 + j\sin 40)$$
$$= 0.613 + j0.514$$

$$\therefore e^{\gamma\ell} = e^{(0.613+j0.514)} = e^{(0.613)}\angle 29.47^\circ = 1.84\angle 29.47^\circ = 1.6 + j0.91$$
$$\therefore e^{-\gamma\ell} = e^{-(0.613+j0.514)} = e^{-(0.613)}\angle -29.47^\circ = 0.54\angle -29.47^\circ = 0.47 - j0.27$$

$$\sinh\gamma\ell = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1.6 + j0.91 - (0.47 - j0.27)}{2} = \frac{1.14 + j1.18}{2}$$
$$= 0.57 + j0.59 = 0.82\angle 46^\circ$$

$$\tanh\frac{\gamma\ell}{2} = \frac{e^{\gamma l/2} - e^{-\gamma l/2}}{e^{\gamma l/2} + e^{-\gamma l/2}} = \frac{e^{\gamma l} - 1}{e^{\gamma l} + 1} = \frac{1.6 + j0.91 - 1}{1.6 + j0.91 + 1} = \frac{0.6 + j0.91}{2.6 + j0.91}$$
$$= 0.31 + j0.24 = 0.39\angle 37.87^\circ$$

$$Z_2 = \frac{Z_o}{\sinh\gamma\ell} = \frac{280\angle -30^\circ}{0.82\angle 46^\circ} = 341.5\angle -76^\circ = 82.6 - j331.3$$

$$\Rightarrow R_2 = 82.6\Omega,$$

$$\omega C_2 = 331.3 \Rightarrow C = \frac{331.3}{5000} = 66.26\times 10^{-3}\ Farads$$

$$\frac{Z_1}{2} = Z_o\tanh\frac{\gamma\ell}{2} = \left(280\angle -30^\circ\right)\times\left(0.39\angle 37.87^\circ\right) = 109.2\angle 7.87^\circ$$
$$= 108.1 + j14.95$$

$$\Rightarrow R_1 = 108.1\Omega,$$

$$\omega L_1 = 14.95 \Rightarrow L_1 = \frac{14.95}{5000} = 2.99mH$$

The T – network for the calculated values is shown in Fig. 9

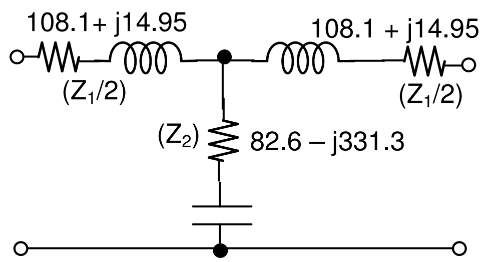


Fig 9

Q.18. Design an unbalanced π - attenuator with loss of 20 dBs to operate between 200 ohms and 500 ohms. Draw the attenuator. **(8)**

Ans:
It is given that,

$R_{i1} = 200$, therefore, $G_{i1} = \frac{1}{200} = 5mS$

$R_{i2} = 500, therefore, G_{i2} = \frac{1}{500} = 2\ mS$

D = 20 dB.

Converting decibels into nepers, we have

$A_i = 20 \times 0.115 = 2.3\ nepers$ $\left(\because A_i = 10\log_{10}\left(\frac{P_1}{P_2}\right) dB = \log_e\left(\frac{P_1}{P_2}\right) nepers\right)$

We know that

$$G_3 = \frac{\sqrt{G_1 G_2}}{\sinh A_i} = \sqrt{\frac{1}{200} \times \frac{1}{500}} \times \frac{1}{\sinh 2.3} = \frac{10^{-2}}{\sqrt{10}} \times \frac{1}{4.94} mho = \frac{10^{-2}}{3.1623 \times 4.94} mho$$

$\therefore R_3 = 3.1623 \times 4.94 \times 10^2 = 1563\Omega$ Similarly

$$G_1 = \frac{G_{i1}}{\tanh A_i} - G_3 = \frac{1}{200 \times \tanh 2.3} - \frac{10^{-2}}{15.63} = 10^{-2}\left[\frac{1}{2 \times 0.98} - \frac{1}{15.63}\right]$$

$$= 10^{-2}\left[\frac{1}{1.96} - \frac{1}{15.63}\right] = 10^{-2} \times \frac{13.67}{1.96 \times 15.63} \quad mhos$$

$$\therefore R_1 = \frac{1.96 \times 15.63}{13.67} \times 10^2\,\Omega = 2.242 \times 10^2\,\Omega = 224.2\Omega$$

Likewise

$$G_2 = \frac{G_{i2}}{\tanh A_i} - G_3 = \frac{1}{500 \times \tanh 2.3} - \frac{10^{-2}}{15.63} = 10^{-2}\left[\frac{1}{5 \times 0.98} - \frac{1}{15.63}\right]$$

$$= 10^{-2}\left[\frac{1}{4.90} - \frac{1}{15.63}\right] = 10^{-2}\left[\frac{1}{4.90 \times 15.63}\right]$$

$$\therefore R_2 = \frac{4.9 \times 15.63}{10.73} \times 10^2\,\Omega = 7.141 \times 10^2\,\Omega = 714.1\Omega$$

And the desired π attenuator is as shown in Fig 11.b

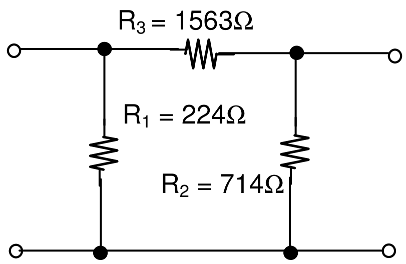


Fig 11.b

Q.19. The current in a conductor varies according to the equation $i = \left(3e^{-t}\text{amp}\right) \times u(t)$
Find the total charge in coulomb that passes through the conductor. **(7)**

Ans:
Given i = $3e^{-t} \times u(t)$ Amp.
We know that

$$\frac{dq}{dt} = i = 3e^{-t} \times u(t)$$

The charge q that passes through the conductor is 3 Coulombs.

$$\Rightarrow q = \int_0^\infty i.dt = \int_0^\infty 3e^{-t}dt$$

$$= -3e^{-t}\Big|_0^\infty = [0 - 3(-1)] = 3\,\text{Coloumbs}$$

Q.20. A current I = 10t A flows in a condenser C of value 10 μF. Calculate the voltage, charge and energy stored in the capacitor at time t= 1 sec. **(7)**

Ans:
Given I = 10t Amp.
C = 10μF,

$$\frac{dQ}{dt} = I = 10t$$, Where Q is the charge across the condenser

$$Q = \int_0^\infty I.dt = \int_0^\infty 10t\ dt$$

The voltage across the condenser is given by

$$V = \frac{1}{C}Q = \frac{1}{C}\int I.dt = \int 10t.\,dt$$

$$= \frac{1}{10\times10^{-6}}\times10\frac{t^2}{2} = 5t^2\times10^5 \text{ Volts}$$

When t = 1,
The voltage across the condenser is
V = 5 × 10^5 Volts
The charge across the condenser is given by

$$Q = \int I.\,dt = \int 10t.\,dt = 10\frac{t^2}{2} = 5t^2 \text{ Coloumbs}$$

At t = 1, the charge is given by
Q = 5 x 1 = 5 Coulombs
The energy stored in the capacitor is given by

$$E = \int P.\,dt = \int V.\,I\,.dt \text{ Joules}$$

$$= \int \frac{5t^2}{C}.\,I.\,dt = \int \frac{5t^2}{C}.10t.\,dt = \int \frac{50t^3}{C}.\,dt = \frac{50}{C}\times\frac{t^4}{4}$$

$$= \frac{50\times t^4}{10^{-5}\times4} = 125\times10^4\times t^4 \text{ Joules}$$

At t = 1, the energy stored in the capacitor is given by
E = 125 x 10^4 Joules.

Q.21. Define Laplace transform of a time function x(t) u(t). Determine Laplace transforms for

(i) δ(t) (the impulse function)
(ii) u(t) (the unit step function)
(iii) $t^n e^{at}$, n +ve integer **(7)**

Ans:
(i) The Laplace transform of impulse function

$$F(s) = L(f(t)) = \lim_{\alpha\to\infty}\left[L(g'(t))\right]$$

$$= \lim_{\alpha\to\infty}\left[L(\alpha\, e^{-\alpha t})\right]$$

$$= \lim_{\alpha\to\infty}\left[\frac{\alpha}{s+\alpha}\right] = 1$$

∴F(s) = 1

(ii) The Laplace transform of unit step function, u(t) is given by

$$F(s) = L(u(t)) = \int_0^\infty e^{-st}dt = \left[\frac{-1}{s}e^{-st}\right]_0^\infty = \frac{1}{s}$$

(iii) The Laplace transform of $t^n e^{at}$, n + integer is given by

$$F(s) = L(t^n) = \int_0^\infty t^n . e^{-st} dt = -\left[\frac{t^n}{s} e^{-st}\right]_0^\infty + \int_0^\infty \frac{1}{s} nt^{n-1} e^{-st} dt$$

$$= \frac{n}{s} \int_0^\infty t^{n-1} e^{-st} dt = \frac{n}{s} L(t^{n-1})$$

similarly $\quad L(t^{n-1}) = \frac{(n-1)}{s} L(t^{n-2})$

$$\therefore L(t^n) = \frac{n}{s} . \frac{n-1}{s} . \frac{n-2}{s} \cdots \frac{2}{s} . \frac{1}{s} L(t^{n-n})$$

$$= \frac{\underline{|n}}{s^n} \times \frac{1}{s} = \frac{\underline{|n}}{s^{n+1}}$$

According to the theorems & replacement of parameters s by (s – b) where b is a constant.

$$L\left[e^{at} . t^n\right] = \frac{\underline{|n}}{(s - a)^{n+1}}$$

Q.22. Find the Inverse Laplace transform for

(i) $\frac{2s + 3}{s^2 + 3s}$

(ii) $\frac{3s^2 + 4}{s(s^2 + 4)}$

(3+4)

Ans:

(i)

$$\frac{2s + 3}{s^2 + 3s} = \frac{2s + 3}{s(s + 3)}$$

According to the partial fraction method

$$\frac{2s + 3}{s(s + 3)} = \frac{K_1}{s} + \frac{K_2}{s + 3} = \frac{K_1(s + 3) + K_2 s}{s(s + 3)}$$

$$K_1(s + 3) + K_2 s = 2s + 3$$

$$When\ s = 0, \qquad 3K_1 = 3$$

$$\Rightarrow K_1 = 1$$

$$When\ s = -3, \qquad -3K_2 = -6 + 3 = -3$$

$$\Rightarrow K_2 = 1$$

$$\therefore \frac{2s+3}{s(s+3)} = \frac{1}{s} + \frac{1}{s+3}$$

$$L^{-1}\left[\frac{2s+3}{s(s+3)}\right] = L^{-1}\left[\frac{1}{s} + \frac{1}{s+3}\right]$$

$$= \left[1 + e^{-3t}\right]u(t)$$

(ii)

$$\frac{3s^2+4}{s(s^2+4)} = \frac{3s^2+4}{s(s+j2)(s-j2)}$$

According to the partial fraction method

$$\frac{3s^2+4}{s(s+j2)(s-j2)} = \frac{K_1}{s} + \frac{K_2}{s+j2} + \frac{K_3}{s-j2}$$

$$= \frac{K_1(s+j2)(s-j2) + K_2 s(s-j2) + K_3 s(s+j2)}{s(s+j2)(s-j2)}$$

$$\Rightarrow 3s^2 + 4 = K_1(s+j2)(s-j2) + K_2 s(s-j2) + K_3 s(s+j2)$$

$$When\ s = 0, \qquad 4K_1 = 4$$

$$\Rightarrow K_1 = 1$$

$$When\ s = +j2, \qquad K_3(j4)(j2) = -12 + 4 = -8$$

$$\Rightarrow K_3 = \frac{-8}{j4 \times j2} = \frac{8}{8} = 1$$

$$When\ s = -j2, \qquad K_2(-j4 \times -j2) = 3(-4) + 4 = -12 + 4 = -8$$

$$\Rightarrow K_2 \times (-8) = -8$$

$$\therefore K_2 = \frac{-8}{-8} = 1$$

$$\therefore \frac{3s^2+4}{s(s+j2)(s-j2)} = \frac{1}{s} + \frac{1}{s+j2} + \frac{1}{s-j2}$$

$$L^{-1}\left[\frac{3s^2+4}{s(s+j2)(s-j2)}\right] = L^{-1}\left[\frac{1}{s} + \frac{1}{s+j2} + \frac{1}{s-j2}\right]$$

$$= \left[1 + e^{-j2t} + e^{+j2t}\right]u(t)$$

$$= \left[1 + 2\cos 2t\right]u(t)$$

Q.23. For the circuit shown, at Fig.4 the switch K is closed at t=0. Initially the circuit is fully dead (zero current and no charge on C). Obtain complete particular solution for the current i(t). **(14)**

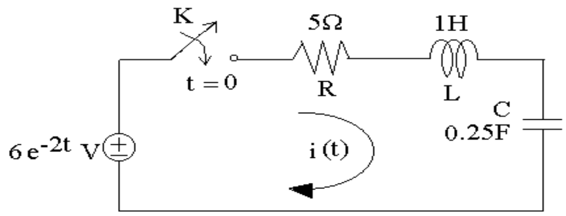


Fig. 4

Ans:
On Applying Kirchoff's voltage law,

$$\frac{di}{dt} + 5i + \frac{1}{0.25}\int_{-\infty}^{0} i\,dt + \frac{1}{0.25}\int_{0}^{t} i\,dt = 6e^{-2t} \cdots\cdots\cdots Eq.1$$

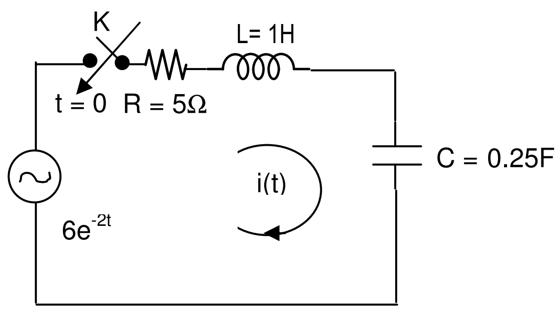


Fig.4

Applying Laplace transformation to Eq. 1

$$[s.I(s) - i(0+)] + 5I(s) + \frac{1}{0.25}\times\frac{q(0+)}{s} + \frac{1}{0.25}\times\frac{I(s)}{s} = \frac{6}{s+2}\cdots\cdots\cdots Eq.2$$

At time t = 0+, current i(0+) must be the same as at time t = 0 – due to the presence of the inductor L.
$\therefore$ i(0+) = 0
At t = 0+, charge q(0+) across capacitor must be the same as at time t = 0 –
$\therefore$ q(0+) = 0
Substituting the initial conditions in *Eq. 2*

$$I(s)[s + 5 + \frac{1}{0.25s}] = \frac{6}{(s+2)}$$

$$I(s)[s^2 + 5s + 4] = \frac{6s}{(s+2)}$$

$$I(s) = \frac{6s}{(s+2)(s^2+5s+4)} = \frac{6s}{(s+2)(s+1)(s+4)}$$

$$\text{Let } \frac{6s}{(s+1)(s+2)(s+4)} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+2)} + \frac{K_3}{(s+4)}$$

$$= \frac{K_1(s+2)(s+4) + K_2(s+1)(s+4) + K_3(s+1)(s+2)}{(s+1)(s+2)(s+4)}$$

$$K_1(s+2)(s+4) + K_2(s+1)(s+4) + K_3(s+1)(s+2) = 6s$$

$$When\ s=-1, \quad 3K_1 = -6 \quad \Rightarrow K_1 = -2$$

$$When\ s=-2, \quad -2K_2 = -12 \quad \Rightarrow K_2 = 6$$

$$When\ s=-4, \quad 6K_3 = -24 \quad \Rightarrow K_3 = -4$$

$$\therefore \frac{6s}{(s+1)(s+2)(s+4)} = \frac{-2}{(s+1)} + \frac{6}{(s+2)} + \frac{-4}{(s+4)}$$

On inverse laplace transformation

$$L^{-1}\left[\frac{6s}{(s+1)(s+2)(s+4)}\right] = L^{-1}\left[\frac{-2}{(s+1)} + \frac{6}{(s+2)} + \frac{-4}{(s+4)}\right]$$

$$= -2e^{-t} + 6e^{-2t} - 4e^{-4t}$$

The current i(t) is given by

$$i(t) = \left[-2e^{-t} + 6e^{-2t} - 4e^{-4t}\right]u(t)$$

Q.24. Derive necessary and sufficient condition for maximum power transfer from a voltage source, with source impedance $R_S + jX_S$, to a load $Z_L = R_L + jX_L$. What is the value of the power transferred in this case? **(7)**

Ans:

Given $Z_S = R_S + j X_S$ and $Z_L = R_L + j X_L$

The power P in the load is $I_L^2 R_L$, where I_L is the current flowing in the circuit, which is given by,

$$I = \frac{V}{Z_S + Z_L} = \frac{V}{R_S + jX_S + R_L + jX_L} = \frac{V}{(R_S + R_L) + j(X_S + X_L)}$$

$\therefore$ Power to the load is $P = I_L^2 R_L$

$$P = \frac{V^2}{(R_S + R_L)^2 + (X_S + X_L)^2} \times R_L \qquad (1)$$

for maximum power, we vary X_L such that

$$\frac{dP}{dX_L} = 0$$

$$\Rightarrow \frac{-2V^2 R_L (X_S + X_L)}{\left[(R_S + R_L)^2 + (X_S + X_L)^2\right]^2} = 0$$

$$\Rightarrow (X_S + X_L) = 0$$

$$\text{i.e. } X_S = -X_L$$

This implies the reactance of the load impedance is of the opposite sign to that of the source impedance. Under this condition $X_L + X_S = 0$
The maximum power is

$$P = \frac{V^2 R_L}{(R_S + R_L)^2}$$

For maximum power transfer, now let us vary R_L such that

$$\frac{dP}{dR_L} = 0$$

$$\Rightarrow \frac{V^2 (R_S + R_L)^2 - 2V^2 R_L (R_S + R_L)}{[R_S + R_L]^4} = 0$$

$$\Rightarrow V^2 (R_S + R_L) = 2V^2 R_L$$

$$\text{i.e.} R_S = R_L$$

The necessary and sufficient condition for maximum power transfer from a voltage source, with source impedance $Z_S = R_S + j\,X_S$ to a load $Z_L = R_L + j\,X_L$ is that the load impedance should be a complex conjugate of that of the source impedance i.e. $R_L = R_S$, $X_L = - X_S$
The value of the power transferred will be

$$P = \frac{V^2 R_L}{[R_S + R_L]^2} = \frac{V^2 R_L}{[2R_L]^2} = \frac{V^2 R_L}{4{R_L}^2} = \frac{V^2}{4R_L}$$

Q.25. By using Norton's theorem, find the current in the load resistor R_L for the circuit shown in Fig. 5. **(7)**

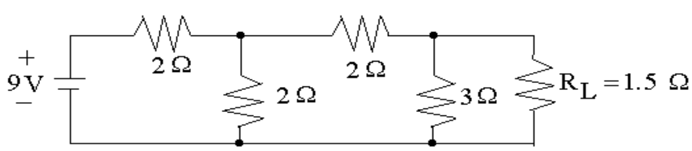


Fig. 5

Ans:
To find the short circuit current I_{sc}, first find the equivalent resistance from Fig 5.b.2.

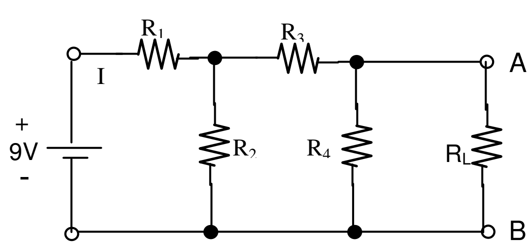


Fig 5.b.1

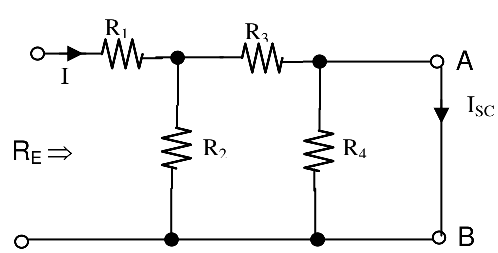


Fig 5.b.2

$$R_E = R_1 + \frac{R_2 R_3}{R_2 + R_3}, \qquad \text{Given } R_1 = 2\Omega, R_2 = 2\Omega, R_3 = 2\Omega, R_4 = 3\Omega, R_L = 1.5\Omega$$

$$= 2 + \frac{2\times 2}{4} = 2 + 1 = 3\Omega$$

$$\therefore I = \frac{V}{R_E} = \frac{9}{3} = 3 Amp.$$

$$\therefore \textit{Short Circuit Current } I_{sc} = I\left[\frac{R_2}{R_2 + R_3}\right] = 3\left[\frac{2}{2+2}\right] = \frac{6}{4} = 1.5 \text{ Amp.}$$

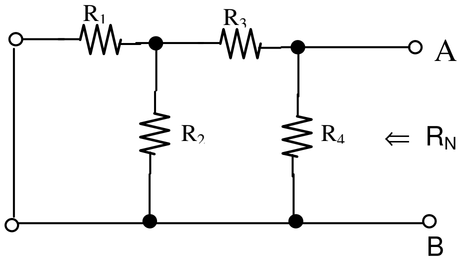


Fig 5.b.3

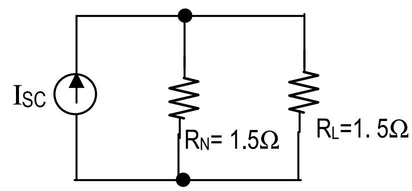


Fig.5.b.4.

To find the Norton's equivalent resistance, short-circuit the voltage source as in Fig 5.b.3.

$$R_N = \frac{R_4\left[R_3 + \frac{R_2 R_1}{R_1 + R_2}\right]}{R_4 + R_3 + \frac{R_2 R_1}{R_1 + R_2}} = \frac{3\times(2 + \frac{2\times 2}{2+2})}{3 + 2 + \frac{2\times 2}{2+2}} = \frac{3\times 3}{6} = \frac{9}{6} = 1.5\Omega$$

Therefore the equivalent circuit is shown in Fig. 5.b.4

$$I_L = I_{SC} \times \frac{R_N}{R_N + R_4} = \frac{1.5\times 1.5}{1.5 + 1.5} = 0.75 \text{Amp.}$$

Q.26. Determine the ABCD parameters for the π-network shown at Fig.6. Is this network bilateral or not? Explain. **(7)**

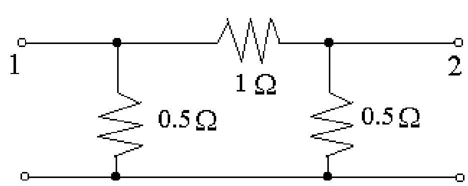


Fig. 6

Ans:

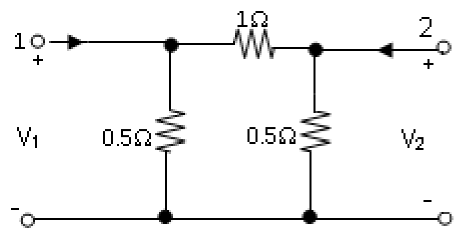


Fig 6.b.1

We know that
$V_1 = AV_2 - B\,I_2$
$I_1 = CV_2 - D\,I_2$

$$\therefore A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$\therefore C = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

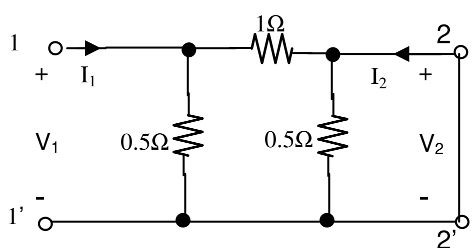


Fig 6.b.2

(To find B and D)

On short circuiting port 2 as in Fig 6.b.2, $V_2 = 0$

$$V_1 = -I_2 \times 1 = -I_2$$

$$\therefore B = \frac{-V_1}{I_2} = 1\Omega$$

$$I_1 = \frac{V_1}{0.5} + \frac{V_1}{1} = 3V_1 = -3I_2$$

$$\therefore D = \frac{-I_1}{I_2} = 3$$

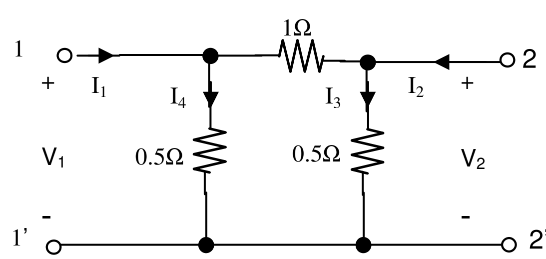


Fig 6.b.3

(To find A and C)

On open circuiting port 2 as in Fig 6.b.3, $I_2 = 0$

$$I_4 = \frac{V_1}{0.5} = 2V_1$$

$$I_3 = \frac{V_1}{1.5} = \frac{2V_1}{3}$$

$$I_1 = I_3 + I_4 = 2V_1(1 + 1\frac{1}{3}) = \frac{8V_1}{3}$$

$$V_2 = I_3 \times 0.5 = \frac{V_1}{3}$$

$$\therefore A = \frac{V_1}{V_2} = 3$$

$$\therefore C = \frac{I_1}{V_2} = \frac{\frac{8V_1}{3}}{\frac{V_1}{3}} = 8 \quad Mhos$$

An element is a bilateral element if the impedance does not change or the magnitude of the current remains the same even if the polarity of the applied EMF if changed. Since the network consists of only resistive elements, the given network is a bilateral network.

Q.27. Solve the differential equation given below and determine the steady state solutions.

(i) $\frac{\mathrm{di}}{\mathrm{dt}} + 3\,\mathrm{i} = 2\mathrm{Sin}3\mathrm{t}$

(ii) $\frac{\mathrm{di}}{\mathrm{dt}} + 2\mathrm{i} = \mathrm{Cos}\ \mathrm{t}$ **(7+7)**

Ans:

$$\frac{di}{dt} + 3i = 2\sin 3t = \frac{e^{j3t} - e^{-3jt}}{j}$$

Consider

$$\frac{di}{dt} + 3i = \frac{e^{j3t}}{j} \qquad (1)$$

Let the steady state current be given by

$$i_{ss1} = A.e^{j3t} \qquad (2)$$

From equation (1) and (2)

$$A.j3e^{j3t} + 3Ae^{j3t} = \frac{e^{j3t}}{j}$$

$$A(3+3j)=\frac{1}{j} \qquad (3)$$

$$A=\frac{1}{(3+3j)j}=\frac{1}{3j-3}$$

Consider

$$\frac{di}{dt}+3i=\frac{e^{-j3t}}{j} \qquad (4)$$

Let the steady state current be given by

$$i_{ss2}=Be^{-j3t} \qquad (5)$$

from equation (4) and (5)

$$B(-j3)e^{-j3t}+3Be^{-j3t}=\frac{e^{-j3t}}{j}$$

$$B(3-j3)=\frac{1}{j}$$

$$B=\frac{1}{j(3-j3)}=\frac{1}{3j+3} \qquad (6)$$

The total steady state current is given by

$$i_{ss}=i_{ss1}-i_{ss2}$$

$$=\frac{1}{3j-3}e^{j3t}-\frac{1}{3j+3}e^{-j3t}$$

$$=\frac{-(3j+3)e^{j3t}+(3j-3)e^{-j3t}}{18}$$

$$=\frac{-3j(e^{j3t}-e^{-j3t})-3(e^{j3t}+e^{-j3t})}{18}$$

$$=2\left[\frac{3\sin 3t-3\cos 3t}{18}\right]$$

$$=2\frac{\sqrt{9+9}}{18}\sin(3t-\tan^{-1}\frac{3}{3})$$

$$=2\frac{\sqrt{18}}{18}\sin(3t-\tan^{-1}(1))$$

$$=\frac{2}{\sqrt{18}}\sin(3t-45°)$$

$$=\frac{2}{\sqrt{18}}\sin(3t-\frac{\pi}{4}) \qquad (\because \textit{angle is in radians})$$

(ii)

$$\frac{di}{dt} + 2i = \cos t = \frac{e^{jt} + e^{-jt}}{2}$$

consider

$$\frac{di}{dt} + 2i = \frac{e^{jt}}{2} \qquad (1)$$

Let the steady state current be given by

$$i_{ss1} = A.e^{jt} \qquad (2)$$

Now equation (1) and (2)

$$A.je^{jt} + 2Ae^{jt} = \frac{e^{jt}}{2}$$

$$A(j+2) = \frac{1}{2}$$

$$A = \frac{1}{(2+j)2} \qquad (3)$$

Consider

$$\frac{di}{dt} + 2i = \frac{e^{-jt}}{2} \qquad (4)$$

Let the steady state current be given by

$$i_{ss2} = Be^{-jt} \qquad (5)$$

from equation (4) and (5)

$$B(-j)e^{-jt} + 2Be^{-jt} = e^{-jt}$$

$$B(2-j) = 1$$

$$B = \frac{1}{(2-j)2} \qquad (6)$$

The total steady state current is given by

$$
\begin{aligned}
i_{ss} &= i_{ss1} + i_{ss2} \\
&= \frac{1}{2}\left[\frac{1}{(2+j)}e^{jt} + \frac{1}{(2-j)}e^{-jt}\right] \\
&= \frac{1}{2}\left[\frac{(2-j)e^{jt} + (2+j)e^{-jt}}{5}\right] \\
&= \frac{1}{2}\left[\frac{2(e^{jt} + e^{-jt}) - j(e^{jt} - e^{-jt})}{5}\right] \\
&= \frac{1}{2}\left[\frac{2\times 2\cos t + 2\sin t}{5}\right] = \frac{2\cos t + \sin t}{5} \\
&= \frac{1}{5}\sqrt{5}\cos\left[t - \tan^{-1}(\frac{1}{2})\right] \\
&= \frac{1}{\sqrt{5}}\cos(t - 26.6°) \\
&= \frac{1}{\sqrt{5}}\cos(t - 0.46) \qquad (\because \textit{angle is in radians})
\end{aligned}
$$

Q.28. A transmission line is terminated by an impedance z_{load}. Measurements on the line show that the standing wave minima are 105 cm apart and the first minimum is 30 cm from the load end of the line. The VSWR is 2.3 and z_0 is 300 Ω. Find the value of z_R. **(7)**

Ans:
The standing wage minima points are the voltage minima points. The two consecutive E_{min} points are separated by 105Cms.

$$\frac{\lambda}{2} = 105 \text{ cms}$$
$$\Rightarrow \lambda = 2.1 \text{ m}$$

The first voltage minimum is given by

$$y_{min} = \frac{\phi + \pi}{2\beta} = \frac{\phi + \pi}{4\pi} \times \lambda = (\frac{\phi + \pi}{4\pi}) \times 2.1$$
$$\therefore \phi + \pi = \frac{y_{min} \times 4\pi}{2.1} = \frac{0.3 \times 4\pi}{2.1} = \frac{4\pi}{7} \quad \text{given } (y_{min} = 0.3 \text{ m})$$
$$\therefore \phi = \frac{4\pi}{7} - \pi = \frac{-3\pi}{7} = -77.14°$$

Given $Z_o = 300\Omega$ and s = 2.3
The magnitude of the reflection coefficient K is given by

$$|k| = \frac{s-1}{s+1} = \frac{2.3-1}{2.3+1} = \frac{1.3}{3.3} = 0.394$$

$$\therefore k = 0.394\angle -77.14°$$

$$= 0.0882 - j0.384$$

The reflection coefficient K in terms of Z_R and R_o is given by

$$k = \frac{Z_R - R_o}{Z_R + R_o}$$

$$\Rightarrow Z_R = \frac{R_o(1+k)}{(1-k)} = 300\left[\frac{1+0.0882 - j0.384}{1-0.0882 + j0.0384}\right]$$

$$= 300\left[\frac{1.0882 - j0.384}{0.9118 + j0.0384}\right] = 300\left[\frac{1.153\angle -19.9°}{0.989\angle 22.8°}\right]$$

$$= 350\angle -42.7° = 257 - j237.5\Omega$$

Q.29. State initial value theorem in the Laplace transform . What is the value of the function at t = 0, if its

$$\mathrm{F}(\mathrm{s}) = \frac{4(\mathrm{s}+25)}{\mathrm{s}(\mathrm{s}+10)}$$ **(4)**

Ans:

The initial value theorem states that

$$f(0) = \lim_{s\to\infty}[sF(s)]$$

where F(s) is the laplace transform of the given function and f(0) is the initial value of the time domain function f(t).

Given $F(s) = \dfrac{4(s+25)}{s(s+10)}$

$$F(s) = \frac{4\times\left[1+\dfrac{25}{s}\right]}{s\times\left[1+\dfrac{10}{s}\right]}$$

$$f(0) = \lim_{s\to\infty}[s\times F(s)]$$

$$\therefore f(0) = \lim_{s\to\infty}\left[s\frac{4\left[1+\dfrac{25}{s}\right]}{s\left[1+\dfrac{10}{s}\right]}\right] = \frac{4\times(1+0)}{(1+0)} = 4$$

$\therefore$The value of the given function at t = 0 is

f(0) = 4.

Q.30. For a series resonant circuit, R=5Ω, L=1H and C=0.25μf. Find the resonance frequency and band width. **(8)**

Ans:
The resonant frequency of a series resonant circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1\times 0.25\times 10^{-6}}} = \frac{10^3}{2\pi\times 0.5} = \frac{1000}{\pi} = 318.5Hz$$

The bandwidth of a series resonant circuit is given by,

B.W = $f_2 - f_1 = \frac{f_0}{Q}$, where f_o is the resonant frequency and Q is the quality factor of the circuit and f_1 and f_2 are the upper and lower half power frequencies.

The quality factor Q is given by, $Q \;\; factor = \frac{\omega_o L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{5}\sqrt{\frac{1}{0.25\times 10^{-6}}} = \frac{2\times 10^3}{5} = 400$$

$$\therefore B.W. = \frac{f_0}{Q} = \frac{318.5}{400} = 0.79625Hz$$

Q.31. Calculate the voltage across the inductor of 2 Henry and the charge in the inductor at time t = 1 sec for the variation of the current as shown in the Fig.1. **(4)**

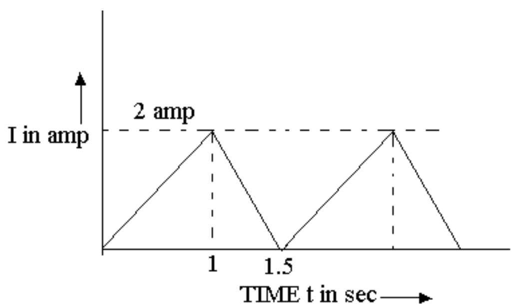


Fig.1

Ans:
From the diagram during t = 0 to t = 1, current i = 2 t amp and

$$\frac{di}{dt} = 2amp/\sec$$

The voltage across the inductor

$$= L\frac{di}{dt} = 2\times 2 = 4volts$$

$$q = \int_{o}^{s} idt = \int_{o}^{t} 2tdt = t^2$$

q = $(1)^2$ = 1 coulomb

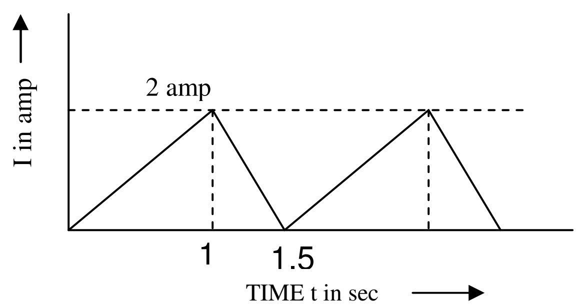


Fig 2.b

Q.32. A 60 Hz sinusoidal voltage V = 100 sin ωt is applied to a series RL circuit. Given R = 10Ω, and L = 0.01H, find the steady state current and its phase angle. **(8)**

Ans:
The driving voltage is given by
$v(t) = 2\sin\omega t = [e^{j\omega t} + e^{-j\omega t}] = e^{j\omega t} + e^{-j\omega t}$ --- eq - 2.c.1

Considering voltage source $e^{-j\omega t}$ and applying kirchoff's voltage law

$$\Rightarrow L\frac{di}{dt} + 3i = e^{j\omega t} \qquad \text{--- eq - 2.c.2}$$

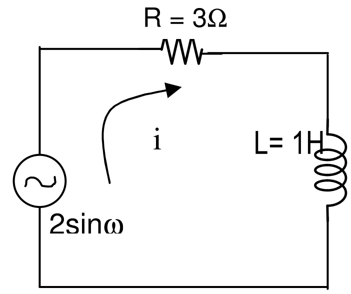


Fig.2.c.

The steady state current is given by
$i_{SS1} = B\, e^{-j\omega t}$

where A is the undetermined coefficient.
From eq – 2.c.1 and 2.c.2
$j\omega A + 3A = 1$

$$A = \frac{1}{3 + j\omega}$$

Considering voltage source B $e^{-j\omega t}$ and applying kirchoff's voltage law

$$\frac{di}{dt} + 3i = e^{-j\omega t} \qquad \text{--- eq - 2.c.3}$$

The steady state current is given by i_{ss2} = B $e^{-j\omega t}$ where B is the undetermined coefficient.
From eq – 2.c.1 and 2.c.3
$-j\omega B + 3B = 1$

$$B = \frac{1}{3 - j\omega}$$

On applying superposition principle, the total steady state current i_{ss} is the summation of the currents i_{ss1} and i_{ss2}.
$\therefore i_{ss} = i_{ss1} + i_{ss2}$
$= i_{ss1,}\ i_{ss2} = A\ e^{j\omega} + B\ e^{j\omega t}$

$$i_{ss} = \frac{e^{j\omega t}}{3 + j\omega} + \frac{e^{-j\omega t}}{3 - j\omega} = \frac{1}{9 + \omega^2}[3\cos\omega t + \omega\sin\omega t]$$

$$i_{ss} = \frac{1}{9 + \omega^2}\cos\left(\omega t - \tan^{-1}\frac{\omega}{3}\right)$$

Q.33. Find the Laplace transform of the functions:
(i) t^n u (t). (ii) cosh ωt u (t). **(4)**

Ans:
(i)

$$F(s) = L(t^n) = \int_0^\infty t^n . e^{-st} dt = \left[\frac{t^n}{s} e^{-st} \right]_0^\infty + \int_0^\infty \frac{1}{s} nt^{n-1} e^{-st} dt$$

$$= \frac{n}{s} \int_0^\infty t^{n-1} e^{-st} dt = \frac{n}{s} L(t^{n-1})$$

$$similllarly L(t^{n-1}) = \frac{(n-1)}{s} L(t^{n-2})$$

$$\therefore L(t^n) = \frac{n}{s} . \frac{n-1}{s} . \frac{n-1}{s} \cdots \frac{2}{s} . \frac{1}{s} L(t^{n-n})$$

$$= \frac{\lfloor n}{s^n} \times \frac{1}{s} = \frac{\lfloor n}{s^{n+1}}$$

(ii) coshωt

$$F(s) = L(\cosh(\omega t)) = \int_0^\infty \cosh(\omega t) . e^{-st} dt = + \int_0^\infty \left[\frac{e^{\omega t} + e^{-\omega t}}{2} \right] e^{-st} dt$$

$$= \frac{1}{2} \int_0^\infty e^{-(s-\omega)t} dt = \frac{1}{2} \int_0^\infty e^{-(s+\omega)t} dt$$

$$= \frac{1}{2} . \frac{1}{(s-\omega)} + \frac{1}{2} . \frac{1}{(s+\omega)}$$

$$= \frac{s}{s^2 - \omega^2}$$

Q.34. Using partial fraction method, obtain the inverse Laplace transform of

$$I(s) = \frac{10^4}{s(s+250)}$$

Ans:
Using partial fraction method, obtain the inverse laplace transform of

$$I(s) = \frac{10^4}{s(s+250)}$$

$$Let \frac{10^4}{s(s+250)} = \frac{P}{s} + \frac{Q}{(s+250)}$$

Multiplying both sides of eq.1 by s and s = 0,

$$P = \frac{10^4}{250} = 40$$

Multiplying both sides of eq.1 by (s + 250) and s = **-** 250,

$$Q = \frac{10^4}{-250} = -40$$

$$\therefore \mathcal{L}^{-1}\left[\frac{10^4}{s(s+250)}\right] = \mathcal{L}^{-1}\left[\frac{40}{s} - \frac{40}{(s+250)}\right]$$

$$= 40 - 40e^{-250t}$$

Q.35. A capacitor of 5μF which is charged initially to 10 V is connected to resistance of 10 KΩ and is allowed to discharge through the resistor by closing of a switch K at t = 0. Find the expression for the discharging current. **(8)**

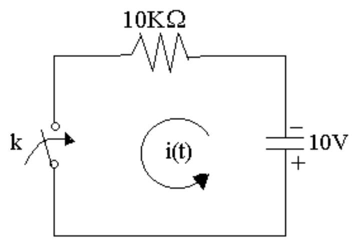


Fig.2

Ans:

When the switch k is closed, the capacitor, which is charged initially to 10V, starts discharging. Let the discharge current be i(t).

On applying Kirchoff's voltage law to the discharging loop as shown in Fig 3.c

$$0 = Ri(t) + \frac{1}{C}\int_0^\infty i(t)dt$$

On applying laplace transformation

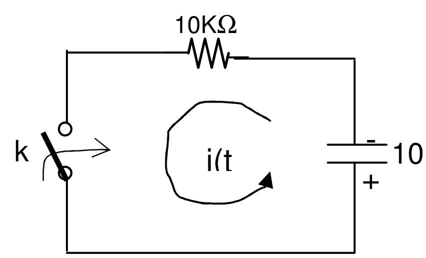


Fig 3.c

$$0 = RI(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{Q_0}{s}\right]$$

$$\Rightarrow -\frac{Q_0}{Cs} = I(s)\left[\frac{1}{Cs} + R\right]$$

$$\Rightarrow I(s) = \frac{-\frac{Q_0}{Cs}}{\left[\frac{1}{Cs} + R\right]}$$

$$\Rightarrow I(s) = \frac{-Q_0}{RC\left[s + \frac{1}{RC}\right]} = \frac{-10}{R\left[s + \frac{1}{RC}\right]} \left[\because \frac{Q_0}{C} = 10\ V\right]$$

$$i(t) = -\frac{10}{R}e^{-\frac{t}{RC}}$$

$$-\frac{10}{10 \times 10^3}e^{-\frac{t}{10\times10^3 \times 5\times10^{-6}}} = -10^{-3}e^{-20t}\ \text{amp}$$

Q.36. State the final value theorem and find the final value of the function where the laplace transform is $I(s) = \frac{s+6}{s(s+3)}$. **(2+3)**

Ans:
Final value theorem states that if function f(t) and its derivative are laplace transformable, then the final value f(∞) of the function f(t) is

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

Applying final value theorem, we get

$$i(\infty) = \lim_{s \to \infty} sF(s) = \lim_{s \to 0} s.\frac{s+6}{s(s+3)} = \lim_{s \to 0}\left[\frac{s+6}{s+3}\right] = \frac{6}{3} = 2$$

Q.37. A symmetric T section has an impedance of j100Ω in each series arm and an impedance of j400Ω in each shunt arm. Find the characteristic impedance and propagation constant of the network. **(5)**

Ans:

$$Z_{oc} = j100 - j400 = -j300\ \Omega$$

$$Z_{sc} = j100 + \frac{j100\,(-j400)}{j100 - j400}$$

$$Z_{sc} = j100 + \frac{(-j^2\,40000)}{-j300}$$

$$Z_{sc} = j100 + j133.33$$

$$Z_{sc} = j233.33\,\Omega$$

$$Z_o = \sqrt{Z_{sc} \times Z_{oc}}$$

$$Z_o = \sqrt{-j300 \times j233.33}$$

$$Z_o = 264.57\,\Omega$$

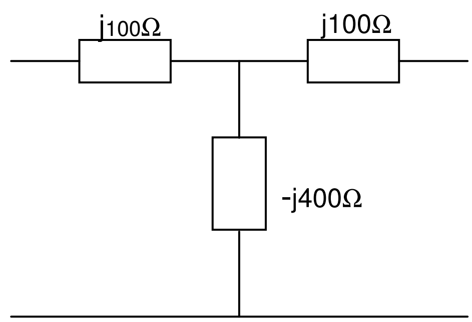


Fig.4.b

Q.38. A symmetrical T section has the following O.C. and S.C. impedances: Zo/c = 800 ohms Zsc = 600 ohms Determine T section parameters to represent the two port network. **(8)**

Ans:
Given Z_{oc} = 800 Ω and Z_{sc} = 600Ω

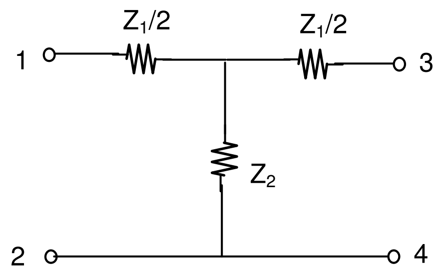


Fig 7.a

The network elements of a T section are given by

$$\boldsymbol{Z_1 = 2[Z_{OC} - \sqrt{Z_{OC}[Z_{OC} - Z_{SC}]}}$$

$$\boldsymbol{= 2[800 - \sqrt{800(800 - 600)}]}$$

$$= 2[800 - 400] = 2 \times 400$$

$$\Rightarrow \frac{Z_1}{2} = 400\ \Omega$$

$$Z_2 = \sqrt{Z_{OC}(Z_{OC} - Z_{SC})}$$

$$= \sqrt{800(800 - 600)}$$

$$\Rightarrow Z_2 = 400\ \Omega$$

Q.39. State Thevenin's theorem. Using Thevenin's, find the current through 5Ω resistor as shown in the Fig. 3 below. **(8)**

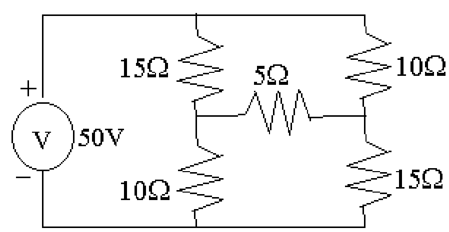


Fig.3

Ans:

Thevenin's theorem: It states that any two terminal networks consisting of linear impedances and generators may be replaced by an e.m.f. in series with an impedance. The e.m.f is the open circuit voltage at the terminals and the impedance is the impedance viewed at the terminals when all the generators in the network have been replaced by impedances equal to their internal impedance.

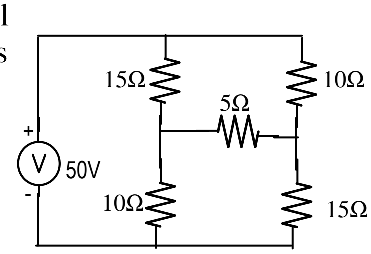


Fig.5.a.1

On removing 5Ω resistor as in fig.5.a.2

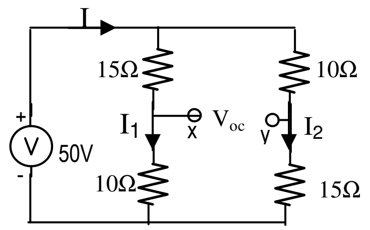


Fig.5.a.2

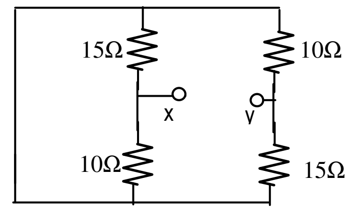


Fig.5.a.3

$$I = \frac{50(25+25)}{25\times 25} = \frac{50\times 50}{25\times 25} = 4A$$

Since both branches have equal resistance

$\therefore I_1 = I_2 = I / 2 = 2A$

$V_y = 50 - 2 \times 10 = 30$ V

$V_x = 50 - 2 \times 15 = 20$ V

Hence $V_{oc} = V_x - V_y = 10V$

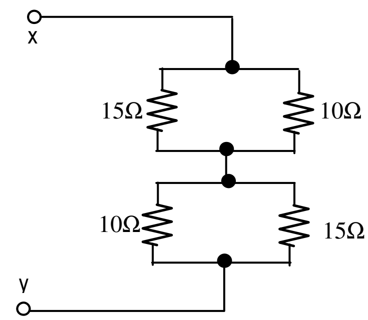


Fig.5.a.

On removing the voltage source and looking from the point x – y as in fig.5.a.3 and fig.5.a.4, the resistance is

$R_{th} = (10 \parallel 15) + (15 \parallel 10)$

$= 6 + 6$

$= 12\Omega$

The equivalent circuit is shown in Fig.5.a.5

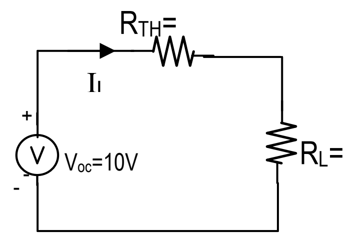


Fig.5.a.5.

$$I_L = \frac{10}{12+5} = 0.59A$$

The current through 5Ω resistor is 0.59 A.

Q.40. State the superposition theorem. Using this theorem find the voltage across the 16Ω resistor. **(8)**

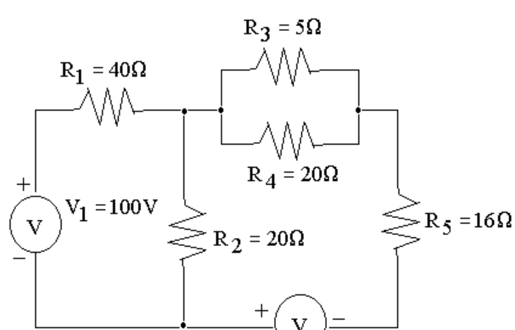


Fig.4

Ans:

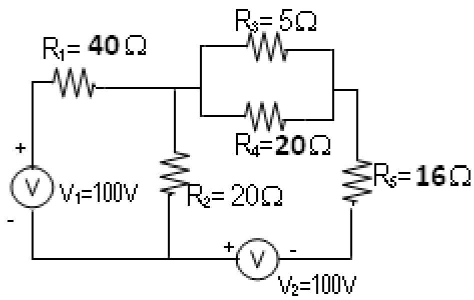


Fig 5.b.1

Superposition theorem: It states that 'if a network of linear impedance contains more than one generator, the current which flows at any point is the vector sum of all currents which would flow at that point if each generator was considered separately and all other generators are replaced at that time by impedance equal to their internal impedances"

Considering only voltage source V_1 and removing voltage source V_2 as in fig 5.b.2

The equivalent resistance is

$R_{eq} = [((20 \| 5) + 16) \| 20] + 40 = [(4 + 16) \| 20] + 40$

$R_{eq} = [20 \| 20] + 40 = 10 + 40 = 50\Omega$

Current through $R_1 = I_{R1}'$

$$I_{R1}' = \frac{V_1}{R_{eq}} = \frac{100}{50} = 2A$$

$$I_{R2}' = \frac{V_x}{R_2} = \frac{20}{20} = 1A$$

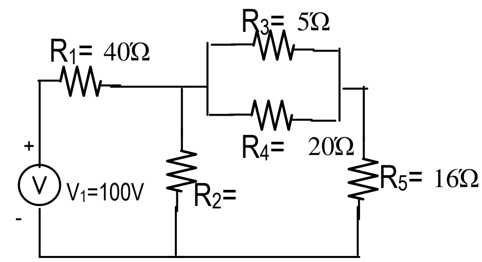


Fig 5.b.2

Voltage at x is $V_x = 100 - 2 \times 40 = 20$ V

Current through $R_2 = I_{R2}'$

Considering only voltage source V_2 and removing voltage source V_1 as in fig 5.b.3

The equivalent resistance is

$R_{eq1} = (40 \| 20) + (5 \| 20) + 16$

$R_{eq1} = 13.33 + 4 + 16$

$R_{eq1} = 33.33\Omega$

Current drawn from V_2

$$\frac{E_2}{R_{eq1}} = \frac{100}{33.33} = 3A$$

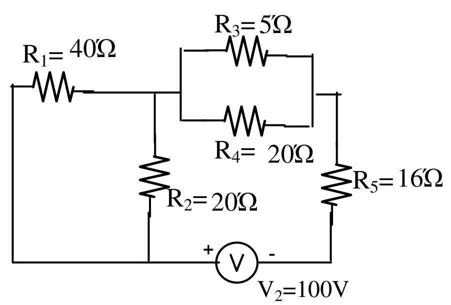


Fig 5.b.3

Current through $R_2 = I_{R2}''$

$$I_{R2}'' = 3 \times \frac{R_1}{R_1 + R_2} = 3 \times \frac{40}{40 + 20} = 2A$$

The direction of current in R_2 due to V_2 is upwards, while due to V_1 is downwards.

Hence by superposition theorem the net current in R_2 is $I_{R2} = I_{R2}' + I_{R2}'' = 1 - 2 = -1A$

The current through R_2 is 1A upwards.

Q.41. Calculate the driving point admittance of the network shown in the Fig.5. **(8)**

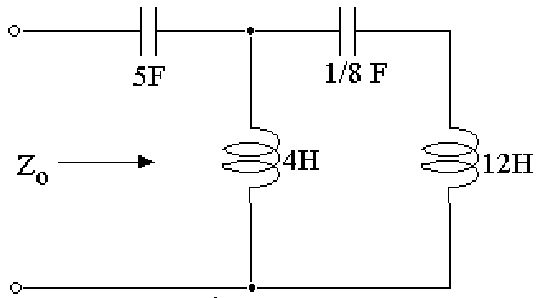
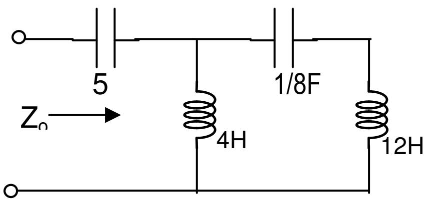


Fig.5

Ans:

The network of fig.6.b.1 is transformed to fig.6.b.2



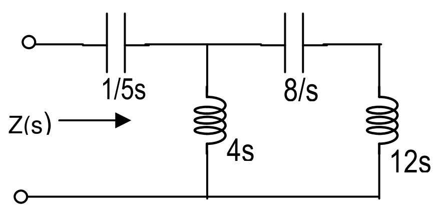


Fig.6.b.1 Fig.6.b.2

$$Z(s) = \frac{1}{5s} + \frac{4s\left(\frac{8}{s} + 12s\right)}{4s + \frac{8}{s} + 12s} = \frac{1}{5s} + \frac{s(32 + 48s^2)}{4s^2 + 12s^2 + 1}$$

$$Z(s) = \frac{1}{5s} + \frac{s(6s^2 + 4)}{2s^2 + 1} = \frac{30s^4 + 22s^2 + 1}{5s(2s^2 + 1)}$$

$$\frac{1}{Y(s)} = \frac{30s^4 + 22s^2 + 1}{5s(2s^2 + 1)}$$

$$\Rightarrow Y(s) = \frac{5s(2s^2 + 1)}{30s^4 + 22s^2 + 1}$$

Q.42. A sinusoidal voltage of rms value 20V and frequency equal to frequency of resonance is applied to a series RLC circuit having resistance $R = 20\Omega$, inductance L = 0.05H and capacitance $C = 0.05\mu F$. Calculate the value of current and voltages across R, L and C. **(8)**

Ans:

R = 20Ω, L = 0.05H, C = 0.05μF

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.05\times0.05\times10^{-6}}} = \frac{10^4}{\pi} = \frac{10000}{3.14} = 3184.7Hz$$

At f = f_o

$$I = \frac{V}{R} = \frac{20}{20} = 1amp$$

$V_R = I \times R = 1 \times 20 = 20$ V.
$V_L = I \times \omega L = 1 \times 2\pi \times 3184.7 \times 0.05 = 0.1 \times 10000 = 1000$ V.
$V_C = V_L = 1000$ V.

Q.43. Two coupled coils $(L_1 = 0.5H \text{ and } L_2 = 0.6H)$ have a coefficient of coupling, k = 0.9. Find the mutual inductance (μ), and the turn's ratio (n). **(4)**

Ans:
The mutual inductance, M is given by

$$M = K\sqrt{L_1 L_2} = 0.9\sqrt{0.5\times0.6} = 0.9\sqrt{0.3} = 0.9\times0.55 = 0.49H$$

Where K is the coefficient of coupling which is given by

$$K = \frac{\Phi_{12}}{\Phi_{11}}$$

The mutual inductance, M is also given by

$$M = \frac{N_2\Phi_{12}}{i_1} = K.\frac{N_2}{N_1}.\frac{N_1\Phi_{11}}{i_1} = K.\frac{N_2}{N_1}.L_1$$

$$\therefore \frac{N_2}{N_1} = \frac{KL_1}{M} = \frac{0.9\times0.5}{0.49} = 0.92$$

Where N_2/N_1 is the turns ratio.

Q.44. Design a T type symmetrical attenuator which offers 40 dB attenuation with a load of 400 Ω. **(8)**

Ans:
N is the attenuation in dB 20 log, N = attenuation
Load Resistance, $R_O = 400\Omega$(given)
N = Antilog(40/20) = Antilog(2) = 100
Series Arm Resistance

$$R_1 = R_O \frac{(N-1)}{(N+1)} = 400\frac{(100-1)}{(100+1)} = 400\times.98 = 392.07\ \Omega$$

Shunt Arm Resistance

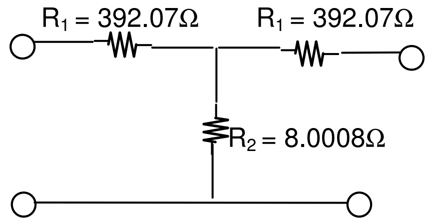


Fig 6.b

$$R_1 = R_O \frac{(N-1)}{(N+1)} = 400\frac{(100-1)}{(100+1)} = 400 \times .98 = 392.07\ \Omega$$

Shunt Arm Resistance

$$R_2 = R_O \frac{2N}{(N^2-1)} = 400\left(\frac{2\times100}{(100)^2-1}\right) = 400\left(\frac{200}{10000-1}\right)$$

$$= 400\times0.02 = 8.0008\,\Omega$$

Q.45. A generator of 1V, 1000Hz supplies power to 1000 Km long open wire line terminated in its characteristic impedance (Z_0) and having the following parameters. $R = 15\Omega$, L=0.004H, $C = 0.008\mu F$, $G = 0.5\mu mhos$. Calculate the characteristic impedance, propagation constant and the phase velocity. **(8)**

Ans:

Given R = 15Ω, L = 0.004H, C = 0.008μF, G = 0.5μmhos.

$\omega = 2\pi f = 2 \times 1000\pi = 2000 \times 3.14 = 6280$ rad/sec

$Z = R + j\omega L = 15 + j \times 6280 \times 0.004 = 15 + j\,25.13 = 29.26\angle 59^0$

$Y = G + j\omega C = 0.5 \times 10^{-6} + j \times 6280 \times .008 \times 10^{-6} = 10^{-6}\,(0.5 + j \times 50.24)$

$Y = 50.25 \times 10^{-6}\ \angle 89.43^0$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{29.26}{50.25\times10^{-6}}}\ \frac{1}{2}\angle 59^0 - 89.43^0 = 763\angle -15.22^0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{29.26\times50.25\times10^{-6}}\ \frac{1}{2}\angle 59^0 + 89.43^0$$

$$\gamma = 0.038\angle 74.22^0 = 0.038(\cos 74.22^0 + j\sin 74.22^0) = 0.0103 + j0.04 = \alpha + j\beta$$

α = 0.0103 Np/Km , β = 0.04 rad/Km

Phase velocity (v_p)

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 1000}{0.04} = 157080\ km\ /\sec$$

Q.46. Design a constant K band pass filter section having cut off frequencies of 2 KHz and 5 KHz and a nominal impedance of 600Ω. Draw the configuration of the filter. **(8)**

Ans:

According to the design equations

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} = \frac{600}{\pi(5000 - 2000)} = 63.68mH$$

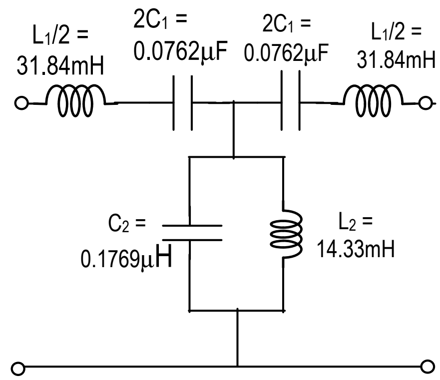
$$\frac{L_1}{2} = \frac{63.68}{2} = 31.8mH$$

$$C_1 = \frac{(f_2 - f_1)}{4\pi R_0 f_1 f_2} = \frac{5000 - 2000}{4 \times \pi \times 600 \times 2000 \times 5000} = 0.0381\mu F$$

$$2C_1 = 2 \times 0.0381 = 0.0762\mu F$$

$$L_2 = \frac{R_0(f_2 - f_1)}{4\pi f_1 f_2} = \frac{600(5000 - 2000)}{4 \times \pi \times 2000 \times 5000} = 14.33mH$$

$$C_2 = \frac{1}{\pi R_0 (f_2 - f_1)} = \frac{1}{\pi \times 600(5000 - 2000)} = 0.1769\mu F$$



Q.47. Using current to voltage transformation, find the current flowing through the resistor $R_L = 80\Omega$ as shown in Fig.1. **(4)**

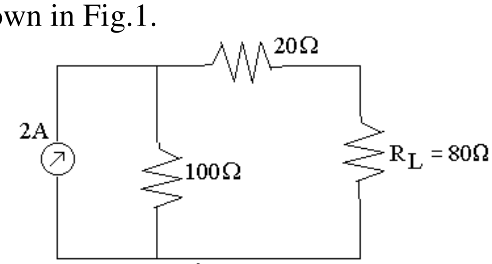


Fig.1

Ans:

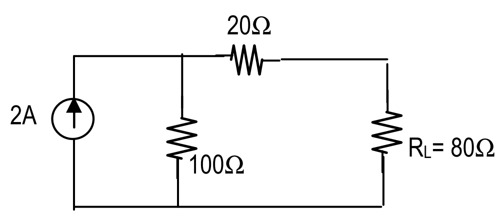


Fig. 2.a.1

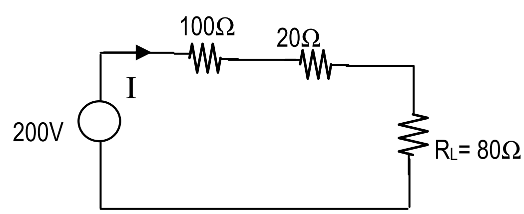


Fig. 2.a.2

Using the current to Voltage transformation, the circuit in Fig 2.a.1 can be replaced by the circuit in Fig.2.a.2.

Let the current flowing through the circuit of Fig 2.a.2 be I. Applying Kirchoff's voltage law

$100I + 20I + 80I = 200$

$\Rightarrow 200I = 200$

$\therefore I = 1$ Ampere

Q.48. A current of $I = e^{2t}$ A flows in a capacitor of value $C = 0.22\mu F$. Calculate the voltage, charge, and energy stored in the capacitor at time t =2 sec. **(4)**

Ans:

Given $I = e^{2t}$ Amp.

$C = 0.22\mu F$,

$\frac{dQ}{dt} = I = e^{2t}$, Where Q is the charge across the capacitor

$$Q = \int_0^{\infty} I.dt = \int_0^{\infty} e^{2t}\ dt$$

The voltage across the capacitor is given by

$$V = \frac{1}{C}Q = \frac{1}{C}\int I.dt = \int e^{2t}.\,dt$$

$$= \frac{1}{0.22\times10^{-6}} \times \frac{e^{2t}}{2} = \frac{e^{2t}}{0.44\times10^{-6}} = 2.27\times10^{6}\times e^{2t} \qquad \text{Volts}$$

When t = 2,
The voltage across the capacitor is
$V = 2.27 \times 10^6 \times e^4$ Volts
The charge across the capacitor is given by

$$Q = \int_0^{\infty} I.dt = \int_0^{\infty} e^{2t}\ dt$$

$$= \frac{e^{2t}}{2} = 0.5\times e^{2t} \text{ Coulombs}$$

At t = 2, the charge is given by
$Q = 0.5 \times e^4$ Coulombs
The energy stored in the capacitor is given by

$$E = \int P.\,dt = \int V.\,I\ .dt \quad \text{Joules}$$

$$= \int \frac{e^{2t}}{C}.\,I.\,dt = \int \frac{e^{2t}}{C}.\,e^{2t}.\,dt = \int \frac{e^{4t}}{C}.\,dt = \frac{e^{4t}}{4\times C}$$

$$= \frac{e^{4t}}{4\times C} = \frac{e^{4t}}{4\times0.22\times10^{-6}} = 1.136\times10^{6}\times e^{4t}$$

At t = 2, the energy stored in the capacitor is given by
$E = 1.136\times10^{6}\times e^{8}$ Joules

Q.49. Find the sinusoidal steady state solution i_{SS} for a parallel RL circuit. **(8)**

Ans:
Consider a parallel RL circuit, where the driving current is given by

$$i(t) = I\cos\omega t = \frac{I}{2}[e^{j\omega t} + e^{-j\omega t}] \qquad \text{--- Eq - 1}$$

Considering current source $Ie^{j\omega t} / 2$ and applying kirchoff's current law

$$\frac{1}{L}\int_{-\infty}^{t} V\ dt + \frac{V}{R} = I\frac{e^{j\omega t}}{2} \qquad \text{--- Eq – 2}$$

The steady state voltage is given by $v_{ss1} = A\ e^{j\omega t}$ where A is the undetermined coefficient

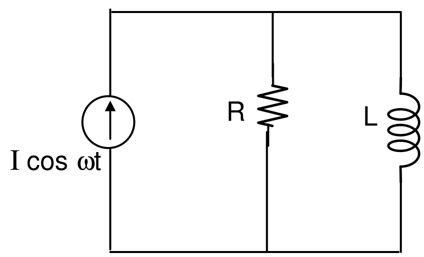


Fig. 2.c

From Eq - 1 and Eq - 2

$$\frac{A}{j\omega L}+\frac{A}{R}=\frac{I}{2}$$

$$A=\frac{I/2}{\frac{1}{R}+\frac{1}{\mathrm{j}\omega L}}$$

Considering current source $Ie^{-j\omega t}/2$ and applying kirchoff's current law

$$\frac{1}{L}\int_{-\infty}^{t}V\ dt+\frac{V}{R}=I\frac{\mathrm{e}^{-j\omega t}}{2} \qquad \text{--- Eq - 3}$$

The steady state voltage is given by $v_{ss2} = B\ e^{-j\omega t}$ where B is the undetermined coefficient.

From Eq -1 and Eq - 3

$$-\frac{B}{j\omega L}+\frac{B}{R}=\frac{I}{2}$$

$$B=\frac{I/2}{\frac{1}{R}-\frac{1}{\mathrm{j}\omega L}}$$

On applying superposition principle, the total steady state voltage v_{ss} is the summation of the voltages v_{ss1} and v_{ss2}.

$\therefore v_{ss} = v_{ss1} + v_{ss2}$

$= A\ e^{j\omega t} + B\ e^{-j\omega t}$

$$v_{ss}=\frac{I}{2}\left[\frac{e^{j\omega t}}{\frac{1}{R}+\frac{1}{\mathrm{j}\omega L}}+\frac{e^{-j\omega t}}{\frac{1}{R}-\frac{1}{\mathrm{j}\omega L}}\right]$$

$$v_{ss}=\frac{I/2}{\frac{1}{R^2}+\frac{1}{\omega^2L^2}}\left[\frac{1}{R}\left\{e^{j\omega t}+e^{-j\omega t}\right\}+\frac{1}{\omega L}\left\{e^{j\omega t}-e^{-j\omega t}\right\}\right]$$

$$v_{ss}=\frac{I}{\frac{1}{R^2}+\frac{1}{\omega^2L^2}}\left[\frac{1}{R}\cos\omega t-\frac{1}{\omega L}\sin\omega t\right]$$

$$v_{ss} = \frac{I}{\sqrt{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}}} \cos\left(\omega t + \tan^{-1}\frac{\omega L}{R}\right)$$

Q.50. Find the Laplace transform of the functions:
(i) sinh ωt . (ii) t cos 4t . **(4)**

Ans:

(i) $F(s) = L(\sinh(\omega t)) = \int_0^\infty \sinh(\omega t).e^{-st}\,dt = \int_0^\infty \left[\frac{e^{\omega t} - e^{-\omega t}}{2}\right] e^{-st}\,dt$

$= \frac{1}{2}\int_0^\infty e^{-(s-\omega)t}\,dt - \frac{1}{2}\int_0^\infty e^{-(s+\omega)t}\,dt$

$= \frac{1}{2} \times \frac{1}{(s-\omega)} - \frac{1}{2} \times \frac{1}{(s+\omega)}$

$= \frac{1}{2} \times \frac{2\omega}{s^2 - \omega^2} = \frac{\omega}{s^2 - \omega^2}$

(ii) $f(t) = t \ \cos 4t$

Let $f_1(t) = \cos 4t$

$F_1(s) = L(f_1(t)) = L(\cos 4t)$

$\Rightarrow F_1(s) = \frac{s}{s^2 + 16}$

$\therefore F(s) = L[t \ f_1(t)] = L(t \ \cos 4t)$

$F(s) = -\frac{d}{ds}\left[\frac{s}{s^2 + 16}\right]$

$\therefore F(s) = -\frac{(s^2 + 16) - s \times 2s}{(s^2 + 16)^2} = \frac{s^2 - 16}{(s^2 + 16)^2}$

Q.51. Find the convolution of $f_1(t)$ and $f_2(t)$ when $f_1(t) = e^{-at}$ and $f_2(t) = t$. **(4)**

Ans:
The convolution integral is given by

$f_1(t) * f_2(t) = \int_0^t f_2(\tau).f_1(t - \tau)d\tau$

$= \int_0^t e^{-a(t-\tau)} \cdot \tau \, d\tau \ = e^{-at} \int_0^t e^{a\tau} \cdot \tau \, d\tau$

$= e^{-at}\left[\frac{\tau e^{a\tau}}{a} - \left(\int_o^t \frac{e^{a\tau}}{a} d\tau\right)\right]_0^t = e^{-at}\left[\frac{\tau e^{a\tau}}{a} - \frac{e^{a\tau}}{a^\tau}\right]_0^t$

$$= e^{-at}\left[\frac{t\,e^{at}}{a} - \frac{e^{at}}{a^2} + \frac{1}{a^2}\right] = \frac{e^{-at}}{a^2}\left[at\,e^{at} - e^{at} + 1\right]$$

Q.52. A unit impulse is applied as input to a series RL circuit with R = 4Ω and L = 2H. Calculate the current i(t) through the circuit at time t = 0. **(8)**

Ans:
Applying kirchoff's voltage law with a unit impulse as driving voltage,

$$L\frac{di}{ds} + Ri = \delta(t) \qquad \text{--- Eq.1}$$

On Laplace transformation,

$$L[s\times I(s) - i(0+)] + R\times I(s) = 1 \qquad \text{--- Eq.2}$$

But $i(0+) = 0$

$\therefore (Ls + R)\times I(s) = 1$

Given, R = 4□ and L = 2H

$$\text{Or } \therefore I(s) = \frac{1}{(Ls+R)} = \frac{1}{(2s+4)} = \frac{1}{2}\times\frac{1}{(s+2)} \qquad \text{--- Eq.3}$$

On inverse Laplace transformation,

$$i(t) = \frac{1}{2}\cdot e^{-2t}$$

Q.53. Define the image impedances of an asymmetric network. Determine the image impedances of the L section shown in Fig.2. **(6)**

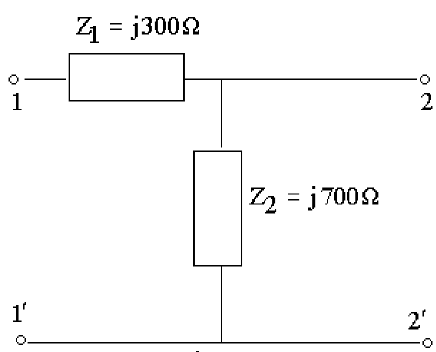
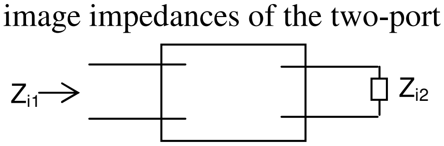
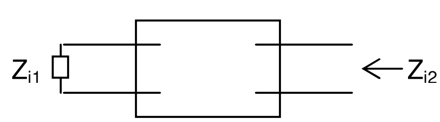


Fig.2

Ans:
Image impedance is that impedance, which when connected across the appropriate pair of terminals of the network, the other is presented by the other pair of terminals. If the driving point impedance at the input port with impedance Z_{i2} is Z_{i1} and if the driving point impedance at the output port with impedance Z_{i1} is Z_{i2}, then Z_{i1} and Z_{i2} are the image impedances of the two-port network.





From the Fig 4.a

On open circuiting the output terminals 2 – 2',

$$Z_{oc1} = j300 + j700 = j1000\ \Omega$$

On short circuiting the output terminals 2 – 2',

$$Z_{sc1} = j300\ \Omega$$

On open circuiting the input terminals 1 – 1',

$$Z_{oc2} = j700\ \Omega$$

On short circuiting the input terminals 1 – 1',

$$Z_{sc2} = \frac{j300 \times j700}{j300 + j700} = \frac{j^2 210000}{j1000} = j210\ \Omega$$

The image impedance at terminal 2 – 2',

$$Z_{o1} = \sqrt{Z_{oc1} \times Z_{sc1}} = \sqrt{j1000 \times j300} = j100\sqrt{3}\ \Omega$$

The image impedance at terminal 1 – 1',

$$Z_{o2} = \sqrt{Z_{oc2} \times Z_{sc2}} = \sqrt{j700 \times j210} = j70\sqrt{30}\ \Omega$$

Q.54. State Norton's theorem and using Norton's theorem find the current flowing through the 15Ω resistor. **(2+6)**

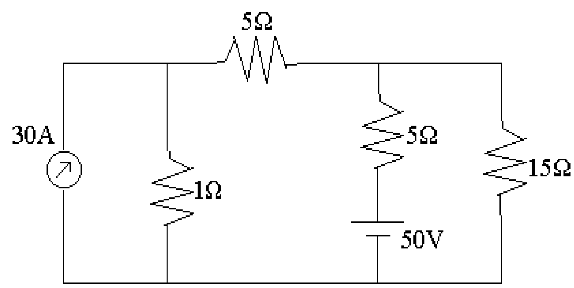


Fig.3

Ans:

Norton's theorem states that the current in any load impedance Z_L connected to the two terminals of a network is the same as if this load impedance Z_L were connected to a current source (called Norton's equivalent current source) whose source current is the short circuit current at the terminals and whose internal impedance (in shunt with the current source) is the impedance of the network looking back into the terminals with all

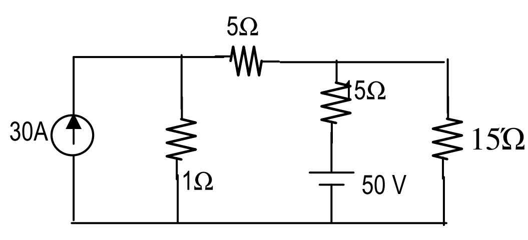


Fig. 5.a.1

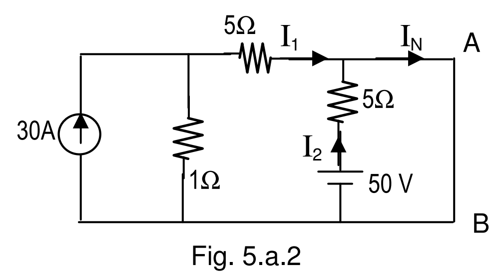


Fig. 5.a.2

the sources replaced by impedances equal to their internal impedances.

Applying Norton's theorem, remove R_L(15 Ω and short circuit the terminals A and B as shown in Fig 5.a.2. Since there are two sources due to which current will flow between the terminals A and B, therefore

$$I_N = I_1 + I_2$$

Considering the current source only, the current I_1 in the 5 Ω resistor is given by

$$I_1 = \frac{30 \times 1}{1+5} = \frac{30}{6} = 5\text{ A}$$

Considering the voltage source only, the current I_2 in the 5Ω resistor is given by

$$I_2 = \frac{50}{5} = 10\text{ A}$$

Applying Superposition theorem, the Norton's current is given by

$$I_N = I_1 + I_2 = 5 + 10 = 15\text{ A}$$

On open circuiting the current source and short circuiting the voltage source as shown

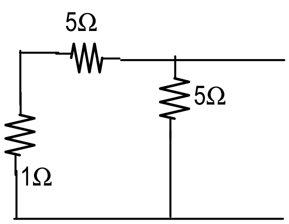


Fig. 5.a.3

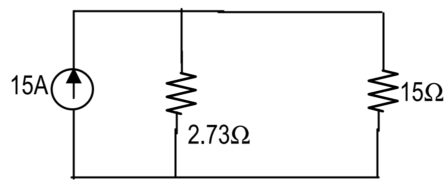


Fig. 5.a.4

in Fig 5.a.3, the Norton's resistance is given by

$$R_N = \frac{(1+5)\times 5}{1+5+5} = \frac{30}{11} = 2.73\ \Omega$$

The Norton's equivalent circuit is shown in Fig 5.a.4. The current through the 15 Ω resistor is given by

$$I_L = I_N \times \frac{R_N}{R_N + R_L} = 15 \times \frac{2.73}{2.73 + 15} = \frac{15 \times 2.73}{15 + 2.73} = 2.31\text{ Amperes}$$

Q.55. Find the current in resistor R_3 of the network using Millman's theorem. **(8)**

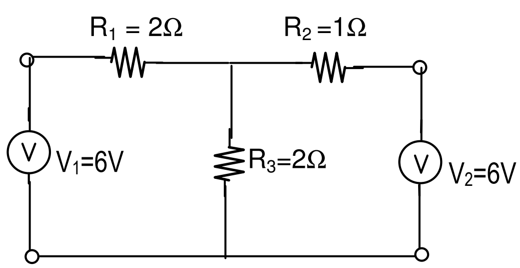


Fig 5.b

Ans:

The two voltage sources V_1 and V_2 with series resistances R_1 and R_2 are combined into

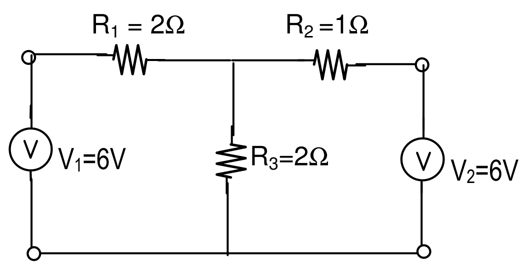


Fig 5.b

one voltage sources V_m with series resistances R_m.

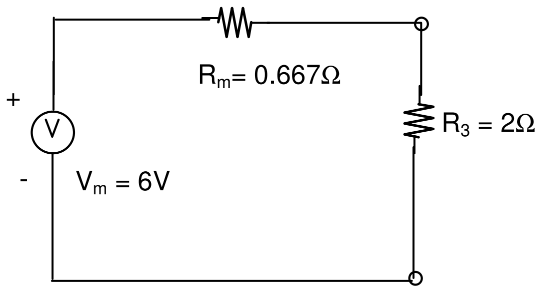


Fig 5.b.2

The equivalent circuit is given by Fig 5.b.2
According to the Mill man theorem,

$$V_m = \frac{V_1Y_1 + V_2Y_2}{Y_1 + Y_2} = \frac{6\times0.5 + 6\times1}{0.5+1} = 6 \text{ volts}$$

$$Z_m = \frac{1}{Y_1 + Y_2} = \frac{1}{0.5+1} = 0.667\ \Omega$$

Hence the current through R_3 is given by

$$I_3 = \frac{V_m}{R_m + R_3} = \frac{6}{0.667+2} = 2.25 \quad amps$$

Q.56. Find the z parameters of the given network. From the z parameters, find the h parameters equivalent and the ABCD parameters equivalent. **(10)**

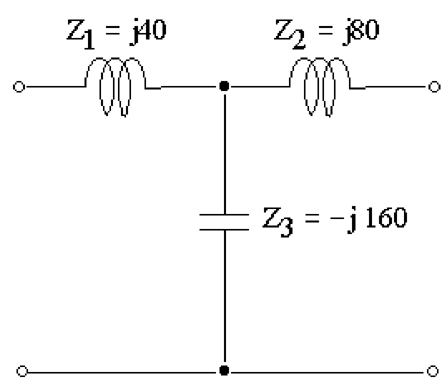


Fig.5

Ans:

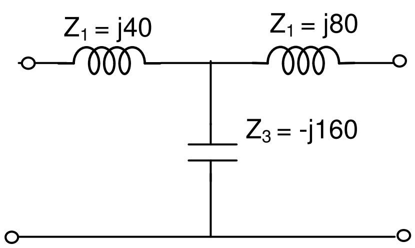


Fig 6.a

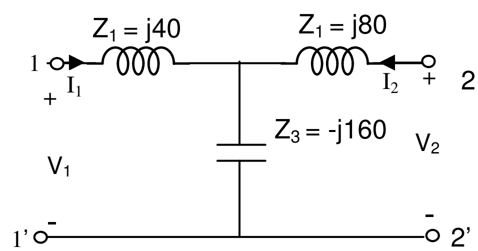


Fig 6.a.1

Assuming open circuit at the output terminals 2 - 2', of Fig 6.a.1
The voltage equations in the first loop are given by

$V_1 = I_1 \times j(40-160) = -I_1 \times j120$ ---Eq.1

$V_2 = -I_1 \times j160$ ---Eq.2

From Eq.1, $\therefore Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = -j120\ \Omega$

From Eq.2, $\therefore Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0} = -j160\ \Omega$

Assuming open circuit at the input terminals 1 - 1', of Fig 6.a.1
The voltage equations in the first loop are given by

$V_1 = -I_2 \times j160$ ---Eq.3

$V_2 = I_2 \times j(80-160) = -I_2 \times j80$ ---Eq.4

From Eq.4, $\therefore Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0} = -j80\ \Omega$

From Eq.3, $\therefore Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0} = -j160\ \Omega$

$Z_{11} = -j120\ \Omega$, $Z_{21} = -j160\ \Omega$, $Z_{22} = -j80\ \Omega$, $Z_{12} = -j120\ \Omega$

Where Z_{11}, Z_{21}, Z_{22}, Z_{12} are the Z- parameters of a network.

The h parameters in terms of Z parameters are given by

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{-j^2 16000}{-j80} = j200\ \Omega \text{, Where } \Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

$$h_{22} = \frac{1}{Z_{22}} = \frac{1}{-j80} \text{ mhos}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} = -\frac{-j160}{-j80} = -2$$

$$h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{-j160}{-j80} = 2$$

The transmission (ABCD) parameters in terms of Z parameters are given by

$$B = \frac{\Delta Z}{Z_{21}} = \frac{-j^2 16000}{-j160} = j100\ \Omega \text{, Where } \Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

$$A = \frac{Z_{11}}{Z_{21}} = \frac{-j120}{-j160} = 0.75$$

$$C = \frac{1}{Z_{21}} = \frac{1}{-j160} \text{ mhos}$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{-j80}{-j160} = 0.5$$

Q.57. Find the transform impedance Z(s) of the one port network. **(6)**

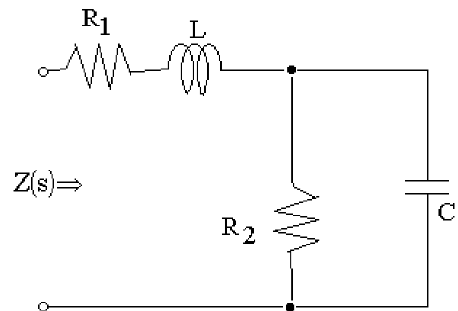


Fig.6

Ans:

Let, $Z_1(s) = R + Ls$

And $Z_2(s) = \left(\frac{R_2 \times (1/Cs)}{R_2 + (1/Cs)}\right) = \left(\frac{R_2}{R_2Cs + 1}\right)$

The transform impedance is given by

$$Z(s) = Z_1(s) + Z_2(s) = (R_1 + Ls) + \left(\frac{R_2}{R_2Cs + 1}\right)$$

$$\Rightarrow Z(s) = \frac{(R_1 + Ls)(R_2Cs + 1) + R_2}{R_2Cs + 1}$$

$$\therefore Z(s) = \frac{(R_1 + R_2) + R_2LCs^2 + R_1R_2Cs + Ls}{R_2Cs + 1}$$

Q.58. Calculate the half power frequencies of a series resonant circuit whose resonant frequency is 150 KHz and the band width is 75 KHz. Derive the relations used. **(10)**

Ans:

If f_1 and f_2 are the half power frequencies, the bandwidth of a series resonant circuit, B.W is given by

$B.W = f_2 - f_1 = 75$ KHz

The resonant frequency f_o is given by

$f_o = \sqrt{f_1 f_2} = 150$ KHz

It is given that

$$(f_1 + f_2)^2 = (f_2 - f_1)^2 + 4f_1f_2$$

$$(f_1 + f_2)^2 = (75)^2 + 4\times(150)^2 = 5625 + 90000 = 95625$$

$f_1 + f_2 = \sqrt{95625} \cong 310\,\text{KHz}$ --- Eq.1

$f_2 - f_1 = 75\,\text{KHz}$ --- Eq.2

Adding Eq.1 and Eq.2

$2f_2 = 385$

$\Rightarrow f_2 = 192.5$ KHz
Subtracting Eq.2 and Eq.1
$2f_1 = 235$
$\Rightarrow f_1 = 117.5$ KHz

Q.59. The combined inductance of two coils connected in series is 0.6 H and 0.1 H depending on the relative directions of the currents in the coils. If one of the coils, when isolated, has a self inductance of 0.2 H, calculate the mutual inductance and the coefficient of coupling. **(6)**

Ans:
Let the self inductances of the two coils be L_1 and L_2, the coefficient of coupling be K and the mutual inductance be μ.
Given $L_1 = 0.2$H
$L_1 + L_2 + 2\mu = 0.6$ --- Eq.1
$L_1 + L_2 - 2\mu = 0.1$ --- Eq.2
Subtracting Eq.1 from Eq.2, we get
$4\mu = 0.5$ --- Eq.3
$\Rightarrow \mu = 0.125$
$0.2 + L_2 + 0.25 = 0.6$
$\Rightarrow L_2 = 0.15$
The coefficient of coupling is given by

$$K = \frac{\mu}{\sqrt{L_1 L_2}} = \frac{0.125}{\sqrt{0.2 \times 0.15}} = \frac{0.125}{\sqrt{0.030}} = \frac{0.125}{0.173} = 0.722$$

Q.60. A loss less line of characteristic impedance 500 Ω is terminated in a pure resistance of 400Ω. Find the value of standing wave ratio. **(4)**

Ans:
In a loss less line, the reflection coefficient is given by

$$|K| = \frac{|Z_o| - |Z_R|}{|Z_o| + |Z_R|} = \frac{500 - 400}{500 + 400} = \frac{100}{900} = 0.11$$

The standing wave ratio of a loss less transmission line is given by

$$S = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.11}{1 - 0.11} = \frac{1.11}{0.89} = 1.25$$

Q.61. A loss less transmission line has an inductance of 1.5 mH/Km and a capacitance of 0.02 μF/Km. Calculate the characteristic impedance and phase constant of a transmission line. Assume $\omega = 5000$ rad/sec. **(6)**

Ans:
Given R = 0, G = 0, since the line is lossless.
$L = 1.2 \times 10^{-3}$ H/Km
$C = 0.05 \times 10^{-6}$ F/Km
The characteristic impedance, Z_O is given by

$$Z_O = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\Rightarrow Z_O = \sqrt{\frac{L}{C}} = \sqrt{\frac{1.2\times10^{-3}}{0.05\times10^{-6}}} = 154.92\ \Omega$$

The propagation constant is given by

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC}$$

Since the line is lossless, $\alpha = 0$, and assuming ω= 5000 rad/sec

$$\gamma = \alpha + j\beta$$

$$\Rightarrow j\beta = j\omega\sqrt{LC} \qquad \Rightarrow \beta = \omega\sqrt{LC}$$

$$\Rightarrow \beta = \omega\sqrt{LC} = 5000\sqrt{1.2\times10^{-3}\times0.05\times10^{-6}} = 0.0387$$

Q.62. Design a symmetrical bridge T attenuator with an attenuation of 40 dB and an impedance of 600 Ω. **(8)**

Ans:

Given $R_o = 600\Omega$, D = 40dB

$$N = Anti\log\left(\frac{D}{20}\right) = Anti\log\left(\frac{40}{20}\right) = 100$$

We know that

$R_2R_3 = R_1^2 = R_0^2$

$\Rightarrow R_1 = R_0 = 600\Omega$

$\therefore R_1 = 600\Omega$

$$R_2 = \frac{R_0}{(N-1)} = \frac{600}{100-1} = 6.06\Omega$$

$R_3 = R_0(N - 1)$

$= 600(100 - 1) = 59.4\Omega$

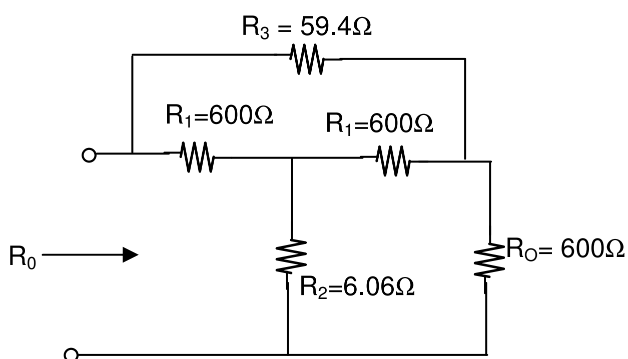


Fig 9.a

The required symmetrical bridge T attenuator is shown in Fig.9.a

Q.63. Show that the input impedance at the sending end is Z_o for an infinite length of the transmission line.

Ans: The input impedance at the sending end of the transmission line is given by

$$Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh \gamma l}{Z_L \tanh \gamma l + Z_o}\right]$$

Where Z_o is the characteristic impedance and Z_L is the load impedance.

For an infinite length of the transmission line, $l = \infty$

$$\therefore Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh \infty}{Z_L \tanh \infty + Z_o}\right] = Z_o$$

Q.64. A transmission line is terminated by an impedance Z_R. Measurements taken on the line show that the standing wave minima are 105 cm apart and the first minimum is 30 cm from the load end of the line. The VSWR is 2.3 and $Z_o = 300\Omega$. Find Z_R. **(7)**

Ans:
The standing wage minima points are the voltage minima points. The two consecutive E_{min} points are separated by 105 cms.

$$\frac{\lambda}{2} = 105 \text{ cms}$$
$$\Rightarrow \lambda = 2.1 \text{ m}$$

The first voltage minimum is given by

$$y_{\min} = \frac{\phi + \pi}{2\beta} = \frac{\phi + \pi}{4\pi}\times\lambda = (\frac{\phi + \pi}{4\pi})\times 2.1$$

$$\therefore \phi + \pi = \frac{y_{\min}\times 4\pi}{2.1} = \frac{0.3\times 4\pi}{2.1} = \frac{4\pi}{7} \quad \text{given } (y_{\min} = 0.3 \text{ m})$$

$$\therefore \phi = \frac{4\pi}{7} - \pi = \frac{-3\pi}{7} = -77.14°$$

Given $Z_o = 300\Omega$ and s = 2.3
The magnitude of the reflection coefficient K is given by

$$|k| = \frac{s-1}{s+1} = \frac{2.3-1}{2.3+1} = \frac{1.3}{3.3} = 0.394$$
$$\therefore k = 0.394\angle -77.14°$$
$$= 0.0882 - j0.384$$

The reflection coefficient K in terms of Z_R and R_o is given by

$$k = \frac{Z_R - R_o}{Z_R + R_o}$$

$$\Rightarrow Z_R = \frac{R_o(1+k)}{(1-k)} = 300\left[\frac{1 + 0.0882 - j0.384}{1 - 0.0882 + j0.0384}\right]$$

$$= 300\left[\frac{1.0882 - j0.384}{0.9118 + j0.0384}\right] = 300\left[\frac{1.153\angle -19.9°}{0.989\angle 22.8°}\right]$$

$$= 350\angle -42.7° = 257 - j237.5\Omega$$

Q.65. A capacitor of $10\,\mu F$ capacitance is charged to a potential difference of 200 V and then connected in parallel with an uncharged capacitor of $30\,\mu F$. Calculate

(i)The potential difference across the parallel combination

(ii)Energy stored by each capacitor. **(4)**

Ans:

We know that, Q = CV, where Q is the charge on the capacitor and V is the potential difference across the capacitance C.

$\therefore$ Charge on the capacitor 10 μF = $10 \times 10^{-6} \times 200$C.

When this capacitor is connected in parallel with 30 μF, let the common voltage be V volts.

Hence, $Q_1 = C_1V = 10 \times 10^{-6} \times V$C.

And $Q_2 = C_2V = 30 \times 10^{-6} \times V$C.

$Q = Q_1 + Q_2$

$\because$ $10 \times 10^{-6} \times 200 = (10 \times 10^{-6} + 30 \times 10^{-6})$ V.

Or $V = \dfrac{10\times10^{-6}\times200}{40\times10^{-6}} = 50\, Volts$

(ii) $\because$ Energy stored by capacitor $= \dfrac{1}{2}CV^2$,

Energy stored by 10 μF capacitor $= \dfrac{1}{2}\times10\times10^{-6}\times(50)^2 = 0.0125J$

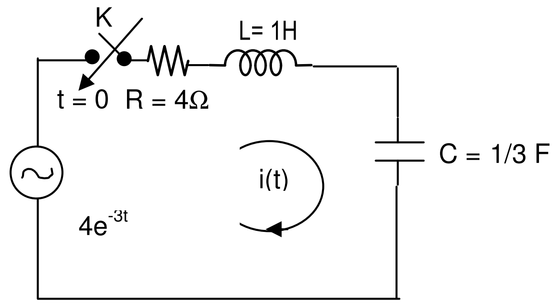
Energy stored by 30 μF capacitor $= \dfrac{1}{2}\times30\times10^{-6}\times(50)^2 = 0.0375J$

Q.66. An exponential voltage $v(t) = 4e^{-3t}$ is applied at time t = 0 to a series RLC circuit comprising of $R = 4\Omega$, L = 1H and $C = \frac{1}{3}$F. Obtain the complete particular solution for current i(t). Assume zero current through inductor L and zero charge across capacitance C before application of exponential voltage. **(8)**

Ans:

On Applying Kirchoff's voltage law,

$$\frac{di}{dt} + 4i + 3\int_{-\infty}^{0} i\,dt + 3\int_{0}^{t} i\,dt = 4e^{-3t} \cdots\cdots\cdots Eq.1$$



Applying Laplace transformation to Eq. 1

$$[s.I(s) - i(0+)] + 4I(s) + 3\times\frac{q(0+)}{s} + 3\times\frac{I(s)}{s} = \frac{4}{s+3}\cdots\cdots\cdots Eq.2$$

At time t = 0+, current i(0+) must be the same as at time t = 0 – due to the presence of the inductor L.

$\therefore$ i(0+) = 0

At t = 0+, charge q(0+) across capacitor must be the same as at time t = 0 –

$\therefore$ q(0+) = 0

Substituting the initial conditions in *Eq. 2*

$$I(s)[s+4+\frac{3}{s}] = \frac{4}{(s+3)}$$

$$I(s)[s^2+4s+3] = \frac{4s}{(s+3)}$$

$$I(s) = \frac{4s}{(s+3)(s^2+4s+3)} = \frac{4s}{(s+3)^2(s+1)} \qquad \ldots Eq.3$$

By partial fraction method,

$$\text{Let } \frac{4s}{(s+1)(s+3)^2} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+3)} + \frac{K_3}{(s+3)^2} \qquad \cdots Eq.4$$

$$\Rightarrow \frac{4s}{(s+1)(s+3)^2} = \frac{K_1(s+3)^2 + K_2(s+1)(s+3) + K_3(s+1)}{(s+1)(s+3)^2}$$

$$\Rightarrow K_1(s+3)^2 + K_2(s+1)(s+3) + K_3(s+1) = 4s \qquad \cdots Eq.5$$

$When\ s = -1, \quad 4K_1 = -4 \quad \Rightarrow \quad K_1 = -1$

$When\ s = -3, \quad -2K_3 = -12 \quad \Rightarrow \quad K_3 = 6$

To find the coefficient $k_{2,}$ Multiplying Eq.4 with (s+3)2 and differentiating with respect s = -1

$$\frac{d}{ds}\left(\frac{4s}{s+1}\right) = K_1\frac{d}{ds}\left(\frac{(s+3)^2}{s+1}\right) + K_2\frac{d}{ds}(s+3)$$

$$\frac{4(s+1)-4s}{(s+1)^2} = K_1\left[\frac{(s+1)\times 2\times (s+3)-(s+3)^2}{(s+1)^2}\right] + K_2$$

$$\frac{4}{(s+1)^2} = K_1\left[\frac{(s+3)(2s+2-s-3)}{(s+1)^2}\right] + K_2$$

$$\frac{4}{(s+1)^2} = K_1\left[\frac{(s+3)(s-1)}{(s+1)^2}\right] + K_2$$

When s = - 3,

$$\frac{4}{4} = K_2 \qquad \Rightarrow K_2 = 1$$

$$\frac{4s}{(s+1)(s+3)^2} = \frac{-1}{(s+1)} + \frac{1}{(s+3)} + \frac{6}{(s+3)^2}$$

On inverse laplace transformation

$$i(t) = L^{-1}\left[\frac{4s}{(s+1)(s+3)^2}\right] = L^{-1}\left[\frac{-1}{(s+1)} + \frac{1}{(s+3)} + \frac{6}{(s+3)^2}\right]$$

$$i(t) = -e^{-t} + e^{-3t} - 6t\,e^{-3t}$$

The current i(t) is given by

$$i(t) = (-e^{-t} + e^{-3t} - 6t\,e^{-3t})u(t)$$

Q.67. Find the Laplace transform of the functions:

(i) $\cos^2$ t (ii) t sin 2t. **(4)**

Ans:

(i) $f(t) = \cos^2 t$

$$F(s) = L(\cos^2 t) = L\left[\frac{1+\cos 2t}{2}\right]$$

$$F(s) = L\left[\frac{1}{2}\right] + L\left[\frac{\cos 2t}{2}\right]$$

$$F(s) = \frac{1}{2s} + \frac{1}{2}\left[\frac{s}{s^2+4}\right]$$

$$\therefore\ F(s) = \frac{2s^2+4}{2s(s^2+4)}$$

(ii) $f(t) = t\ \sin 2t$

Let $f_1(t) = \sin 2t$

$F_1(s) = L(f_1(t)) = L(\sin 2t)$

$\Rightarrow F_1(s) = \dfrac{2}{s^2+4}$

$\therefore F(s) = L[t\ f_1(t)] = L(t \sin 2t)$

$F(s) = -\dfrac{d}{ds}\left[\dfrac{2}{s^2+4}\right]$

$\therefore F(s) = \dfrac{4s}{(s^2+4)^2}$

Q.68. Find the value of v(t) given its laplace transform $V(s) = \dfrac{s^2+7s+14}{(s^2+3s+2)}$. **(4)**

Ans:
Since the degree of the numerator is equal to the degree of denominator, dividing the numerator by denominator we get

$V(s) = 1 + \dfrac{4s+12}{(s^2+3s+2)}$

By partial fraction method

Let $V_1(s) = \dfrac{4s+12}{(s+2)(s+1)}$

$\therefore \dfrac{4s+12}{(s+2)(s+1)} = \dfrac{A}{(s+2)} + \dfrac{B}{(s+1)}$

$\dfrac{4s+12}{(s+2)(s+1)} = \dfrac{B(s+2)+A(s+1)}{(s+2)(s+1)}$

$\Rightarrow 4s+12 = B(s+2)+A(s+1)$

When s = -1, B = 8
When s = -2, A = - 4

$\therefore V_1(s) = \dfrac{4s+12}{(s+2)(s+1)} = \dfrac{-4}{(s+2)} + \dfrac{8}{(s+1)}$

$\therefore V(s) = 1 + V_1(s) = 1 + \dfrac{-4}{(s+2)} + \dfrac{8}{(s+1)}$

On taking Inverse Laplace transform

$\therefore V(t) = 1 + V_1(t) = 1 - 4e^{-2t} + 8e^{-t}$

Q.69. Find the sinusoidal steady state solution i_{ss} for a series RC circuit. **(8)**

Ans:
The driving voltage is given by

$v(t) = V\cos\omega t = \dfrac{V}{2}[e^{j\omega t} + e^{-j\omega t}]$ --- Eq.1

Considering voltage source $Ve^{j\omega t}/2$ and applying Kirchoff's voltage law

$$R\,i+\frac{1}{C}\int i\;dt=V\,\frac{\mathrm{e}^{\mathrm{j}\omega t}}{2} \qquad \text{--- Eq.2}$$

Differentiating Eq.2 with respect to dt

$$\frac{1}{C}i+R\frac{di}{dt}=\frac{V}{2}\,j\,\omega e^{j\omega t} \qquad \text{--- Eq.3}$$

The steady state current is given by i_{ss1} = A $e^{j\omega t}$ where A is the undetermined coefficient.

From Eq.1 and Eq.3

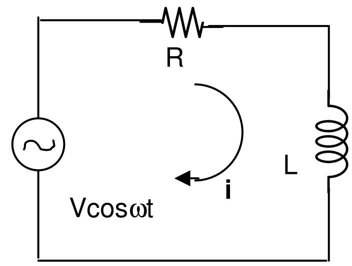


Fig.3.c.

$$\frac{Ae^{\,j\omega t}}{C}+j\omega RAe^{\,j\omega t}=\frac{V}{2}\,j\,\omega e^{\,j\omega t}$$

$$\frac{A}{C}+j\omega RA=j\omega\frac{V}{2}$$

$$A=\frac{V/2}{R+\dfrac{1}{\mathrm{j}\omega C}} \qquad \text{--- Eq.4}$$

Considering voltage source $Ve^{-j\omega t}$ / 2and applying Kirchoff's voltage law

$$R\,i+\frac{1}{C}\int i\;dt=V\,\frac{\mathrm{e}^{-\mathrm{j}\omega t}}{2} \qquad \text{--- Eq - 5}$$

Differentiating Eq.5 with respect to dt

$$\frac{1}{C}i+R\frac{di}{dt}=-\frac{V}{2}\,j\,\omega e^{j\omega t} \qquad \text{--- Eq.6}$$

The steady state current is given by i_{ss2} = B $e^{-j\omega t}$ where B is the undetermined coefficient.

From Eq -1 and Eq - 6

$$\frac{Be^{\,j\omega t}}{C}-j\omega RBe^{\,j\omega t}=-\frac{V}{2}\,j\,\omega e^{-j\omega t}$$

$$\frac{B}{C}-j\omega RB=-j\omega\frac{V}{2}$$

$$B=\frac{V/2}{R-\dfrac{1}{\mathrm{j}\omega C}}$$

On applying superposition principle, the total steady state current i_{ss} is the summation of the currents i_{ss1} and i_{ss2}.

$\therefore i_{ss} = i_{ss1} + i_{ss2}$

$= i_{ss1} + i_{ss2} = A\, e^{j\omega t} + B\, e^{-j\omega t}$

$$i_{ss}=\frac{V}{2}\left[\frac{e^{j\omega t}}{R+\dfrac{1}{\mathrm{j}\omega C}}+\frac{e^{-j\omega t}}{R-\dfrac{1}{\mathrm{j}\omega C}}\right]=\frac{V}{R^2+\dfrac{1}{\omega^2C^2}}\left[R\cos\omega t+\frac{1}{\omega C}\sin\omega t\right]$$

$$i_{ss}=\frac{V}{\sqrt{R^2+\dfrac{1}{\omega^2C^2}}}\cos\left(\omega t-\tan^{-1}\frac{1}{\omega CR}\right)$$

Q.70. A symmetrical Π section is given in Fig.1. Find out its equivalent T-network. **(6)**

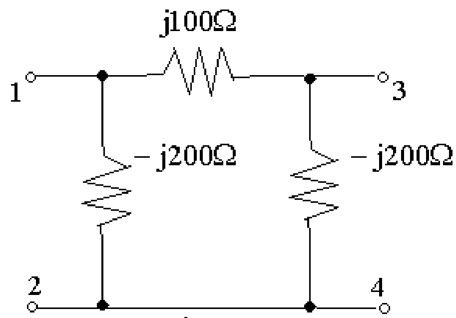


Fig.1

Ans:

Given R_1 = - j 200Ω, R_2 = j 100Ω R_3 = - j 200 Ω

Let the arms of the T – network be R_{12}, R_{23} and R_{31}. The values of R_{12}, R_{23} and R_{31} are given by

$$R_{12} = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{-j200 \times j100}{-j200 - j200 + j100} = \frac{-j^2 20000}{-j300} = \frac{j200}{3}\Omega$$

$$R_{23} = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{-j200 \times j100}{-j200 - j200 + j100} = \frac{-j^2 20000}{-j300} = \frac{j200}{3}\Omega$$

$$R_{31} = \frac{R_3 R_1}{R_1 + R_2 + R_3} = \frac{-j200 \times -j200}{-j200 - j200 + j100} = \frac{j^2 40000}{-j300} = \frac{-j400}{3}\Omega$$

Q.71. What is the input impedance at the sending end if the transmission line is loaded by a load $Z_L = Z_o$?

Ans:

The input impedance at the sending end of the transmission line is given by

$$Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh \gamma l}{Z_L \tanh \gamma l + Z_o}\right]$$

Where Z_o is the characteristic impedance and Z_L is the load impedance.

When $Z_L = Z_o$,

$$Z_{in} = Z_o\left[\frac{Z_o + Z_o \tanh \gamma l}{Z_o \tanh \gamma l + Z_o}\right] = Z_o$$

Q.72. A loss free transmission line has an inductance of 1.2 mH/km and a capacitance of 0.05μF/km. Calculate the characteristics impedance and propagation constant of the line. **(8)**

Ans:
Given R = 0, G = 0, since the line is lossless.
$L = 1.2 \times 10^{-3}$ H/Km
$C = 0.05 \times 10^{-6}$ F/Km
The characteristic impedance, Z_O is given by

$$Z_O = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\Rightarrow Z_O = \sqrt{\frac{L}{C}} = \sqrt{\frac{1.2\times10^{-3}}{0.05\times10^{-6}}} = 154.92\ \Omega$$

The propagation constant is given by

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC}$$

Since the line is lossless, $\alpha = 0$, and assuming $\omega = 5000$ rad/sec

$$\gamma = \alpha + j\beta$$

$$\Rightarrow j\beta = j\omega\sqrt{LC} \qquad \Rightarrow \beta = \omega\sqrt{LC}$$

$$\Rightarrow \beta = \omega\sqrt{LC} = 5000\sqrt{1.2\times10^{-3}\times0.05\times10^{-6}} = 0.0387$$

Q.73. In the network shown in Fig.2, find the value of Z_L, so that it draws the maximum power from the source. Also determine the maximum power. **(8)**

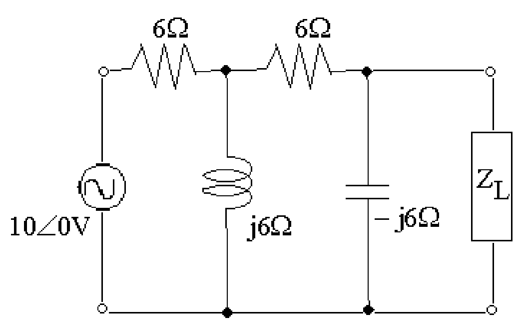


Fig.2

Ans:

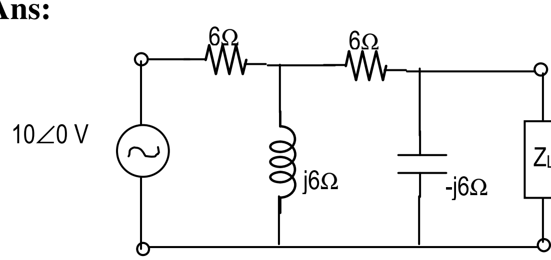


Fig 5.b.1

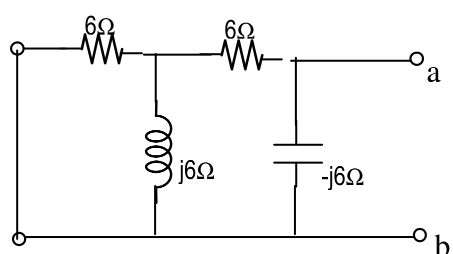


Fig 5.b.2

$$V_{TH} = V_{ab} = \left[\frac{10\angle 0°}{6 + \frac{j6(6-j6)}{j6+(6-j6)}} \right] \times \frac{j6}{j6+(6-j6)}(-j6)$$

$$V_{TH} = \frac{360\angle 0°}{72 + j36} = 4.472\angle -26.56°$$

$$Z_{TH} = Z_{ab} = \frac{\left[6 + \frac{6\times j6}{6+j6} \right](-j6)}{6 + \frac{6\times j6}{6+j6} - j6}$$

$$= \frac{12-j6}{2+j} = 6\angle -53.12° = 3.6 - j4.8\,\Omega$$

Thevenin's equivalent circuit is shown in Fig 5.b.2

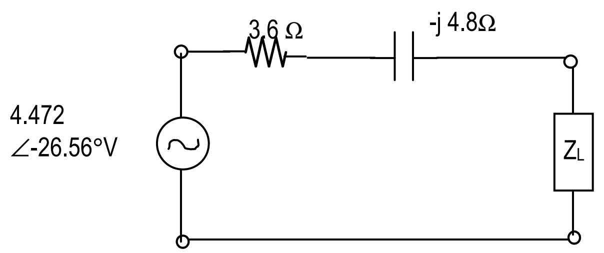


Fig 5.b.2

For maximum power transfer $Z_L = 3.6 + j\,4.8$

$$I = \frac{4.472\angle -26.56°}{3.6 + j4.8 + 3.6 - j4.8} = \frac{4.472\angle -26.56°}{7.2}$$

$$I = 0.621\angle -26.56°\ A$$

Power transferred $|I|^2 R_L = (0.621)^2 \times 3.6 = 1.398\,W$

Q.74. Find the transmission parameters of the network shown in Fig.3. Determine whether the given circuit is reciprocal and symmetric or not. **(8)**

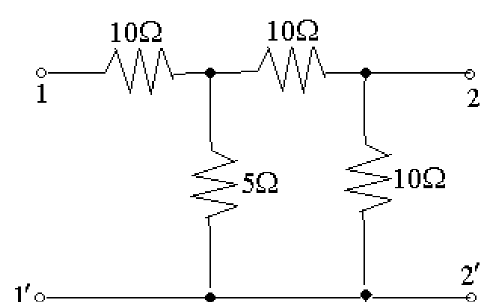


Fig.3

Ans:

When the output is open circuited, $I_2 = 0$,

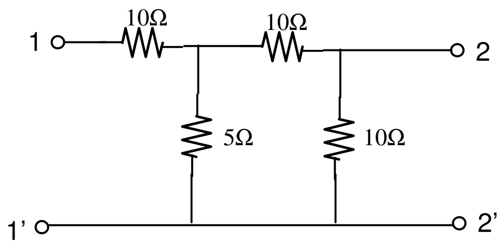


Fig 6.a

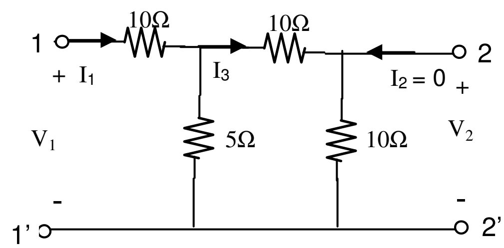


Fig 6.a.1

$V_1 = 10\ I_1 + (I_1 - I_3)\ 5 = 15\ I_1 - 5\ I_3$ … eq.1

And $0 = (\ I_3 - I_1)\ 5 + 20\ I_3$

Or $5\ I_1 = 25\ I_3$

$\therefore I_3 = \frac{1}{5} I_1$ … eq.2

From eq.1 and eq.2

$V_1 = 14\ I_1$

With $I_2 = 0$, $V_2 = 10$ and $I_3 = 2\ I_1$

$$\therefore A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 7 \quad \therefore C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{2} \text{ mho}$$

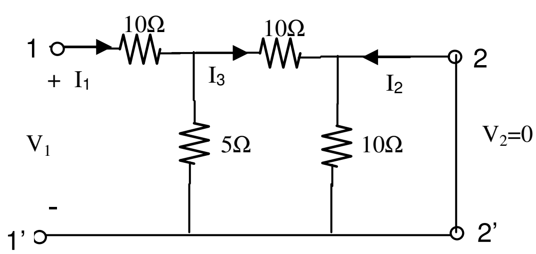


Fig 6.a.2

When the output port is short circuited, $V_2 = 0$, $I_2 = I_3$

$V_1 = 10I_1 + (I_1 - I_3)5 = 15\ I_1 - 5I_2$ ------- Eq.3

And $0 = (I_3 - I_1)\ 5 + 10\ I_3$

Or $5\ I_1 = -15\ I_3$ .. eq.4

$$\therefore D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 3$$

From eq.3 and eq.4

$V_1 = 5\ I_2 - 45\ I_2 = 40\ I_2$

$$\therefore B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 40\ \Omega$$

AD – BC = 7 x 3 – 40 x $\frac{1}{2}$ = 21 – 20 = 1

$\therefore$ A ≠ D

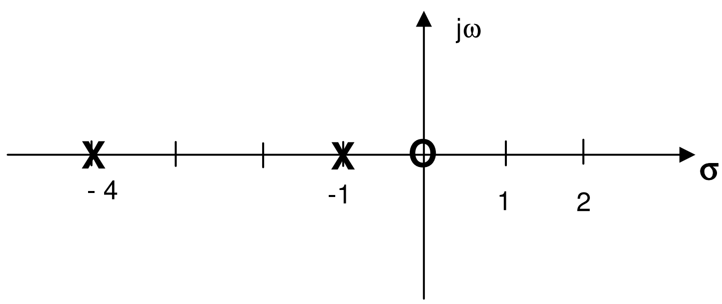
$\therefore$The network is reciprocal and is not symmetric.

Q.75. A transform voltage is given by $V(s) = \frac{3s}{(s+1)(s+4)}$. Plot the pole zero plot in the s-plane and obtain the time domain response. **(8)**

Ans:

The transform voltage is given by $V(s) = \frac{3s}{(s+1)(s+4)}$

From the function it is clear that the function has poles at – 1 and – 4 and a zero at the origin. The plot of poles and zeros in shown below:



(s+1) and (s+4) are factors in the denominators, the time domain response is given by $i(t) = K_1\ e^{-t} + K_2\ e^{-4t}$

To find the constants K_1 and K_2

From the pole zero plot

M_{01} = 1 and $\phi_{01} = 180^0$

Q_{21} = 3 and $\theta_{21} = 0^0$

$$K_1 = F\frac{M_{01}e^{j\phi_{01}}}{Q_{21}e^{j\theta_{21}}} = 3\times\frac{1}{3}\times\frac{e^{j180°}}{e^{j0°}} = e^{j180°} = -1$$

Where ϕ_{01} and ϕ_{02}, θ_{21} and θ_{12} are the angles of the lines joining the given pole to other finite zeros and poles.

Where M_{01} and M_{02} are the distances of the same poles from each of the zeros.

Q_{21} and Q_{12} are the distances of given poles from each of the other finite poles.

Similarly,

M_{02} = 4 and $\phi_{02} = 180^0$

Q_{12} = 3 and $\theta_{12} = 180^0$

$$K_2 = F\frac{M_{02}e^{j\phi_{02}}}{Q_{12}e^{j\theta_{12}}} = 3\times\frac{4\times e^{j180°}}{3\times e^{j180°}} = 3$$

Substituting the values of K_1 and K_2, the time domain response of the current is given by

$i(t) = -e^{-t} + 4e^{-4t}$

Q.76. A 100 mH inductor with $500\,\Omega$ self-resistance is in parallel with a 5nF capacitor. Find the resonant frequency of the combination. Find the impedance at resonance, quality factor of the circuit and the half power bandwidth. **(10)**

Ans:
Given L = 100 mH , R = 500 Ω., C = 5nF
The resonant frequency of the parallel combination is given by

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{500\times10^{-12}} - \left(\frac{500}{100\times10^{-3}}\right)^2}$$

$$= \frac{1}{2\pi}\sqrt{\frac{10^{10}}{5} - (5000)^2}$$

$$= \frac{1}{2\pi}\sqrt{2000\times10^6 - 25\times10^6}$$

$$= \frac{1}{2\pi}\sqrt{1975\times10^6} = 7.07\,KHz$$

Impedance at resonance $= \dfrac{L}{CR} = \dfrac{100\times10^{-3}}{500\times5\times10^{-9}} = \dfrac{1}{25}\times10^6 = 400\,K\Omega$

Q factor $= \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{500}\sqrt{\dfrac{100\times10^{-3}}{5\times10^{-9}}} = \dfrac{2000\sqrt{5}}{500} = 4\sqrt{5}$

Half power bandwidth $B.W = f_2 - f_1 = \dfrac{f_r}{Q} = \dfrac{7.07\times10^3}{4\times\sqrt{5}} = 789.06\,Hz$

Q.77. When the far end of the transmission line is open circuited, the input impedance is $650\angle-12^\circ$ ohms and when the line is short-circuited, the input impedance is $312\angle-8^\circ$ ohms. Find the characteristic impedance of the line. **(4)**

Ans:
Given $Z_{oc} = 650\angle-12^0$ ohms and $Z_{sc} = 312\angle-8^0$ ohms.
The characteristic impedance, Z_o is given by

$$Z_o = \sqrt{Z_{oc}\times Z_{sc}} = \sqrt{650\times312}\angle((-12+-8)/2)$$

$$Z_o = \sqrt{650\times312}\angle-10$$

$$Z_o = 450.33\angle-10\ \Omega$$

Q.78. A lossless line has a characteristic resistance of 50Ω. The line length is $1.185\,\lambda$. The load impedance is $110 + j80\,\Omega$. Find the input impedance. **(4)**

Ans:
Given $R_o = 50\Omega$, $l = 1.185\lambda$, $Z_L = 110 + j80\ \Omega$.

The input impedance is given by, $Z_{in} = Z_o \dfrac{Z_R + jZ_o \tan\beta l}{Z_o + jZ_R \tan\beta l}$

$$\beta l = \frac{2\pi}{\lambda} \times l = \frac{2\pi}{\lambda} \times 1.185\lambda = 7.44\ rad = 426° \quad or \quad 66°$$

$$Z_{in} = 50\left[\frac{(110 + \mathrm{j}80) + j50 \times \tan 66°}{50 + j(110 + \mathrm{j}80) \times \tan 66°}\right]$$

$$Z_{in} = 50 \times \left[\frac{(110 + \mathrm{j}80) + j50 \times 2.25}{50 + j(110 + \mathrm{j}80) \times 2.25}\right]$$

$$Z_{in} = 50 \times \left[\frac{110 + \mathrm{j}80 + j112.5}{50 + j247.5 - 180}\right] = 50 \times \left[\frac{110 + \mathrm{j}192.5}{-130 + j247.5}\right]$$

$$Z_{in} = 50 \times \left[\frac{110 + \mathrm{j}1925}{-130 + j247.5}\right]$$

Q.79. Design an m-derived T section (high pass) filter with a cut off frequency $f_c = 20\text{kHz}$, $f_\infty = 16\text{kHz}$ and a design impedance $R_o = 600\Omega$. **(8)**

Ans:
For m-derived filter

$$m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2} = \sqrt{1 - \left(\frac{16000}{20000}\right)^2} = 0.6$$

For a prototype low pass filter

$$L = \frac{R_o}{4\pi f_c} = \frac{600}{4 \times \pi \times 20000} H = 2.39mH$$

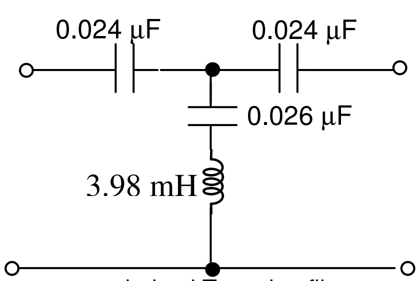
$$C = \frac{1}{4\pi R_o f_c} = \frac{1}{4 \times \pi \times 20000 \times 600} F = 0.007\mu F$$

In the T-section, m-derived high pass filter, the values of the elements are

$$\frac{L}{m} = \frac{2.39}{0.6} = 3.98mH$$

$$\frac{2C}{m} = \frac{2 \times 0.007}{0.6} = 0.024\mu F$$

$$\left(\frac{4m}{1 - m^2}\right)C = \left[\frac{4 \times 0.6}{1 - (0.6)^2}\right] \times 0.007 = .026\mu F$$



m-derived T-section filter

Q.80. A sinusoidal current, I = 100 cos2t is applied to a parallel RL circuit. Given R=5Ω and L=0.1H, find the steady state voltage and its phase angle. **(8)**

Ans:
The driving current is given by

$$i(t) = I\cos 2t = \frac{I}{2}[e^{j\omega t} + e^{-j\omega t}] \qquad \text{--- Eq - 1}$$

Considering current source $Ie^{j\omega t}/2$ and applying kirchoff's current law

$$\frac{1}{L}\int_{-\infty}^{t} V\, dt + \frac{V}{R} = I\frac{e^{j\omega t}}{2} \qquad \text{--- Eq - 2}$$

The steady state voltage is given by v_{ss1} = A $e^{j\omega t}$ where A is the undetermined coefficient.

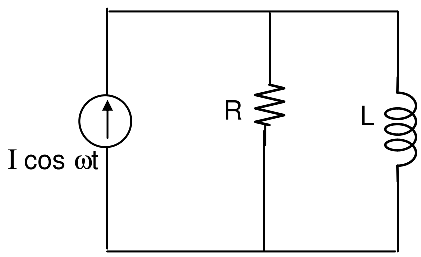


Fig. 2.b

From Eq - 1 and Eq - 2

$$\frac{A}{j\omega L} + \frac{A}{R} = \frac{I}{2}$$

$$A = \frac{I/2}{\frac{1}{R} + \frac{1}{j\omega L}}$$

Considering current source $Ie^{-j\bullet t}/2$ and applying kirchoff's current law

$$\frac{1}{L}\int_{-\infty}^{t} V\, dt + \frac{V}{R} = I\frac{e^{-j\omega t}}{2} \qquad \text{--- Eq - 3}$$

The steady state voltage is given by v_{ss2} = B $e^{-j\omega t}$ where B is the undetermined coefficient.

From Eq -1 and Eq - 3

$$-\frac{B}{j\omega L} + \frac{B}{R} = \frac{I}{2}$$

$$B = \frac{I/2}{\frac{1}{R} - \frac{1}{j\omega L}}$$

On applying superposition principle, the total steady state voltage v_{ss} is the summation of the voltages v_{ss1} and v_{ss2}.

$$\therefore v_{ss} = v_{ss1} + v_{ss2}$$

$$= A\, e^{j\omega t} + B\, e^{-j\omega t}$$

$$v_{ss} = \frac{I}{2}\left[\frac{e^{j\omega t}}{\frac{1}{R}+\frac{1}{\mathrm{j}\omega L}} + \frac{e^{-j\omega t}}{\frac{1}{R}-\frac{1}{\mathrm{j}\omega L}}\right] = \frac{I/2}{\frac{1}{R^2}+\frac{1}{\omega^2 L^2}}\left[\frac{1}{R}\left\{e^{j\omega t}+e^{-j\omega t}\right\}+\frac{1}{\omega L}\left\{e^{j\omega t}-e^{-j\omega t}\right\}\right]$$

$$v_{ss} = \frac{I}{\frac{1}{R^2}+\frac{1}{\omega^2 L^2}}\left[\frac{1}{R}\cos\omega t - \frac{1}{\omega L}\sin\omega t\right]$$

$$v_{ss} = \frac{I}{\sqrt{\frac{1}{R^2}+\frac{1}{\omega^2 L^2}}}\cos\left(\omega t + \tan^{-1}\frac{\omega L}{R}\right) \qquad \text{--- Eq.4}$$

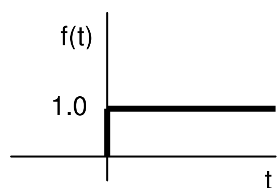
Given R = 5Ω L = 0.1H, I = 100A and ω = 2
The steady state voltage is given by

$$v_{ss} = \frac{100}{\sqrt{\frac{1}{25}+\frac{1}{0.04}}}\cos\left(\omega t + \tan^{-1} 0.04\right)$$

$$v_{ss} = \frac{100}{1/5}\cos(\omega t + 2.29°) = 500\cos(\omega t + 2.29°)$$

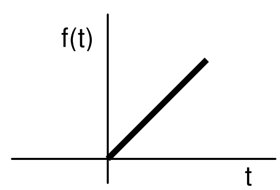
Q.81. Define the unit step, ramp and impulse function. Determine the Laplace transform for these functions. **(6)**

Ans:
The unit step function is defined as
u(t) = 0 t ≤ 0
1 t > 0



The laplace transform is given by

$$F(s) = Lu(t) = \int_0^\infty e^{-st}dt = \left[\frac{-1}{s}e^{-st}\right]_0^\infty = \frac{1}{s}$$



The ramp function is given by
f(t) = t
The laplace transform is given by

$$F(s) = Lf(t) = \int_0^\infty t.e^{-st}dt = \left[\frac{t}{s}e^{-st}\right]_0^\infty + \int_0^\infty \frac{1}{s}.e^{-st}dt$$

$$= 0 + \frac{1}{s}\int_0^\infty e^{-st}dt = \frac{1}{s}.\frac{1}{s} = \frac{1}{s^2}$$

$$F(s) = \frac{1}{s^2}$$

Unit impulse f(t) is defined as

$$\int_0^\infty f(t)dt = 1$$

The unit impulse function is given by

$$f(t) = \lim_{\Delta t \to 0} \left[\frac{u(t) - u(t - \Delta t)}{\Delta t} \right]$$

Which is nothing but the derivative of the unit step function.

$$f(t) = \frac{d}{dt} u(t)$$

f(t) has the value zero for t > 0 and ∝ at t = 0.
Let g(t) = 1 - $e^{-\alpha t}$
g(t) approaches f(t) when α is very large.

$$g'(t) = \frac{d}{dt} g(t) = \alpha e^{-\alpha t}$$

$$\int_0^\infty g'(t)dt = \int_0^\infty \alpha . e^{-\alpha t} dt = 1$$

On applying laplace transform

$$F(s) = Lf(t) = \lim_{\alpha \to \infty} \left[Lg'(t) \right]$$

$$= \lim_{\alpha \to \infty} \left[L\alpha e^{-\alpha t} \right]$$

$$= \lim_{\alpha \to \infty} \left[\frac{\alpha}{s + \alpha} \right] = 1$$

∴F(s) = 1

Q.82. Find the inverse Laplace transform of

$$\mathrm{F(s)} = \frac{7\mathrm{s} + 2}{\mathrm{s}^3 + 3\mathrm{s}^2 + 2\mathrm{s}}$$ **(4)**

Ans:

Let $\frac{7s + 2}{s^3 + 3s^2 + 2s} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)}$

$$= \frac{A(s+1)(s+2) + Bs(s+2) + Cs(s+1)}{s(s+1)(s+2)}$$

$$\Rightarrow A(s+1)(s+2) + Bs(s+2) + Cs(s+1) = 7s + 2$$

When $s = 0, \quad 2A = 2 \quad \Rightarrow A = 1$

When $s = -1, \quad -B = -5 \quad \Rightarrow B = 5$

When $s = -2, \quad 2C = -12 \quad \Rightarrow C = -6$

$$\therefore \frac{7s + 2}{s(s+1)(s+2)} = \frac{1}{s} + \frac{5}{(s+1)} - \frac{6}{(s+2)}$$

On applying inverse Laplace transform

$$L^{-1}\left[\frac{7s+2}{s(s+1)(s+2)}\right] = L^{-1}\left[\frac{1}{s}+\frac{5}{(s+1)}-\frac{6}{(s+2)}\right]$$
$$= \left[1+5e^{-t}-6e^{-2t}\right]u(t)$$

Q.83. Design a symmetrical T section having parameters of $Z_{oc} = 1000\,\Omega$ and $Z_{sc} = 600\,\Omega$. **(6)**

Ans:
Given $Z_{oc} = 1000\Omega$ and $Z_{sc} = 600\Omega$
The network elements of a T section are given by

$$Z_1 = 2\left[Z_{OC} - \sqrt{Z_{OC}(Z_{OC}-Z_{SC})}\right]$$
$$= 2\left[1000 - \sqrt{1000(1000-600)}\right]$$
$$= 2[1000-632.46] = 2\times 367.54$$
$$\Rightarrow \frac{Z_1}{2} = 367.54\ \Omega$$
$$Z_2 = \sqrt{Z_{OC}(Z_{OC}-Z_{SC})}$$
$$= \sqrt{1000(1000-600)}$$
$$\Rightarrow Z_2 = 632.46\ \Omega$$

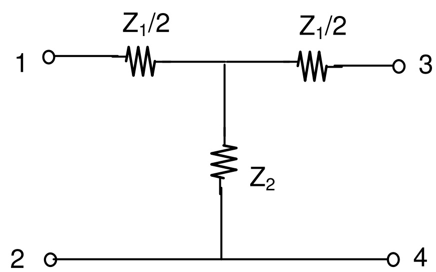


Fig 4.a

Q.84. A transmission line has the following primary constants per Km loop, R=26 Ω, L=16mH, C=0.2μF and G=4μ mho. Find the characteristic impedance and propagation constant at ω=7500 rad/sec. **(4)**

Ans:
$R + j\,\omega L = 26 + j \times 7500 \times 16 \times 10^{-3} = 26 + j \times 120 = 122\ \angle 77°$
$G + j\,\omega C = 4 \times 10^{-6} + j \times 7500 \times 0.2 \times 10^{-6} = 4 \times 10^{-6} + j \times 1.5 \times 10^{-3} = 0.0015\ \angle 89°$

The characteristic impedance is given by, $Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$= \sqrt{\frac{122\angle 77°}{1.5\times 10^{-3}\angle 89°}}$$
$$Z_o = 285\times\frac{1}{2}\angle -12° = 285\angle -6°\ \Omega$$

The Propagation constant is given by, $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$= \sqrt{122\angle 77°(1.5\times 10^{-3}\angle 89°)}$$

$$= 0.427\times\frac{1}{2}\angle 166° = 0.427\angle 83°$$

Q.85. Find out the Z parameters and hence the ABCD parameters of the network shown in Fig 5.a. Check if the network is symmetrical or reciprocal. **(10)**

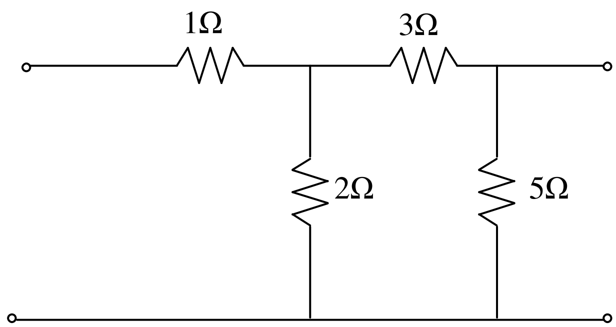


Fig.5.a

Ans:

On open circuiting the terminals 2-2' as in Fig 5.a.2

Applying Kirchoff's voltage law (KVL) for the first loop

$V_1 = I_1 + 2(I_1 - I_3) = 3I_1 - 2I_3$ --- (1)

Applying KVL for the second loop

$0 = 3I_3 + 5I_3 + 2(I_3 - I_1)$

$0 = 10I_3 - 2I_1$

$10I_3 = 2I_1$

$$I_3 = \frac{2I_1}{10} = \frac{I_1}{5} \quad \text{---(2)}$$

From (1) and (2)

$$V_1 = 3I_1 - \frac{2}{5}I_1 = \frac{13}{5}I_1 \quad \text{--- (3)}$$

$$V_2 = 5I_3 = I_1 \quad \text{--- (4)}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 2.6\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{I_1}{I_1} = 1\ \Omega$$

On open circuiting the terminals 1-1' as in Fig 5.a.3

Applying Kirchoff's voltage law (KVL) for the first loop of Fig 5.a.3

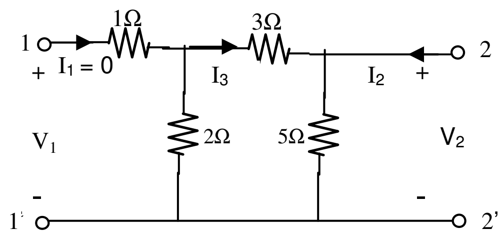


Fig 5.a.3

$V_1 = 2I_3$ --- (5)

Applying KVL for the second loop

$0 = 2I_3 + 3I_3 + 5(I_3 + I_2) = 10I_3 + 5I_2$

$I_2 = -2I_3$ --- (6)

$$I_3 = -\frac{1}{2}I_2$$

From (5) and (6)

$$V_1 = -2\times I_3 = \frac{1}{2}\times 2\times I_2 = I_2 \quad \text{--- (7)}$$

$$V_2 = 5(I_3 + I_2) = 5(-\frac{1}{2}+1)I_2 = \frac{5}{2}I_2 = 2.5I_2 \quad \text{--- (8)}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 2.5\,\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 1\,\Omega$$

$$\therefore A = \frac{Z_{11}}{Z_{21}} = \frac{2.6}{1} = 2.6, \quad B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{2.6\times 2.5 - 1\times 1}{1} = \frac{6.5-1}{1} = 5.5\,\Omega,$$

$$\therefore C = \frac{1}{Z_{21}} = 1\,\text{mhos}, \qquad D = \frac{Z_{22}}{Z_{11}} = \frac{2.5}{2.6} = 0.96$$

$\Rightarrow A \neq D$

$AD - BC = 2.6\times 0.96 - 5.5\times 1 = -3.004$

$\therefore AD - BC \neq 1$

$\therefore$ The circuit is neither reciprocal nor symmetrical.

Q.86. Calculate the driving point impedance Z(s) of the network shown in Fig 5.b. Plot the poles and zeros of the driving point impedance function on the s-plane. **(6)**

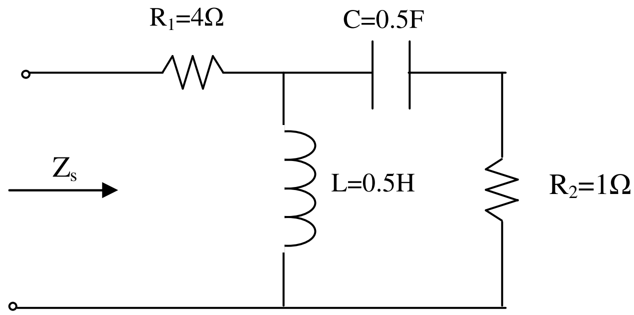


Fig 5.b

Ans:

Using Laplace transformations

$$Z(s) = R_1 + \frac{\left(R_2 + \frac{1}{sC}\right)sL}{R_2 + \frac{1}{sC} + sL}$$

$$= 4 + \frac{\left(1+\frac{2}{s}\right)\frac{s}{2}}{1+\frac{2}{s}+\frac{s}{2}}$$

$$= 4 + \frac{(s+2)s}{2s+s^2+4}$$

$$= \frac{4s^2+8s+16+s^2+2s}{s^2+2s+4} = \frac{5s^2+10s+16}{s^2+2s+4}$$

$$= \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

Where z_1, z_2= $\dfrac{-10 \pm \sqrt{100-320}}{10} = -1 \pm j\sqrt{2.2}$ i.e z_1 = -1 + j $\sqrt{2.2}$, z_2 = -1 – j $\sqrt{2.2}$

Where p_1, p_2 = $\dfrac{-2 \pm \sqrt{4-16}}{2} = -1 \pm j\sqrt{3}$ i.e p_1 = -1 + j $\sqrt{3}$, z_2 = -1 – j $\sqrt{3}$

Q.87. State Thevenin’s theorem. Using Thevenin’s theorem, calculate the current in the branch XY, for the circuit given in Fig.6.a. **(2+6)**

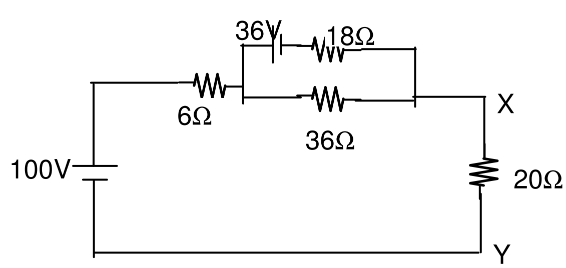


Fig 6.a.1

Ans:

Thevenin’s theorem states that ‘the current I_L which flows from a given network (active) A to another network (passive) B usually referred to as load, is the same as if this network B were connected to an equivalent network whose EMF is the open circuit voltage (V_{oc}) with internally equivalent impedance Z_{TH}. V_{oc} is the open circuit voltage measured across 'a' and 'b' terminals and is the impedance of the network (A) looking back into the terminals 'a' and 'b' with all energy sources on removing R_L the circuit is shown Fig 6.a.2

In the circuit B to X

$-36I - 18I + 36 = 0$

$-54I = -36$

$$\therefore \quad I = \frac{36}{54} = \frac{2}{3} A$$

$$\therefore \quad V_{BC} = \frac{2}{3} \times 36 = 24 \, Volts$$

Since the current flowing through the 6 Ω resistor is zero.

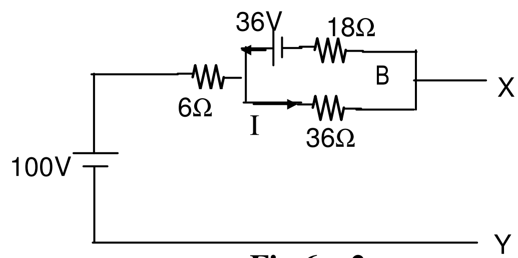


Fig 6.a.2

$$V_{TH} = 100 - 24 = 76 \, Volts.$$

$$R_{TH} = \frac{18 \times 36}{18 + 36} + 6$$

$$= \frac{18 \times 36}{54} + 6 = 12 + 6$$

$$R_{TH} = 18 \, \Omega$$

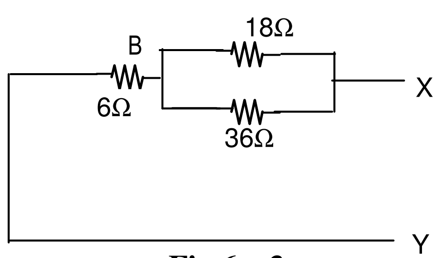


Fig 6.a.3

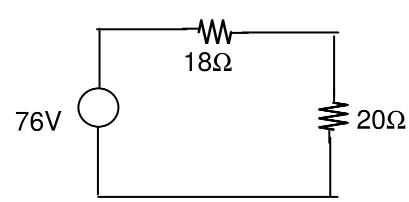


Fig 6.a.4

From Fig 6.a.4

$$I_L = \frac{76}{18 + 20} = \frac{76}{38} = 2 \, A$$

Q.88. Determine the condition for resonance. Find the resonance frequency when a capacitance C is connected in parallel with a coil of inductance L and resistance R. What is impedance of the circuit at resonance? What is the Quality factor of the parallel circuit? **(8)**

Ans:
Consider an anti-resonant RLC circuit as shown in Fig. 7.a.i
When the capacitor is perfect and there is no leakage and dielectric loss.
i.e. $R_C = 0$ and let $R_L = R$ as shown in Fig 7.a.ii

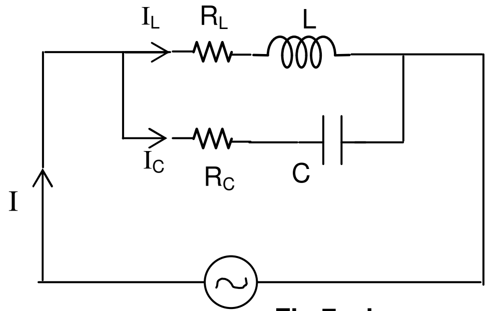


Fig 7.a.i

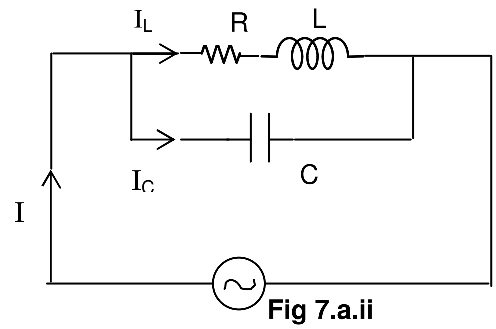


Fig 7.a.ii

The admittance

$$Y_L = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$Y_C = j\omega C$$

$$Y = Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

At resonance, the susceptance is zero.

$$\therefore \omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$$

$$\Rightarrow \omega_0 C = \frac{\omega_0 L}{R^2 + \omega_0^2 L^2}$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\therefore \omega_0^2 = \frac{-R^2}{L^2} + \frac{1}{LC}$$

$$\therefore \omega_0 = \sqrt{\frac{-R^2}{L^2} + \frac{1}{LC}} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R is negligible

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The admittance at resonance is

$$Y_o = \frac{R}{R^2 + \omega_o^2 L^2} = R \times \frac{C}{L} = \frac{RC}{L}$$

Since, $R^2 + \omega_0{}^2 L^2 = \frac{L}{C}$

The impedance at resonance is $Z_o = \frac{L}{RC}$

The quality factor, Q of the circuit is given by

$$Q = \frac{\omega_o L}{R} = \frac{L}{R} \times \frac{1}{\sqrt{LC}} = \frac{1}{R} \times \sqrt{\frac{L}{C}}$$

Q.89. A coil having a resistance of 20Ω and inductive reactance of 31.4Ω at 50Hz is connected in series with capacitor of capacitance of 10 mF. Calculate

i. The value of resonance frequency.

ii. The Q factor of the circuit. **(8)**

Ans:

Given 2µf L = 31.4 and f = 50Hz

$$\therefore L = \frac{31.4}{2 \times \pi \times 50} = 0.1H$$

At resonance, $2\pi \mathrm{f_o} L = \frac{1}{2\pi \mathrm{f_o} C}$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 10 \times 10^{-3}}} = \frac{50}{\pi \times 3.16} = 5.04\ Hz$$

The value of resonance frequency is 5.04 Hz.

$$\Rightarrow \omega_o = 2\pi f_o = \frac{100}{3.16} = 31.65\ rad/\mathrm{sec}$$

$$\therefore Q = \frac{\omega_o \times L}{R} = \frac{31.65 \times 0.1}{20} = 0.158$$

The Q factor of the circuit is 0.158

Q.90. A transmission line with characteristic impedance of 500Ω is terminated by a purely resistive load. It is found by measurement that the minimum value of voltage upon it is 5µV and maximum voltage is 7.55µV. What is the value of load resistance? **(8)**

Ans:

Given V_{max} = 7.55µV, V_{min} = 5µV

$$\mathrm{S} = \frac{V_{\max}}{V_{\min}} = \frac{7.55}{5} = 1.51$$

The standing wave ratio in terms reflection coefficient is given by

$$\mathrm{S} = \frac{1+k}{1-k} = 1.51$$

$$\Rightarrow 1.51 - 1.51k = 1 + k$$

$$k = \frac{0.51}{2.51} = 0.2$$

The reflection coefficient k is given by

$$k = -\frac{Z_L - Z_O}{Z_L + Z_O} = -\frac{Z_L - 500}{Z_L + 500} = \frac{1}{5}$$

$$\Rightarrow -5Z_L + 2500 = Z_L + 500$$

$$\Rightarrow -6Z_L = -2000$$

$$\therefore Z_L = 333.3\ \Omega$$

Q.91. Design a m-derived low pass filter (T and π section) having a design resistance of Ro=500Ω and the cut off frequency (f_c) of 1500 Hz and an infinite attenuation frequency (f_c) of 2000 Hz. **(8)**

Ans:

For m-derived filter

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1-\left(\frac{1500}{2000}\right)^2} = 0.661$$

For a prototype low pass filter

$$L = \frac{R_o}{\pi f_c} = \frac{500}{\pi \times 1500} H = 106.103 mH$$

$$C = \frac{1}{\pi R_o f_c} = \frac{1}{\pi \times 1500 \times 500} F = 0.424 \mu F$$

In the T-section, m-derived low pass filter, the values of the elements are

$$\frac{mL}{2} = \frac{.661 \times 106.103}{2} = 35.067 mH$$

$$mC = 0.424 \times 0.661 = 0.280 \mu F$$

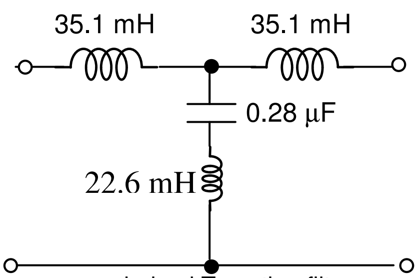
$$\left(\frac{1-m^2}{4m}\right)L = \left[\frac{1-(.661)^2}{4 \times 0.661}\right] \times 106.103 = 22.596 mH$$

In the π-section, m-derived low pass filter, the values of the elements are

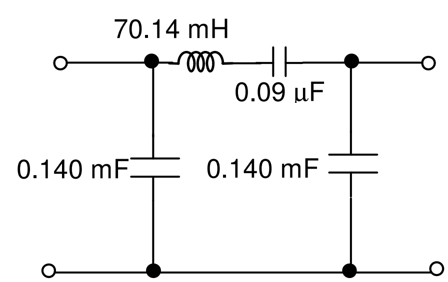
$$\frac{mC}{2} = \frac{.661 \times 0.424}{2} = 0.140 \mu F$$

$$mL = 106.103 \times 0.661 = 70.134 mH$$

$$\left(\frac{1-m^2}{4m}\right)C = \left[\frac{1-(.661)^2}{4 \times 0.661}\right] \times 0.424 = 0.09 \mu F$$



m-derived T-section filter



m-derived Π-section filter

Q. 92. A capacitance of $4\mu F$ is charged to potential difference of 400 V and then connected in parallel with an uncharged capacitor of $2\mu F$ capacitance. Calculate the potential difference across the parallel capacitors. **(8)**

Ans:
Given that, $C_1 = 4\mu F$, $V_1 = 400V$
The charge on the capacitor C_1 is $Q_1 = C_1V_1 = 4 \times 10^{-6} \times 400 = 1600 \times 10^{-6}$
$C_2 = 2\mu F$, $C = C_1 + C_2 = 4 + 2 = 6\mu F$
Charge on the capacitor C_1 will be shared by C_2
$\therefore Q = Q_2 = 16 \times 10^{-4}$ C
$\therefore$Potential difference across the parallel capacitor

$$V_2 = \frac{Q}{C} = \frac{16 \times 10^{-4}}{6 \times 10^{-6}} = 266.7 \text{ V}$$

Q.93. Voltage $v(t) = V_0 \cos(\omega t + \phi)$ is applied to a series circuit containing resistor R, inductor L and capacitor C. Obtain expression for the steady state response.

Ans:
Application of the Kirchoff's voltage law t o the circuit gives the transform equation

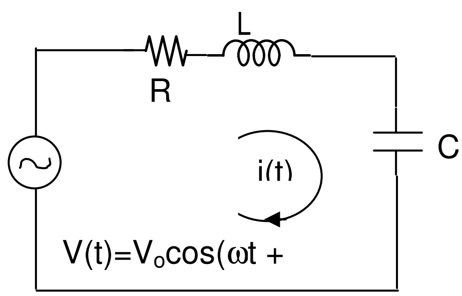


Fig.6.c

$$I(s).Z(s) = V(s) \qquad \text{----- (1)}$$

$$I(s).\left[R + Ls + \frac{1}{Cs}\right] = V(s) \qquad \text{----- (2)}$$

In sinusoidal steady state, (2) in the phasor form can be written as

$$I.\left[R + j\omega L + \frac{1}{j\omega C}\right] = V_o e^{j\phi} \qquad \text{----- (3)}$$

Hence **I =** $Ie^{j\phi 1} = \dfrac{V_o e^{j\phi}}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$ ----- (4)

Hence magnitude **I =** $\dfrac{V_o}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$ ---- (5)

And $\theta_1 = \phi - \tan^{-1}\left(\dfrac{\omega L - \dfrac{1}{\omega C}}{R}\right)$ ----- (6)

Hence the expression for the time domain response is,

$i(t) = I\cos(\omega t + \theta_1)$ ----- (7)

Where I and θ_1 are given by equations (5) and (6) respectively.

Q.94. In a series RLC circuit, $\mathrm{R} = 5\Omega, \mathrm{L} = 1\mathrm{H}$ and C=0.25F and the input driving voltage is $10\mathrm{e}^{-3\mathrm{t}}$. Assume that there is zero current through the inductor (L) and zero charge across capacitor (C) before closing the switch. Find steady state current flowing through circuit **(8)**

Ans:

RLC differential equation $\mathbf{L\dfrac{di}{dt} + Ri(t) + \dfrac{1}{C}\int_{-\infty}^{t} idt = V(t)}$

On Applying Kirchoff's voltage law,

$\dfrac{di}{dt} + 5i + \dfrac{1}{0.25}\int_{-\infty}^{0} i\,dt + \dfrac{1}{0.25}\int_{0}^{t} i\,dt = 10e^{-3t}$ -------- Eq.1

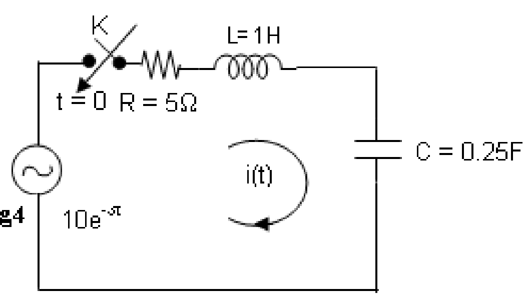


Fig4

Applying Laplace transformation to Eq. 1

$[s.I(s) - i(0+)] + 5I(s) + \dfrac{1}{0.25}\times\dfrac{q(0+)}{s} + \dfrac{1}{0.25}\times\dfrac{I(s)}{s} = \dfrac{10}{s+3}$ --------- Eq.2

At time t = 0+, current i(0+) must be the same as at time t = 0 – due to the presence of the inductor L.

$\therefore$ i(0+) = 0

At t = 0+, charge q(0+) across capacitor must be the same as at time t = 0 –

$\therefore$ q(0+) = 0

Substituting the initial conditions in *Eq. 2*

$$I(s)[s+5+\frac{1}{0.25s}] = \frac{10}{(s+3)}$$

$$I(s)[s^2+5s+4] = \frac{10s}{(s+3)}$$

$$I(s) = \frac{10s}{(s+3)(s^2+5s+4)} = \frac{10s}{(s+3)(s+1)(s+4)}$$

Let $\frac{10s}{(s+1)(s+3)(s+4)} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+3)} + \frac{K_3}{(s+4)}$

$$= \frac{K_1(s+3)(s+4) + K_2(s+1)(s+4) + K_3(s+1)(s+3)}{(s+1)(s+3)(s+4)}$$

$$K_1(s+3)(s+4) + K_2(s+1)(s+4) + K_3(s+1)(s+3) = 10s$$

When s = -1, $6K_1 = -10 \Rightarrow K_1 = \frac{1}{2}$

When s = - 3, $-2K_2 = -30 \Rightarrow K_2 = 15$

When s = - 4, $3K_1 = -40 \Rightarrow K_3 = \frac{-40}{3}$

$$\therefore \frac{10s}{(s+1)(s+3)(s+4)} = \frac{-5}{2(s+1)} + \frac{15}{(s+3)} + \frac{-40}{3(s+4)}$$

On inverse laplace transformation

$$L^{-1}\left[\frac{10s}{(s+1)(s+3)(s+4)}\right] = L^{-1}\left[\frac{-5}{2(s+1)} + \frac{15}{(s+3)} + \frac{-40}{3(s+4)}\right]$$

$$= -2.5\,e^{-t} + 15\,e^{-3t} - 13.33\,e^{-4t}$$

The current i(t) is given by

$$i(t) = \left[-2.5\,e^{-t} + 15\,e^{-3t} - 13.33\,e^{-4t}\right]u(t)$$

Q.95. Define image impedances and iterative impedances of an asymmetric two-port network. For the two port network, calculate the open circuit and short circuit impedances and hence the image impedances. **(4+4)**

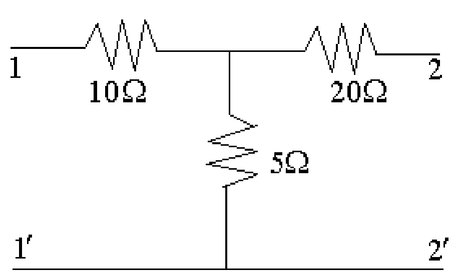
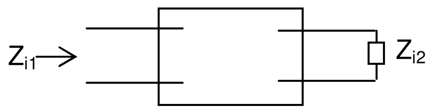
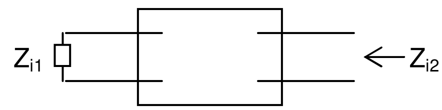


Fig.3

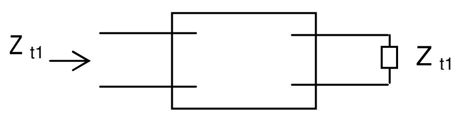
Ans:

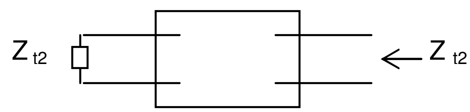
Image impedance is that impedance, which when connected across the appropriate pair of terminals of the network, the other is presented by the other pair of terminals. If the driving point impedance at the input port with impedance Z_{i2} is Z_{i1} and if the driving point impedance at the output port with impedance Z_{i1} is Z_{i2}, Then Z_{i1} and Z_{i2} are the image impedances of the two-port network.





Iterative impedance is that impedance, which when connected across the appropriate pair of terminals of the network, the same is presented by the other pair of terminals. If the driving point impedance at the input port with impedance Z_{t1} connected at the output is Z_{t1} and the driving point impedance at the output port with impedance Z_{t2} connected at the input, is Z_{t2}, Then Z_{t1} and Z_{t2} are the iterative impedances of the two-port network.





For the circuit shown in Fig 5.b

$Z_{oc1} = Z_1 + Z_3 = 10 + 5 = 15\Omega$

$Z_{oc2} = Z_2 + Z_3 = 20 + 5 = 25\Omega$

$$Z_{sc1} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_2 + Z_3}$$

$$Z_{sc1} = \frac{10\times20 + 10\times5 + 20\times5}{20+5} = \frac{350}{15} = 14\,\Omega$$

$$Z_{sc2} = Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3} = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_1 + Z_3}$$

$$Z_{sc2} = \frac{10\times20 + 10\times5 + 20\times5}{10+5} = \frac{350}{15} = 23.33\,\Omega$$

The image impedances in terms of open circuit and short circuit impedances are given by the relations

$$Z_{i1} = \sqrt{Z_{oc1}\times Z_{sc1}} = \sqrt{15\times14} = 14.49\,\Omega$$

$$Z_{i2} = \sqrt{Z_{oc2}\times Z_{sc2}} = \sqrt{25\times23.33} = 24.15\,\Omega$$

Q.96. Find the Laplace transforms of

(i) $f(t) = e^{-\theta t}\cos\omega t$

(ii) $f(t) = u(t-a)$ (Shifted unit step function) **(8)**

Ans:
(i)

$$F(s) = L(e^{-\theta t}\cos(\omega t)) = \int_0^\infty e^{-\theta t}\cos(\omega t).e^{-st}dt$$

$$= \int_0^\infty \cos(\omega t).e^{-(s+\theta)t}dt$$

$$= \left[\frac{-(s+\theta)\cos(\omega t).e^{-(s+\theta)t} + \omega\sin(\omega t).e^{-(s+\theta)t}}{(s+\theta)^2+\omega^2}\right]_0^\infty$$

$$= \frac{s+\theta}{(s+\theta)^2+\omega^2}$$

(ii) f(t) = u(t-a) (shifted unit step function)

$$F(s) = L(u(t-a)) = \int_0^\infty 1.e^{-st}dt$$

$$= \left.\frac{-e^{-st}}{s}\right]_0^\infty = e^{-as}\left(\frac{1}{s}\right)$$

Q.97. Obtain the Y-parameters of the network shown in Fig.1. **(6)**

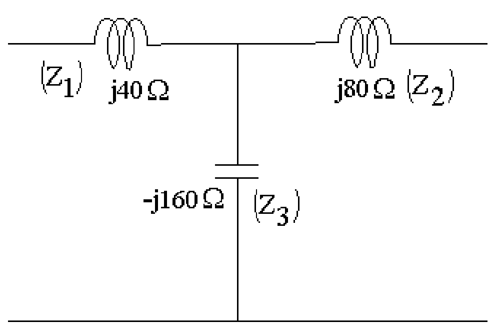


Fig.1

Ans:
On short circuiting the output terminals, $V_2 = 0$

$$V_1 = I_1\left[j40 + \frac{-j160 \times j80}{-j160 + j80} \right] = I_1[j40 + j160] = I_1 j200$$

$$\Rightarrow Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{j200} \quad \text{mhos}$$

$$I_2 = -I_1\left[\frac{-j160}{-j160 + j80} \right] = -I_1\left[\frac{-j160}{-80} \right] = -2I_1$$

$$\Rightarrow Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-2I_1}{V_1} = -2 \times \frac{1}{j200} = -\frac{1}{j100} \quad \text{mhos}$$

On short circuiting the input terminals, $V_1 = 0$

$$V_2 = I_2\left[j80 + \frac{-j160 \times j40}{-j160 + j40} \right] = I_2[j80 + j160/3] = I_2(j400/3)$$

$$\Rightarrow Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{j400} \quad \text{mhos}$$

$$I_1 = -I_2\left[\frac{-j160}{-j160 + j40} \right] = -I_2\left[\frac{-j160}{-j120} \right] = -\frac{4}{3} I_2$$

$$V_2 = \left(j\frac{400}{3} \right)\left(-\frac{3}{4} \right) I_1 = -j100 I_1$$

$$\Rightarrow Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_2=0} = -\frac{1}{j100} \quad \text{mhos}$$

Q.98. Calculate the value of the load resistance R_L for maximum power transfer in the circuit shown in Fig.5. Calculate the value of maximum power. **(8)**

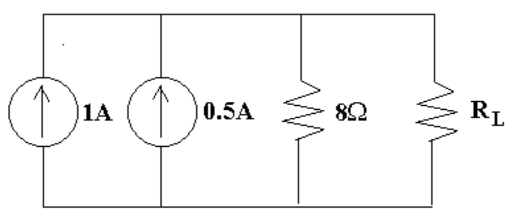


Fig. 5

Ans:
The two current sources are parallel and the supply of current is in the same direction and hence can be replaced by a single current source as in Fig.6.b.2. On removing R_L, the resultant circuit is similar to a Norton's circuit as in Fig 6.b.3.

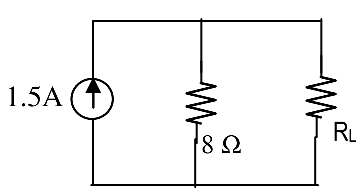


Fig.6.b.2.

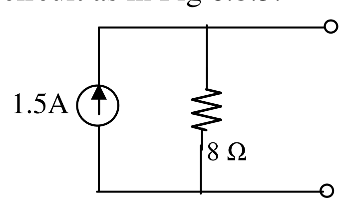


Fig.6.b.3.

Converting the Norton's circuit to equivalent Thevenin's circuit as in Fig 6.b.4,

$E_{Th} = 8 \times 1.5 = 12$ Volts.
$R_{Nort} = R_i = 8\,\Omega$
Therefore for maximum power transfer,
$R_L = R_i = 8\,\Omega$

Maximum power $= \dfrac{(E_{TH})^2}{4 \times R_i} = \dfrac{(12)^2}{4 \times 8} = 4.5$ Watts

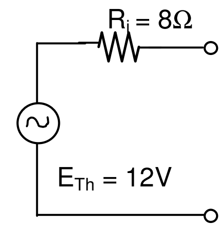


Fig.6.b.4

Q.99. State and prove final value theorem. Find the final value of the function where the Laplace transform is $I(S) = \dfrac{S+9}{S(S+5)}$ **(8)**

Ans:

Final value theorem states that if function f(t) and its derivative are laplace transformable, then the final value f(∞) of the function f(t) is

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

On taking the laplace transform of the derivative and limit is taken as s → 0

$$\lim_{s \to 0} L\left[\frac{df(t)}{dt}\right] = \lim_{s \to 0} \int_0^\infty \frac{df(t)}{dt} e^{-st} dt = \lim_{s \to 0} \{sF(s) - f(0)\}$$

But $\displaystyle \lim_{s \to 0} \int_0^\infty \frac{df(t)}{dt} e^{-st} dt = \int_0^\infty df(t) = f(\infty) - f(0)$

However, f(0) being a constant

$$f(\infty) - f(0) = -f(0) + \lim_{s \to 0} \{sF(s)\}$$

$$\therefore f(\infty) = \lim_{s \to 0} \{sF(s)\}$$

$$\lim_{s \to 0} \{sF(s)\} = \lim_{t \to \infty} f(t)$$

Applying final value theorem, we get

$$i(\infty) = \lim_{s \to \infty} sI(s) = \lim_{s \to 0} s.\frac{s+9}{s(s+5)} = \lim_{s \to 0}\left[\frac{s+9}{s+5}\right] = \frac{9}{5} = 1.8$$

Q.100. Design a symmetrical bridged T-attenuator to provide attenuation of 60dB and to work into a line of characteristic impedance $600\,\Omega$. **(8)**

Ans:

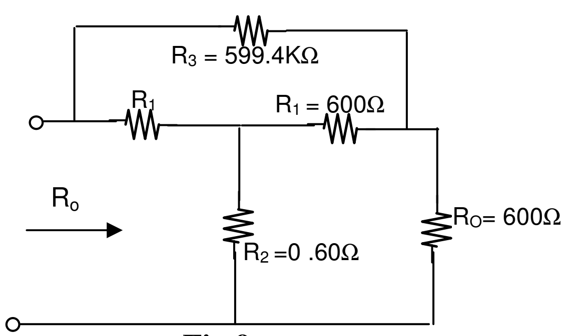


Fig 9.a

Given $R_0 = 600\Omega$

$\therefore R_1 = R_0 = 600\Omega$

Compute N

$20 \log_e N$ = Attenuation in dB

N = 100

$$R_2 = \frac{R_o}{N-1} = \frac{600}{999} = 0.601\Omega$$

$$R_3 = R_o(N-1) = 600 \times 999 = 599400\ \Omega$$

Q.101. Design a Constant K Band Pass filter T-section having cut-off frequencies 2 kHz & 5kHz and a normal impedance of $600\,\Omega$. Draw the configuration of the filter. **(8)**

Ans:

We know that

$$L_1 = \frac{R_o}{\pi(f_2 - f_1)} = \frac{600}{\pi(5000-2000)} = 63.68\ mH$$

$$\Rightarrow \frac{L_1}{2} = 31.84\ mH$$

$$C_1 = \frac{(f_2 - f_1)}{4\pi R_o f_2 f_1} = \frac{(5000-2000)}{4\pi \times 600 \times 5000 \times 2000} = 0.0381\ \mu F$$

$$\Rightarrow 2C_1 = 0.0762\ \mu F$$

$$L_2 = \frac{R_o(f_2 - f_1)}{4\pi f_2 f_1} = \frac{600(5000-2000)}{4\pi \times 5000 \times 2000} = 14.33\ mH$$

$$C_2 = \frac{1}{\pi R_o(f_2 - f_1)} = \frac{1}{\pi \times 600 \times (5000-2000)} = 0.1769\ \mu F$$

The configuration of the filter is given in Fig 9.b.

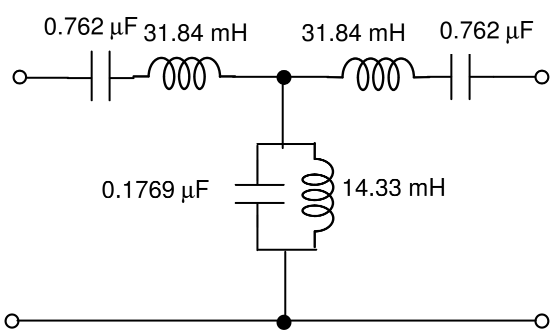


Fig 9.b

Q.102. Calculate the value of R_L which will be drawing maximum power from the circuit of Fig.-1. Also find the maximum power. **(8)**

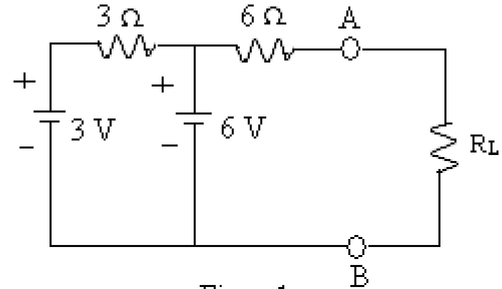


Fig. - 1

Ans:

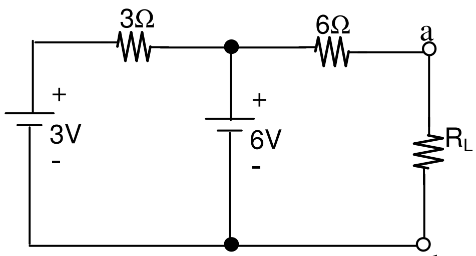


Fig 2.b.1

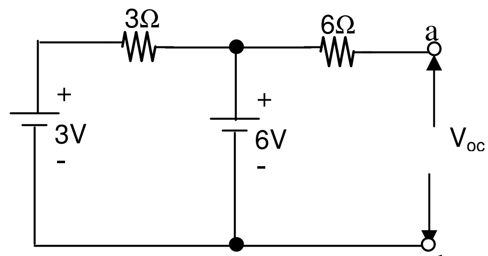


Fig 2.b.2

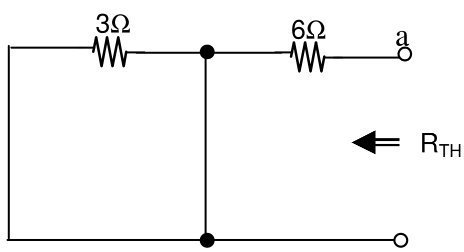


Fig 2.b.3

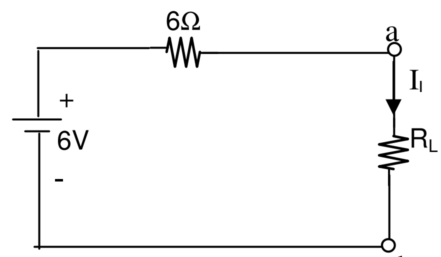


Fig 2.b.4

When the terminals a-b are open circuited as shown in Fig 2.b.2, the open circuit voltage is given by

$V_{oc} = 6\,V$

When the terminals a-b are open circuited and the voltage sources are shorted as shown in Fig 2.b.3, the Thevenin's resistance is given by

$R_{TH} = 6\,\Omega$

The Thevenin's equivalent source is associated with $V_{oc} = 6\,V$ battery with a series resistance $R_{TH} = 6\,\Omega$ to the load resistance as shown in the Fig 2.b.4.

The value load resistance R_L is equal to the value of R_{TH} because R_L draws maximum power from the source.

$\therefore R_L = R_{TH} = 6\,\Omega$

Hence the maximum power $(P_{\max}) = \dfrac{E^2}{4R_L}$

$$\Rightarrow P_{\max} = \frac{(6)^2}{4\times 6} = \frac{36}{24} = 1.5\,\text{Watts}$$

PART – III

DESCRIPTIVES

Q.1. Give the applications of Millman's theorem. **(7)**

Ans:
Applications of Millman's theorem:

- This theorem enables us to combine a number of voltage (current) sources to a single voltage (current) source.
- Any complicated network can be reduced to a simple one by using the Millman's theorem.
- It can be used to determine the load current in a network of generators and impedances with two output terminals.

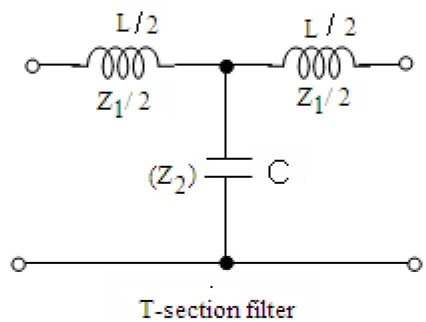
Q.2. Design a prototype low pass filter (L.P.F.), assuming cut off frequency ω_c. **(7)**

Ans:
Consider a constant – K filter, in which the series and shunt impedances, Z_1 and Z_2 are connected by the relation

$Z_1 Z_2 = R_0^2$

Where R_0 is a real constant independent of frequency. R_0 is often termed as design impedance or nominal impedance of the constant – K filter.

Let $Z_1 = j\omega L$ and $Z_2 = 1/j\omega C$



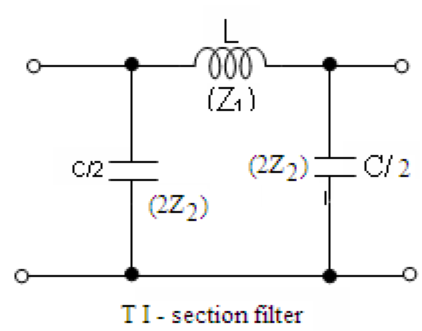
$$Z_1 Z_2 = j\omega L \times \frac{1}{j\omega C} = \frac{L}{C} = R_0^2$$

$$\therefore R_0 = \sqrt{\frac{L}{C}} ---------(eq-3a.1)$$

At cut off-frequency f_c

$$\frac{\omega_c^2 LC}{4} = 1$$

$$fc = \frac{1}{\pi\sqrt{LC}} ---------(eq-3a.2)$$



Given the values of R_0 and ω_c, using the eq 3a.1 and 3a.2 the values of network elements L and C are given by the equations

$$L = \frac{R_o}{\pi f_c} \qquad C = \frac{1}{\pi R_o f_c}$$

Q.3. State advantages of m-derived networks in case of filters. **(7)**

Ans:
Advantages of m - derived filters:

(i) M - derived filters have a sharper cut-off characteristic with steeper rise at f_c (cut-off frequency). The slope of the rise is adjustable by fixing the distance between f_c and f.
(ii) Characteristic impedance (Z_0) of the filter is uniform within the pass band when m - derived half sections, having m = 0.6 are connected at the ends.
(iii) M - Derived filters are used to construct "composite filters" to have any desired attenuation/frequency characteristics.

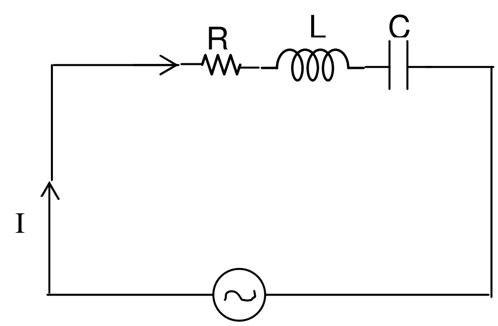
Q.4. For a series R – L – C circuit in resonance, derive values of 'Resonant Frequency', 'Q' of the circuit, current and impedance values at resonance. Give the significance of Q. Why is it called Quality Factor? **(14)**

Ans:
The impedance of the series RLC circuit is given by

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L - \frac{j}{\omega C}$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right) = R + jX$$

The circuit is at resonance when the imaginary part is zero,
i.e. at $\omega = \omega_0$, X = 0
$\therefore$ To find the condition for resonance X = 0.

$$\Rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0$$
$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$
$$\Rightarrow \omega_0{}^2 = \frac{1}{LC}$$
$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \, radians/\sec$$
$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} \, Hz$$



The current at any instant in a series RLC circuit is given by

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At resonance, X = 0
$Z_0 = R$
$I_0 = \frac{V}{R}$ The Q-factor of an RLC series resonant circuit is given as the voltage magnification that the circuit produces at resonance.

$$Voltage\ magnification = Q\ factor = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{1}{2\pi f_0 CR}$$

$$Q\ factor = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\Rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$Bandwidth = f_2 - f_1 = \frac{f_0}{Q}$$

The ability to discriminate the different frequencies is called the Q factor of the circuit. The quality factor of the circuit determines the overall steepness of the response curve. Higher the value of Q of a series resonant circuit, the smaller is the bandwidth and greater is the ability to select or reject a particular narrow band of frequencies.

Q.5. Write short notes on:

(i) Compensation Theorem.
(ii) Stub matching.
(iii) Star delta conversion. **(14)**

Ans:

(i) **Compensation Theorem:** The theorem may be stated as "Any impedance linear or nonlinear, may be replaced by a voltage source of zero internal impedance and voltage source equal to the instantaneous potential difference produced across the replaced impedance by the current flowing through it".

Proof:

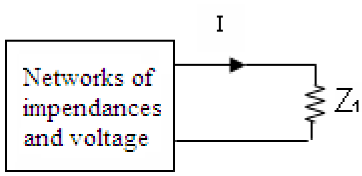


fig 5.ii.(a)

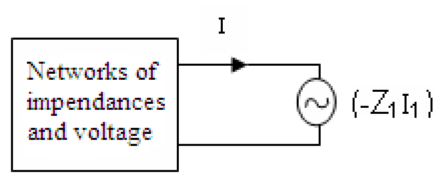


fig 5.ii.(b)

Consider a network of impedance and voltage source, together with the particular impedance Z_1, that is to be replaced, considered as the load.

By Kirchoff's law to fig 5.ii.(b),

$\Sigma I_1 Z_1 + Z\,I = \Sigma V_1$.

Where the summation extends over a number of unspecified impedances in the mesh shown in fig 5.ii.(b).

By Kirchoff's law to fig 5.ii.(a), the equation is the same as for fig 5.ii.(b) except the equation for the right hand side mesh.

$\Sigma Z_1 I_1 = -IZ_1 + \Sigma V_1$.

$\therefore$ All the equations are identical for the two networks, and so are the currents and voltages through out the two networks, that is networks are equivalent.

(ii) Stub matching: When there are no reflected waves, the energy is transmitted efficiently along the transmission line. This occurs only when the terminating impedance is equal to the characteristic impedance of the line, which does not exist practically. Therefore, impedance matching is required. If the load impedance is complex, one of the ways of matching is to tune out the reactance and then match it to a quarter wave transformer. The input impedance of open or short circuited lossless line is purely reactive. Such a section is connected across the line at a convenient point and cancels the reactive part of the impedance at this point looking towards the load. Such sections are called impedance matching stubs. The stubs can be of any length but usually it is kept within quarter wavelength so that the stub is practically lossless at high frequencies. A short circuited stub of length less than $\lambda/4$ offers inductive reactance at the input while an open circuited stub of length less than $\lambda/4$ offers capacitive reactance at the input. The advantages of stub matching are:

- Length of the line remains unaltered.
- Characteristic impedance of the line remains constant.
- At higher frequencies, the stub can be made adjustable to suit variety of loads and to operate over a wide range of frequencies.

(iii) **Star delta conversion**: At any one frequency, a star network can be interchanged to a delta network and vice-versa, provided certain relations are maintained.

Let Z_1, Z_2, Z_3 be the three elements of the star network and Z_A, Z_B, Z_C be the three elements of the delta network as shown in Fig 5.iii.a. and Fig 5.iii.b

The impedance between the terminal 1 and terminal 3 is

$$Z_1 + Z_2 = \frac{(Z_B + Z_C)Z_A}{Z_B + Z_C + Z_A} \qquad \text{......... (1)}$$

The impedance between the terminal 3 and terminal 4 is

$$Z_2 + Z_3 = \frac{(Z_A + Z_B)Z_C}{Z_B + Z_C + Z_A} \qquad \text{.........(2)}$$

The impedance between the terminal 1 and terminal 2 is

$$Z_1 + Z_3 = \frac{(Z_A + Z_C)Z_B}{Z_B + Z_C + Z_A} \qquad \text{..........(3)}$$

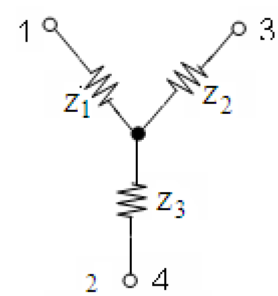


Fig 5.iii.a

Adding eq.1, eq.2 and subtracting eq.3

$$Z_2 = \frac{Z_A Z_C}{Z_B + Z_C + Z_A} \qquad \text{......... (4)}$$

Adding eq.2, eq.3 and subtracting eq.1

$$Z_3 = \frac{Z_B Z_C}{Z_B + Z_C + Z_A} \qquad \text{......... (5)}$$

Adding eq.3, eq.1 and subtracting eq.2

$$Z_1 = \frac{Z_B Z_A}{Z_B + Z_C + Z_A} \qquad \text{......... (6)}$$

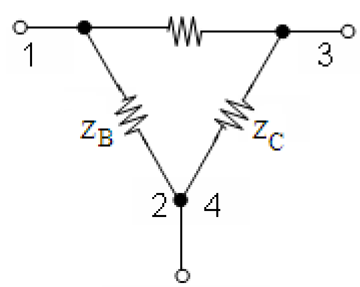


Fig 5.iii.b

Consider $Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = \Sigma Z_1Z_2$

From eq.4, eq.5, eq.6 we get

$$\sum Z_1 Z_2 = \frac{Z_B^{\,2} Z_A Z_C + Z_B Z_A^{\,2} Z_C + Z_B Z_A Z_C^{\,2}}{(Z_B + Z_C + Z_A)^2}$$

$$\sum Z_1 Z_2 = \frac{Z_B Z_A Z_C (Z_B + Z_C + Z_A)}{(Z_B + Z_C + Z_A)^2}$$

$$\sum Z_1 Z_2 = \frac{Z_A Z_B Z_C}{Z_B + Z_C + Z_A} = \frac{Z_A Z_B Z_C}{\sum Z_A}$$

From eq .6

$$Z_1 = \frac{Z_B Z_A}{\sum Z_A} \qquad \therefore Z_C = \frac{\sum Z_1 Z_2}{Z_1}$$

$$\therefore Z_B = \frac{\sum Z_1 Z_2}{Z_2} \qquad \therefore Z_A = \frac{\sum Z_1 Z_2}{Z_3}$$

Q.6. What is an Attenuator? Classify and state its applications. **(7)**

Ans:

An attenuator is a four terminal resistive network connected between the source and load to provide a desired attenuation of the signal. An attenuator can be either symmetrical or asymmetrical in form. It also can be either a fixed type or a variable type. A fixed attenuator is known as pad.

Applications of Attenuators:

(i) Resistive attenuators are used as volume controls in broadcasting stations.

(ii) Variable attenuators are used in laboratories, when it is necessary to obtain small value of voltage or current for testing purposes.

(iii) Resistive attenuators can also be used for matching between circuits of different resistive impedances.

Q.7. What is Line Loading? Why is it required? State methods of loading a transmission line **(7)**

Ans:

The transmission properties of the line are improved by satisfying the condition

$$\frac{L}{R} = \frac{C}{G}$$

Where L is the inductance of the line, R is the resistance, C is the capacitance and G is the capacitance of the line per unit length. The above condition is satisfied either by increasing L or decreasing C. C cannot be reduced since it depends on the construction. The process of increasing the value of L to satisfy the condition

$$\frac{L}{R} = \frac{C}{G}$$

so as to reduce attenuation and distortions of the line is known as “loading of the line”. It is done in two ways. (i) Continuous loading (ii) Lumped loading.

- **Continuous loading:** Continuous loading is done by introducing the distributed inductance throughout the length of the line. Here one type of iron or some other material as mu-metal is wound around the conductor to be loaded thus increasing the permeability of the surrounding medium. Here the attenuation increases uniformly with increase in frequency. It is used in submarine cables. This type of loading is costly.
- **Lumped loading:** Lumped loading is done by introducing the lumped inductances in series with the line at suitable intervals. A lumped loaded line

behaves as a low pass filter. The lumped loading is usually provided in open wire lines and telephone cables. The a.c resistance of the loading coil varies with frequency due to Hysteresis and eddy current losses and hence a transmission line is never free from distortions.

Q.8. Define selectivity and Q of a series RLC circuit. Obtain the relation between the bandwidth, the quality factor and selectivity of a series RLC circuit. **(8)**

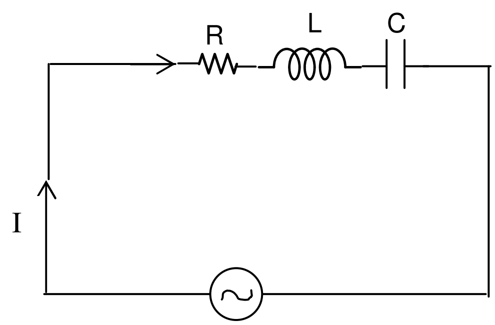
Ans:
The impedance of the series RLC circuit is given by

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L - \frac{j}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right) = R + jX$$

The circuit is at resonance when the imaginary part is zero,
i.e. at $\omega = \omega_0$, X = 0
$\therefore$To find the condition for resonance X = 0.

$$\Rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0$$



$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\Rightarrow \omega_0^{\ 2} = \frac{1}{LC}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} radians/\sec$$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} Hz$$

The Q-factor of an RLC series resonant circuit is given as the voltage magnification that the circuit produces at resonance.

Voltage magnification = Q factor$= \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{1}{2\pi f_0 CR}$

Q factor $= \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$\Rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0$$

Bandwidth = $f_2 - f_1 = \frac{f_0}{Q}$

The ability to discriminate the different frequencies is called the Q factor of the circuit. The quality factor of the circuit determines the overall steepness of the response curve. Higher the value of Q of a series resonant circuit, the smaller is the bandwidth and greater is the ability to select or reject a particular narrow band of frequencies.

Q.9. Derive the relationships between Neper and Decibel units. **(4)**

Ans:
The attenuation in decibel (dB) is given by
1 dB = 20 x $\log_{10}(N)$
where

$$N = \frac{I_S}{I_R} = \frac{V_S}{V_R}$$

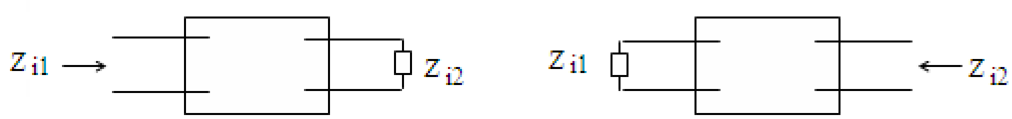
$\therefore$ The attenuation in Neper (Nep) is given by
1 Nep = $\log_e(N)$
The relation between decibel and neper is

$$\begin{aligned} dB &= 20 \times \log_{10}(N) \\ &= 20 \times \log_e(N) \times \log_{10}(e) \\ &= 20 \times \log_e(N) \times 0.434 \qquad (\therefore \log_{10}(e) = 0.434) \\ &= 8.686 \log_e(N) \end{aligned}$$

$\therefore$ Attenuation in decibel = 8.686 x attenuation in Neper.
$\therefore$ Attenuation in Neper = 0.1151 x attenuation in decibel.

Q.10. Explain the terms VSWR and Image Impedance. **(5)**

Ans:
Image impedance is that impedance, which when connected across the appropriate pair of terminals of the network, the other is presented by the other pair of terminals.



VSWR (Voltage Standing Wave Ratio) is defined as the ratio of maximum and minimum magnitudes of voltage on a line having standing waves.

$$VSWR = \frac{|V_{\max}|}{|V_{\min}|}$$

VSWR is always greater than 1. When VSWR is equal to 1, the line is correctly terminated and there is no reflection.

Q.11. State the relationship between reflection Coefficient 'K' and voltage standing wave ratio. **(5)**

Ans:
At the points of voltage maxima,
$|V_{max}| = |V_I| + |V_R|$
where V_I is the r.m.s value of the incident voltage.
where V_R is the r.m.s value of the reflected voltage.
Here the incident voltages and reflected voltages are in phase and add up.
At the points of voltage minima,

$|V_{min}| = |V_I| - |V_R|$

Here the incident voltages and reflected voltages are out of phase and will have opposite sign.

VSWR (Voltage Standing Wave Ratio) is defined as the ratio of maximum and minimum magnitudes of voltage on a line having standing waves.

$$VSWR = \frac{|V_{\max}|}{|V_{\min}|}$$

The voltage reflection coefficient, k is defined as the ratio of the reflected voltage to incident voltage.

$$k = \frac{|V_R|}{|V_I|}$$

$$VSWR(s) = \frac{|V_{\max}|}{|V_{\min}|} = \frac{|V_I| + |V_R|}{|V_I| - |V_R|} = \frac{1 + \left|\frac{V_R}{V_I}\right|}{1 - \left|\frac{V_R}{V_I}\right|}$$

$$\Rightarrow s = \frac{1+|k|}{1-|k|} \qquad \therefore k = \frac{s-1}{s+1}$$

Q.12. For the circuit in Fig.4 show the equivalency of Thevenin's and Norton's circuit. **(8)**

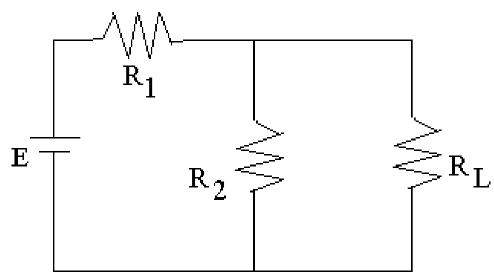


Fig.4

Ans:

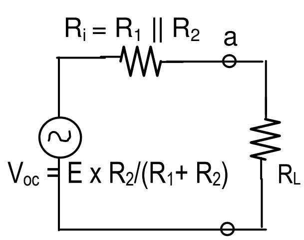


Fig 6.a.2

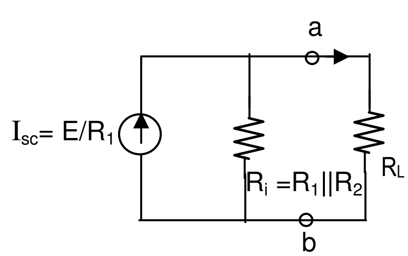


Fig 6.a.3

From the Fig 6.a.2 (Thevenin's equivalent circuit), the load current is given by

$$I_L(\text{Th}) = \frac{V_{oc}}{R_i + R_L} = \frac{\frac{ER_2}{R_1+R_2}}{\frac{R_1R_2}{R_1+R_2} + R_L} = \frac{\frac{ER_2}{R_1+R_2}}{\frac{R_1R_2 + R_1R_L + R_2R_L}{R_1+R_2}} = \frac{ER_2}{R_1R_2 + R_1R_L + R_2R_L}$$

From the Fig 6.a.3 (Norton's equivalent circuit), the load current is given by

$$I_L\,(\text{Nor}) = \frac{I_{sc}R_i}{R_i + R_L} = \frac{\dfrac{E}{R_1}\times\dfrac{R_1R_2}{R_1+R_2}}{\dfrac{R_1R_2}{R_1+R_2}+R_L} = \frac{\dfrac{ER_2}{R_1+R_2}}{\dfrac{R_1R_2+R_1R_L+R_2R_L}{R_1+R_2}} = \frac{ER_2}{R_1R_2+R_1R_L+R_2R_L}$$

$$\therefore I_L\,(\text{Th}) = I_L\,(\text{Nor}) = \frac{ER_2}{R_1R_2+R_1R_L+R_2R_L}$$

Q.13. Derive equation for resonant Frequency of an anti resonant circuit. **(7)**

Ans:
Consider an anti-resonant RLC circuit as shown in Fig 10.b.i
When the capacitor is perfect and there is no leakage and dielectric loss.
i.e. $R_C = 0$ and let $R_L = R$ as shown in Fig 10.b.ii.
The admittance

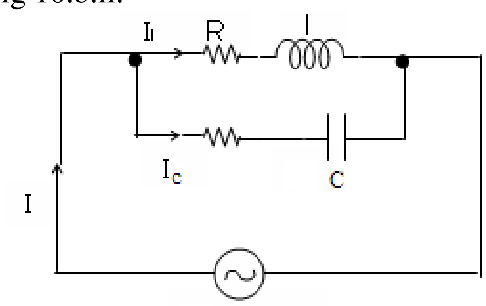


Fig. 10.b.i

$$Y_L = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$Y_C = j\omega C$$

$$Y = Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

At resonance frequency ω_o, the susceptance is zero.

$$\therefore \omega_0 C - \frac{\omega_0 L}{R^2 + {\omega_0}^2 L^2} = 0$$

$$\omega_0 C = \frac{\omega_0 L}{R^2 + {\omega_0}^2 L^2}$$

$$R^2 + {\omega_0}^2 L^2 = \frac{L}{C}$$

$$-R^2 + \frac{L}{C} = {\omega_0}^2 L^2$$

$$\therefore {\omega_0}^2 = \frac{-R^2}{L^2} + \frac{1}{LC}$$

$$\therefore \omega_0 = \sqrt{\frac{-R^2}{L^2} + \frac{1}{LC}}$$

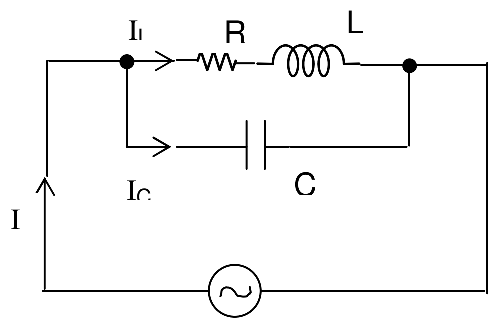


Fig 10.b.ii.

$$\therefore \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R is negligible

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}}$$

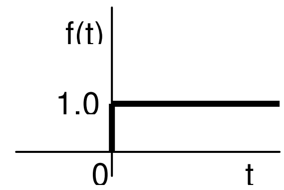
It is possible to have parallel resonance as long as

$$\frac{1}{LC} > \frac{R^2}{L^2} \quad \text{to have } f_o \text{ or } \omega_o \text{ to be real.}$$

Q.14. Define unit step, ramp and impulse function. Derive the Laplace transforms for these functions. **(7)**

Ans:

The unit step function is defined as

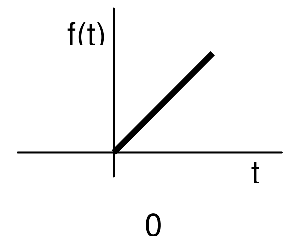


$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

The Laplace transform is given by

$$F(s) = Lu(t) = \int_0^\infty e^{-st} dt = \left[\frac{-1}{s} e^{-st}\right]_0^\infty = \frac{1}{s}$$

The ramp function is given by



f(t) = t, t ≥ 0

The Laplace transform is given by

$$F(s) = Lf(t) = \int_0^\infty t.e^{-st} dt = \left[\frac{-t}{s} e^{-st}\right]_0^\infty + \int_0^\infty \frac{1}{s}.e^{-st} dt$$

$$= 0 + \frac{1}{s}\int_0^\infty e^{-st} dt = \frac{1}{s}.\frac{1}{s} = \frac{1}{s^2}$$

$$F(s) = \frac{1}{s^2}$$

The unit impulse function is given by

$$\delta(t) = \lim_{\Delta t \to 0} \left[\frac{u(t) - u(t - \Delta t)}{\Delta t}\right]$$

Which is nothing but the derivative of the unit step function.

$$\delta(t) = \frac{d}{dt} u(t)$$

δ(t) has the value zero for t > 0 and unity at t = 0.

Let g(t) = 1 - $e^{-\alpha t}$

g(t) approaches δ(t) when α is very large.

$$g'(t) = \frac{d}{dt} g(t) = \alpha e^{-\alpha t}$$

$$\int_0^\infty g'(t) dt = \int_0^\infty \alpha . e^{-\alpha t} dt = 1$$

On applying Laplace transform

$$F(s) = L\delta(t) = \lim_{\alpha \to \infty} [L\, g'(t)]$$
$$= \lim_{\alpha \to \infty} \left[L\alpha\, e^{-\alpha t}\right]$$
$$= \lim_{\alpha \to \infty} \left[\frac{\alpha}{s+\alpha}\right] = 1$$

$\therefore$F(s) = 1 for impulse function.

Q.15. What is convolution in time domain? What is the Laplace transform of convolution of two time domain functions? **(7)**

Ans:
Consider the two functions $f_1(t)$ and $f_2(t)$ which are zero for t < 0.
The convolution of $f_1(t)$ and $f_2(t)$ in time domain is normally denoted by $f_1(t) * f_2(t)$ and is given by

$$f_2(t) * f_1(t) = \int_0^t f_1(\tau)\,.\,f_2(t-\tau)\,d\tau \quad \text{and}$$

$$f_1(t) * f_2(t) = \int_0^t f_2(\tau).f_1(t-\tau)\,d\tau$$

Where τ is a dummy variable part
The Laplace transform of convolution of two time domain functions is given by convolution theorem. The convolution theorem states that the Laplace transform of the convolution of $f_1(t)$ and $f_2(t)$ is the product of individual Laplace transforms.
L [$f_1(t) * f_2(t)$] = $F_1(s) * F_2(s)$.

Q.16. State and prove the superposition theorem with the help of a suitable network. **(8)**

Ans:
Superposition theorem: It states that "if a network of linear impedances contains more than one generator, the current which flows at any point is the vector sum of all currents which would flow at that point if each generator was considered separately and all other generators are replaced at that time by impedance equal to their internal impedances"

$I_1 = I_1' + I_1''$
$I_2 = I_2' + I_2''$

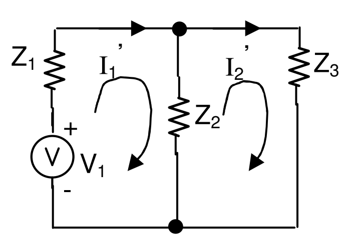


Fig 2.a.1

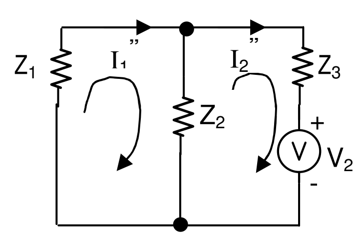


Fig 2.a.2

Let the currents due to V_1 alone be I_1' and I_2' and currents due to V_2 alone be I_1'' and I_2'' and the currents due to V_1 and V_2 acting together be I_1 and I_2.
Applying kirchoff's voltage law

When V_1 is acting alone, as in Fig 2.a.1, then

$I_1'(Z_1 + Z_2) - I_2'Z_2 = V_1$ ----- (1)

$I_2'(Z_2 + Z_3) - I_1'Z_2 = 0$ ----- (2)

When V_2 is acting alone, as in Fig 2.a.2, then

$I_1''(Z_1 + Z_3) - I_2''Z_2 = 0$ ----- (3)

$I_2''(Z_2 + Z_3) - I_1''Z_2 = -V_2$ ----- (4)

When both V_1, and V_2 are acting, as in Fig 2.a.3, then

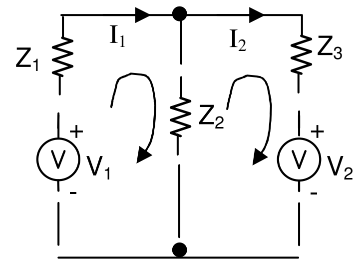


Fig 2.a.3

$I_1(Z_1 + Z_2) - I_2Z_2 = V_1$ ----- (5)

$I_2(Z_2 + Z_3) - I_1Z_2 = -V_2$ ----- (6)

Adding equations (1) and (3),

$(I_1' + I_1'')(Z_1 + Z_2) - (I_2' + I_2'')Z_2 = V_1$ ----- (7)

Adding equations (2) and (4),

$(I_2' + I_2'')(Z_1 + Z_2) - (I_1' + I_1'')Z_2 = -V_2$ ----- (8)

Comparing equations (5), (7) and (6), (8)

$I_1 = I_1' + I_1''$

$I_2 = I_2' + I_2''$

which proves the superposition theorem.

Q.17. Derive the expression for characteristic impedance of a symmetrical Bridged-T-network. **(8)**

Ans:

Consider the symmetrical Bridged -T network shown in Fig 3.a

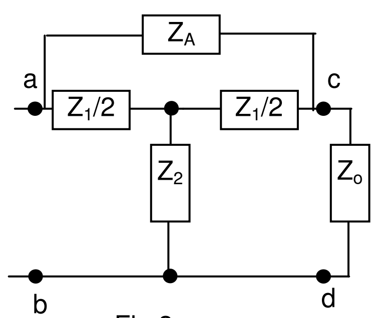


Fig 3.a

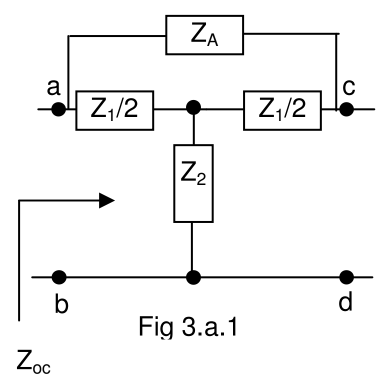


Fig 3.a.1

Let the terminals c-d be open circuited as shown in fig 3.a.1

$$Z_{OC} = Z_2 + \frac{\frac{Z_1}{2}(Z_A + \frac{Z_1}{2})}{Z_A + Z_1}$$

$$= Z_2 + \frac{Z_1(Z_1 + 2Z_A)}{4(Z_A + Z_1)}$$

$$Z_{OC} = \frac{4Z_2(Z_A + Z_1) + Z_1(Z_1 + 2Z_A)}{4(Z_A + Z_1)}$$

Let the terminals c-d be short circuited as shown in Fig 3.a.2

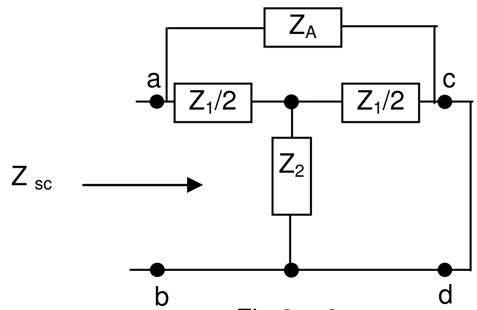


Fig 3.a.2

$$Z_{SC} = \frac{Z_A\left(\frac{Z_1 Z_2}{Z_1 + 2Z_2} + \frac{Z_1}{2}\right)}{Z_A + \frac{Z_1 Z_2}{Z_1 + 2Z_2} + \frac{Z_1}{2}}$$

$$Z_{SC} = \frac{Z_A\left(\frac{2Z_1 Z_2 + Z_1(Z_1 + 2Z_2)}{2(Z_1 + 2Z_2)}\right)}{Z_A + \frac{2Z_1 Z_2 + Z_1(Z_1 + 2Z_2)}{2(Z_1 + 2Z_2)}}$$

$$= \frac{Z_A\left(2Z_1 Z_2 + Z_1(Z_1 + 2Z_2)\right)}{2Z_A(Z_1 + 2Z_2) + 2Z_1 Z_2 + Z_1(Z_1 + 2Z_2)}$$

$$= \frac{Z_A\left(Z_1^{\ 2} + 4Z_1 Z_2\right)}{2Z_A Z_1 + 4Z_A Z_2 + Z_1^{\ 2} + 4Z_1 Z_2}$$

$$Z_{SC} = \frac{Z_A Z_1\left(Z_1 + 4Z_2\right)}{Z_1(2Z_A + Z_1) + 4Z_2(Z_A + Z_1)}$$

Since the characteristic impedance is given by

$$Z_O = \sqrt{Z_{OC} \times Z_{SC}}$$

$$= \sqrt{\frac{4Z_2(Z_A + Z_1) + Z_1(Z_1 + 2Z_A)}{4(Z_A + Z_1)} \times \frac{Z_A Z_1\left(Z_1 + 4Z_2\right)}{Z_1(2Z_A + Z_1) + 4Z_2(Z_A + Z_1)}}$$

$$= \sqrt{\frac{Z_A Z_1\left(Z_1 + 4Z_2\right)}{4(Z_A + Z_1)}}$$

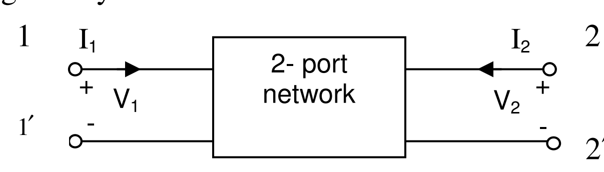
Q.18. Define the h-parameters of a two port network. Draw the h-parameter equivalent circuit. Where are the h-parameters used mostly? **(6)**

Ans:

In a hybrid parameter model, the voltage of the input port and the current of the output port are expressed in terms the current of the input port and the voltage of the output port. The equations are given by

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$



When the output terminal is short circuited, $V_2 = 0$

$V_1 = h_{11}I_1$

$I_2 = h_{21}I_1$

$$h_{11} = \frac{V_1}{I_1}\ ohms \qquad h_{21} = \frac{I_2}{I_1}$$

Where h_{11} is the input impedance expressed in ohms and h_{21} is the forward current gain.
When the input terminal is open circuited, $I_1 = 0$

$V_1 = h_{12}V_2$

$I_2 = h_{22}V_2$

$$h_{12} = \frac{V_1}{V_2} \qquad h_{22} = \frac{I_2}{V_2} mhos$$

Where h_{12} is the reverse voltage gain and h_{22} is the output admittance expressed in mhos. The equivalent circuit of the hybrid parameter representation is shown in Fig 4.a $h_{12}V_2$ is the controlled voltage source and $h_{21}I_1$ is the controlled current source.

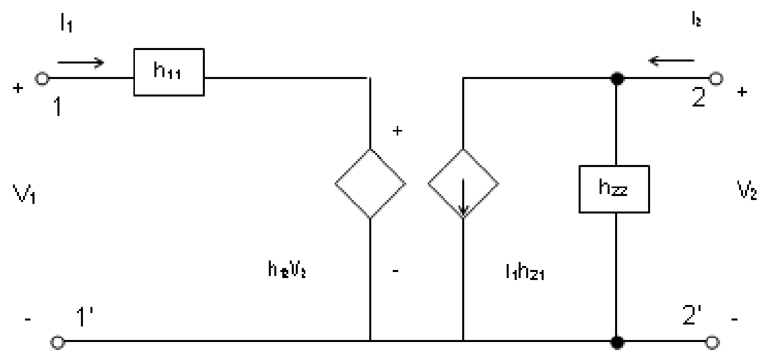
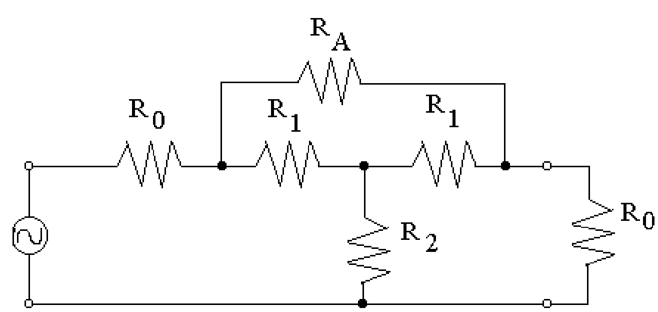


Fig 4.a

h- parameters are widely used in modeling of electronic components and circuits particularly transistors where both the open circuit and circuit conditions are utilized.

Q.19. Design a symmetrical bridged-T attenuator shown below use necessary assumptions for simplification. **(6)**



where R_o is the characteristic impedance and $N = e^{\alpha}, \alpha$ is the attenuation constant.

Ans:
Consider the loop *abcde* of the circuit in Fig 5.a

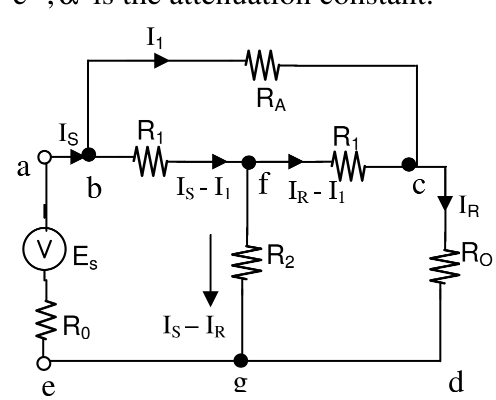


Fig. 5.a

$E_s = I_1R_A + I_RR_O + I_SR_O$ ------(i)

Consider the loop afcde in Fig.5a

$E_S = (I_S - I_1)R_1 + (I_R - I_1)R_1 + I_RR_o + I_SR_o$ ---------(ii)

Consider the loop *abfcde* of the circuit in Fig 5.a

$E_s = (I_S - I_1)R_1 + (I_R - I_1)R_1 + I_RR_O + I_SR_O$ ------(iii)

Solving (i) and (ii) & (iii) we obtain

$$\frac{I_S}{I_R} = \frac{\left(R_1 + R_O + \frac{2R_1R_O}{R_A}\right)}{\left(R_O - R_1 + \frac{2R_1R_O}{R_A}\right)} = N$$

Usually the load and arm impedances are selected in such a way that

$R_1^2 = R_O^2 = R_2R_A$

$$\Rightarrow R_1 = R_O = \sqrt{R_2R_A}$$

$$N = \frac{\left(R_1 + R_O + \frac{2R_1R_O}{R_A}\right)}{\left(R_O - R_1 + \frac{2R_1R_O}{R_A}\right)} = \frac{\left(2R_O + \frac{2R_O^2}{R_A}\right)}{\left(\frac{2R_O^2}{R_A}\right)}$$

$$N = \frac{1 + \frac{R_O}{R_A}}{\frac{R_O}{R_A}} = 1 + \frac{R_A}{R_O}$$

$$N = 1 + \frac{R_A}{R_O}$$

$$\Rightarrow \frac{R_A}{R_O} = N - 1$$

$$\Rightarrow R_A = R_O(N-1)$$

$$\therefore N = 1 + \frac{R_O}{R_2} \qquad \because R_O^2 = R_2R_A$$

$$\therefore \frac{R_O}{R_2} = N - 1$$

$$\Rightarrow R_2 = \frac{R_O}{N - 1}$$

Q.20. What are the disadvantages of the prototype filters? How are they removed in composite filters? **(8)**

Ans:

The two disadvantages of a prototype filters are:

- The attenuation does not increase rapidly beyond cutoff frequencies (f_c), that is in

the stop band.
- The characteristic impedance varies widely in the pass band from the required value.

In m-derived sections attenuation reaches a very high value at a frequency (f_∞) very close to cut off frequency but decreases for frequencies beyond (f_∞) and the characteristic impedance is more uniform within the pass band.
Composite filters are used to overcome the two disadvantages. A composite filter consists of
- One or more constant K sections to produce a specific cut off frequency.
- One or more m-derived sections to provide infinite attenuation at a frequency close to (f_c).
- Two terminating half sections (m-derived) that provide almost constant input and output impedances.

In a composite filter, the attenuation rises very rapidly with frequency in the range of f_c to f_∞, and falls only marginally with frequencies after f_∞.

Q.21. Draw the equivalent circuit of a section of transmission line. Explain primary and secondary parameters. **(6)**

Ans:
A uniform transmission line consists of series resistance (R), series inductance (L), shunt capacitance (C), and shunt conductance (G). The series resistance is due to the conductors and depends on the resistivity and diameter. The inductance is due to the magnetic field of each of the conductor carrying current. The inductance is in series with resistance since the effect of inductance is to oppose the flow of the current. The shunt capacitance is due to the two conductors placed parallel separated by a dielectric. The dielectric is not perfect and hence a small leakage current flows in between the wires, which results in shunt conductance. All the four parameters are uniformly distributed over the length of the line.
The series impedance Z per unit length is $Z = R + j\,\omega L$ ohms/unit length
The shunt admittance Y per unit length is $Y = G + j\,\omega C$ seimens/unit length
Where $Z \neq 1/Y$.
The parameters R, L, G, C are normally constant for a particular transmission line and are known as primary constants of a transmission line.
The characteristic impedance, Z_o and propagation constant, γ are the secondary constants of a transmission line and indicate the electrical properties of a line. Characteristic impedance, Z_o is the input impedance of a infinite length line. Propagation constant, γ is defined as the natural logarithm of the ratio of the input to the output current.
The equivalent T section of transmission line of length Δx is shown in the Fig 7.a.

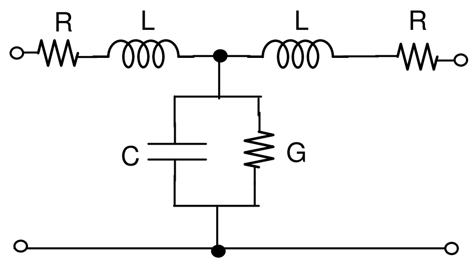


Fig 7.a

Q.22. Define h-parameters and transmission parameters of a two-port network. Determine the relation between them. **(8)**

Ans:
In a hybrid parameter model, the voltage of the input port and the current of the output port are expressed in terms the current of the input port and the voltage of the output port. The equations are given by

$V_1 = h_{11}I_1 + h_{12}V_2$ eq-1

$I_2 = h_{21}I_1 + h_{22}V_2$ eq-2

When the output terminal is short circuited, $V_2 = 0$

$V_1 = h_{11}I_1$ $I_2 = h_{21}I_1$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

Where h_{11} is the input impedance expressed in ohms and h_{21} is the forward current gain.

When the input terminal is open circuited, $I_1 = 0$

$V_1 = h_{12}V_2$ $I_2 = h_{22}V_2$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Where h_{12} is the reverse voltage gain and h_{22} is the output admittance expressed in mhos.

Transmission (ABCD) Parameters: The ABCD parameter equations are given by

$V_1 = AV_2 - B\,I_2$ eq-3

$I_1 = CV_2 - D\,I_2$ eq-4

When the output terminal is short circuited, $V_2 = 0$
Where B is the impedance expressed in ohms.

$V_1 = -BI_2$ $I_1 = -DI_2$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

When the output terminal is open circuited, $I_2 = 0$
Where C is the admittance expressed in mhos.

$V_1 = AV_2$ $I_1 = CV_2$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

To find the relation between h parameters and ABCD parameters

$I_1 = CV_2 - DI_2$

$$\Rightarrow I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2 \qquad \text{----- Eq.5}$$

$$V_1 = AV_2 - BI_2$$

$$\Rightarrow V_1 = AV_2 - B\left[\left(-\frac{1}{D}\right) I_1 + \left(\frac{C}{D}\right) V_2\right]$$

$$V_1 = \left(\frac{AD - BC}{D}\right) V_2 + \left(\frac{B}{D}\right) I_1 \qquad \text{----- Eq.6}$$

Comparing equations Eq.1, Eq.2, and Eq.5, Eq.6,

The h parameters in terms of ABCD parameters are given by

$$h_{11} = \frac{B}{D} \qquad h_{12} = \frac{AD - BC}{D}$$

$$h_{21} = \frac{-1}{D} \qquad h_{22} = \frac{C}{D}$$

Q.23. Describe various types of losses in a transmission line. How these losses are reduced? **(8)**

Ans:
The three types of losses are

- **Radiation loss:** The radiation loss is due to the electromagnetic field around the conductors. The loss of energy is proportional to the square of the frequency and also depends on the spacing between the conductors. These losses are more in open wire lines than that in co-axial cables. The radiation loss increases with frequency and is more evident in high frequency cables. Radiation losses can be reduced by decreasing the spacing between the conductors and allowing only low frequency signals to pass through.
- **Dielectric loss:** Air acts as a dielectric in transmission line and chemical compounds in a coaxial cable. The dielectric medium possesses finite conductivity and there is leakage of current and loss of energy between the conductors. This loss is due to the imperfect dielectric medium. Dielectric loss is proportional to the voltage across the dielectric and inversely proportional to the characteristic impedance of the dielectric medium. Dielectric loss increases with frequency. Dielectric losses can be reduced by choosing a perfect dielectric or by using air as dielectric.

- **Copper loss (thermal loss or conductor heating loss):** Copper loss is the energy loss in the form of heat dissipated in the surrounding medium by the conductors. This loss is due the existence of the resistance in the line conductors. It is expressed as I^2R loss, where I is the current through the conductor and R is the resistance.

Q.24. Find the image impedances of an asymmetrical- π (pi) network. **(6)**

Ans:

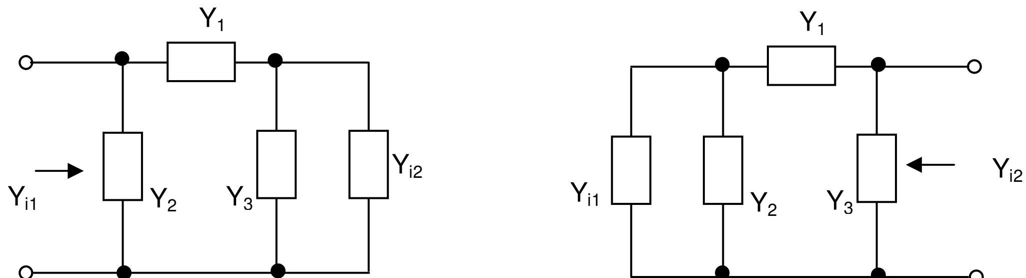


Fig 8.b.1 Fig 8.b.2

Let Y_1, Y_2 and Y_3 be the admittances and Y_{i1} and Y_{i2} be the image admittances of the asymmetric π network.

From Fig 8.b.1

$$Y_{i1} = Y_2 + \frac{Y_1(Y_3 + Y_{i2})}{Y_1 + Y_3 + Y_{i2}} = \frac{Y_1Y_2 + Y_3Y_2 + Y_2Y_{i2} + Y_1Y_3 + Y_1Y_{i2}}{Y_1 + Y_3 + Y_{i2}}$$

$$Y_1Y_{i1} + Y_3Y_{i1} + Y_{i2}Y_{i1} = Y_1Y_2 + Y_3Y_2 + Y_2Y_{i2} + Y_1Y_3 + Y_1Y_{i2} --- (1)$$

From Fig 8.b.2

$$Y_{i2} = Y_3 + \frac{Y_1(Y_2 + Y_{i1})}{Y_1 + Y_2 + Y_{i1}} = \frac{Y_1Y_3 + Y_3Y_2 + Y_3Y_{i1} + Y_1Y_2 + Y_1Y_{i1}}{Y_1 + Y_2 + Y_{i1}}$$

$$Y_1Y_{i2} + Y_2Y_{i2} + Y_{i1}Y_{i2} = Y_1Y_3 + Y_3Y_2 + Y_3Y_{i1} + Y_1Y_2 + Y_1Y_{i1} --- (2)$$

Adding equations (1) and (2)

$$2Y_{i1}Y_{i2} = 2(Y_1Y_3 + Y_3Y_2 + Y_1Y_2)$$

$$\Rightarrow Y_{i1}Y_{i2} = Y_1Y_3 + Y_3Y_2 + Y_1Y_2$$

$$\Rightarrow Y_{i2} = \frac{Y_1Y_3 + Y_3Y_2 + Y_1Y_2}{Y_{i1}} ------- (3)$$

Subtracting equation (1) from equation (2)

$$(Y_1+Y_2)\mathrm{Y}_{i2}-(Y_1+Y_3)Y_{i1}=(Y_1+Y_3)Y_{i1}-(Y_1+Y_2)\mathrm{Y}_{i2}$$

$$\Rightarrow (Y_1+Y_2)\mathrm{Y}_{i2}=(Y_1+Y_3)Y_{i1}$$

$$\Rightarrow \mathrm{Y}_{i2}=\frac{(Y_1+Y_3)Y_{i1}}{(Y_1+Y_2)} --------(4)$$

$$\Rightarrow \frac{Y_1Y_3+Y_3Y_2+Y_1Y_2}{Y_{i1}}=\frac{(Y_1+Y_3)Y_{i1}}{(Y_1+Y_2)}$$

$$\Rightarrow Y_{i1}^{\ 2}=\frac{(Y_1Y_3+Y_3Y_2+Y_1Y_2)(Y_1+Y_2)}{(Y_1+Y_3)} ----------(4A)$$

$$\Rightarrow \frac{1}{Z_{i1}^{\ 2}}=\frac{(\frac{Z_1+Z_2+Z_3}{Z_1Z_2Z_3})(\frac{Z_1+Z_2}{Z_1Z_2})}{\frac{Z_1+Z_3}{Z_1Z_3}}=\frac{(Z_1+Z_2+Z_3)(Z_1+Z_2)}{Z_1Z_2^{\ 2}(Z_1+Z_3)}$$

$$where \quad Y_{i1}=\frac{1}{Z_{i1}},\quad Y_{i2}=\frac{1}{Z_{i2}},\quad Y_1=\frac{1}{Z_1},\quad Y_2=\frac{1}{Z_2},\quad Y_3=\frac{1}{Z_3}$$

$$Z_{i1}^{\ 2}=\frac{Z_1Z_2^{\ 2}(Z_1+Z_3)}{(Z_1+Z_2+Z_3)(Z_1+Z_2)}$$

$$Z_{i1}=\sqrt{\frac{Z_1Z_2^{\ 2}(Z_1+Z_3)}{(Z_1+Z_2+Z_3)(Z_1+Z_2)}}$$

$$\text{we know that}\quad \mathrm{Y_{i2}}=\frac{(\mathrm{Y_1}+\mathrm{Y_3})\mathrm{Y_{i1}}}{(\mathrm{Y_1}+\mathrm{Y_2})}\qquad \text{(from (4))}$$

$$\Rightarrow \mathrm{Y_{i2}}=\frac{(\mathrm{Y_1}+\mathrm{Y_3})}{(\mathrm{Y_1}+\mathrm{Y_2})}\sqrt{\frac{(\mathrm{Y_1Y_3}+\mathrm{Y_3Y_2}+\mathrm{Y_1Y_2})(\mathrm{Y_1}+\mathrm{Y_2})}{(\mathrm{Y_1}+\mathrm{Y_3})}}\qquad \text{(due to (4A))}$$

$$\Rightarrow \mathrm{Y_{i2}}=\sqrt{\frac{(\mathrm{Y_1Y_3}+\mathrm{Y_3Y_2}+\mathrm{Y_1Y_2})(\mathrm{Y_1}+\mathrm{Y_3})}{(\mathrm{Y_1}+\mathrm{Y_2})}}$$

$$\Rightarrow \frac{1}{\mathrm{Z_{i2}}^2}=\frac{(\frac{\mathrm{Z_1}+\mathrm{Z_2}+\mathrm{Z_3}}{\mathrm{Z_1Z_2Z_3}})(\frac{\mathrm{Z_1}+\mathrm{Z_3}}{\mathrm{Z_1Z_3}})}{\frac{\mathrm{Z_1}+\mathrm{Z_2}}{\mathrm{Z_1Z_2}}}=\frac{(\mathrm{Z_1}+\mathrm{Z_2}+\mathrm{Z_3})(\mathrm{Z_1}+\mathrm{Z_3})}{\mathrm{Z_1Z_3}^2(\mathrm{Z_1}+\mathrm{Z_2})}$$

$$\mathrm{Z_{i2}}=\sqrt{\frac{\mathrm{Z_1Z_3}^2(\mathrm{Z_1}+\mathrm{Z_2})}{(\mathrm{Z_1}+\mathrm{Z_2}+\mathrm{Z_3})(\mathrm{Z_1}+\mathrm{Z_3})}}$$

Q.25. In Laplace domain a function is given by

$$\mathrm{F(s)}=\mathrm{M}\left[\frac{(s+\alpha)\sin\theta}{(s^2+\alpha^2)+\beta^2}+\frac{\beta\cos\theta}{(s+\alpha)^2+\beta^2}\right]$$

where α, β, θ & M are constants. Show by initial value theorem

$$\lim_{t\to 0} f(t)=\mathrm{M}\sin\theta$$

(6)

Ans:

As per the initial value theorem

$$\underset{t\to 0}{Lim}\, f(t) = \underset{s\to\infty}{Lim}\, sF(s)$$

$$s\times F(s) = s\times M\left[\frac{(s+\alpha)\sin\theta}{(s^2+\alpha^2)+\beta^2} + \frac{\beta\cos\theta}{(s+\alpha)^2+\beta^2}\right]$$

$$s\times F(s) = M\left[\frac{s(s+\alpha)\sin\theta}{(s^2+\alpha^2)+\beta^2} + \frac{s\beta\cos\theta}{(s+\alpha)^2+\beta^2}\right]$$

$$s\times F(s) = M\left[\frac{(1+\frac{\alpha}{s^2})\sin\theta}{(1+\frac{\alpha^2}{s^2})+\frac{\beta^2}{s^2}} + \frac{\frac{\beta}{s}\cos\theta}{(1+\frac{\alpha^2}{s^2})+\frac{\beta^2}{s^2}}\right]$$

$$\underset{s\to\infty}{Lim}\, sF(s) = \underset{s\to\infty}{Lim}\, M\left[\frac{(1+\frac{\alpha}{s^2})\sin\theta}{(1+\frac{\alpha^2}{s^2})+\frac{\beta^2}{s^2}} + \frac{\frac{\beta}{s}\cos\theta}{(1+\frac{\alpha}{s})^2+\frac{\beta^2}{s^2}}\right] = M\sin\theta$$

$$\underset{s\to\infty}{Lim}\, sF(s) = M\sin\theta$$

$$\Rightarrow \underset{t\to 0}{Lim}\, f(t) = \underset{s\to\infty}{Lim}\, sF(s) = M\sin\theta$$

Q.26. Check, if the driving point impedance Z (s), given below, can represent a passive one port network.

(i) $Z(s) = \frac{s^4+s^2+1}{s^3+2s^2-2s+10}$ (ii) $Z(s) = \frac{s^4-s^3+2s^2}{s+5}$

Also specify proper reasons in support of your answer. **(8)**

Ans:

(i) $$Z(s) = \frac{s^4+s^2+1}{s^3+2s^2-2s+10}$$

The given function is not suitable to represent the impedance of one port network due to following reasons :

- In the numerator, one coefficient is missing.
- In the denominator, one coefficient is negative.

(ii) $$Z(s) = \frac{s^4-s^3+2s^2}{s+5}$$

The given function is not suitable for representing the driving point impedance due to following reasons.

- In the numerator, one coefficient is negative.
- The degree of numerator is 4, while that of denominator is 1. Then a difference of 3

exists between the degree of numerator and denominator and is not permitted.

- The numerator gives $s^2(s^2-s+2)$ that is double zero, at s = 0,

$$s=-0.5+j\sqrt{\frac{7}{4}} \quad and, \quad s=-0.5-j\sqrt{\frac{7}{4}}$$

This not permitted.
The term of the lowest degree in numerator is 2 while that in the denominator is zero.

Q.27. State the Millman theorem and prove its validity by taking a suitable example. **(8)**

Ans:
Millman's theorem states that if n voltage sources V_1, V_2, ---, V_n having internal impedances Z_1, Z_2, ---, Z_n, respectively, are connected in parallel, then these sources may be replaced by a single voltage source V_m having internal series impedance Z_m where V_m and Z_m are given by the equations

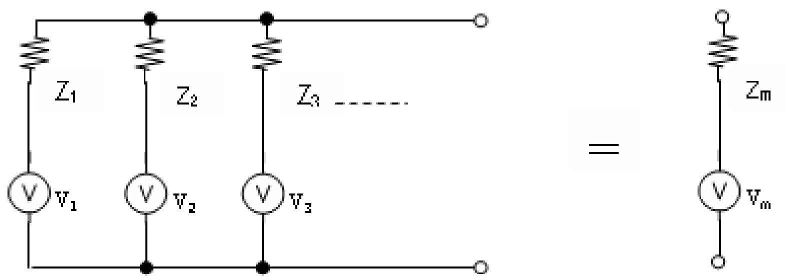


Fig.2.a.1

$$V_m = \frac{V_1Y_1+V_2Y_2+V_3Y_3+---}{Y_1+Y_2+Y_3+---} \qquad Z_m = \frac{1}{Y_1+Y_2+Y_3+---}$$

where Y_1, Y_2, ---, Y_n are the admittances corresponding to Z_1, Z_2, ---, Z_n

Proof:
A voltage source V_1 with series impedance Z_1 can be replaced by a current source V_1Y_1 with shunt admittance $Y_1 = 1/Z_1$

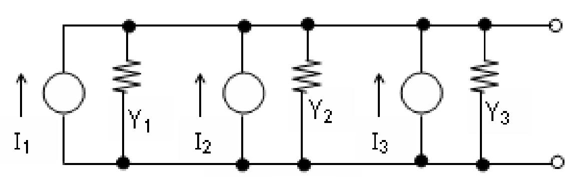


Fig 2.a.2

$\therefore$The network of Fig 2.a.1 can be replaced by its equivalent network as shown in Fig 2.a.2, where

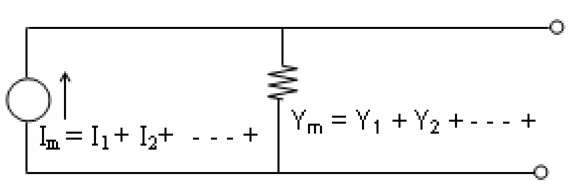


Fig 2.a.3

$I_1= V_1Y_1$, $I_2= V_2Y_2$, - - - $I_n= V_nY_n$, and $Y_1 = 1/Z_1$, $Y_2 = 1/Z_2$, - - -, $Y_n = 1/Z_n$,

Network of Fig 2.a.2 is equivalent to that of Fig 2.a.3 where by
$I_m = I_1 + I_2 + - - - + I_n$ and $Y_m = Y_1 + Y_2 + - - - + Y_n$
Reconverting the current source of Fig 2.a.3 into an equivalent voltage source,
The equivalent circuit of 2.a.4 is obtained where

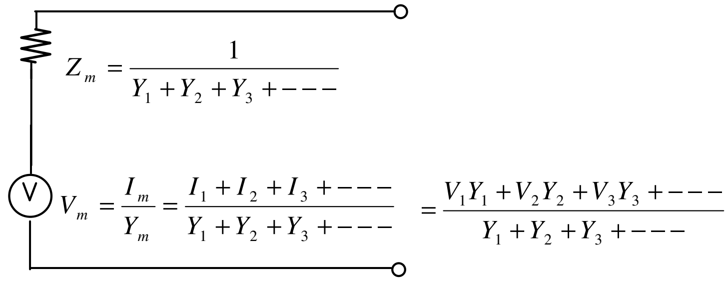


Fig 2.a.4

To find the current in the resistor R_3 in the Fig 2.a.5 using Millmans theorem

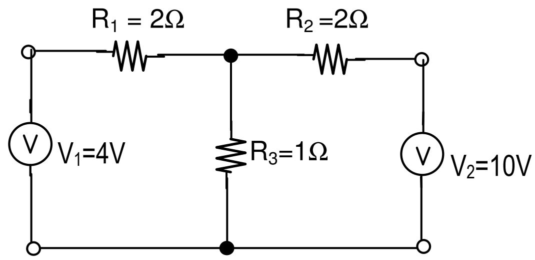


Fig 2.a.5

The two voltage sources V_1 and V_2 with series resistances R_1 and R_2 are combined into one voltage source V_m with series resistances R_m.
According to the Millman theorem,

$$V_m = \frac{V_1 Y_1 + V_2 Y_2}{Y_1 + Y_2} = \frac{4 \times 0.5 + 10 \times 0.5}{0.5 + 0.5} = 7 \text{ volts}$$

$$Z_m = \frac{1}{Y_1 + Y_2} = \frac{1}{0.5 + 0.5} = 1\Omega$$

Hence the current through R_3 is given by

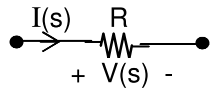
$$I_3 = \frac{V_m}{R_m + R_3} = \frac{7}{1+1} = 3.5 \quad amps$$

Q.28. State the advantages of using Laplace transform in networks. Give the 's' domain representations for resistance, inductance and capacitance. **(7)**

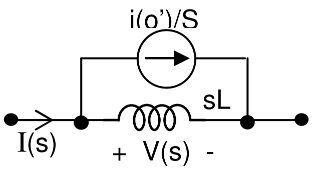
Ans:
The advantages of using Laplace transform in networks are:
- The solution is easy and simple.
- It gives the total solution i.e. complementary function and particular integral as a single entity.

- The initial conditions are automatically included as one of the steps rather that at the end in the solution.
- It provides the direct solution of non-homogenous differential equations.

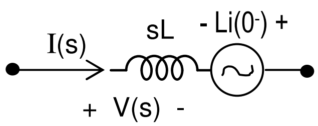
  The ‘s’ domain representations for resistance, inductance and capacitance are

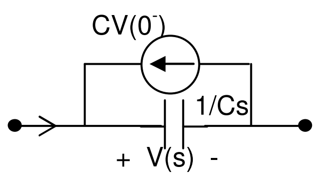


V(s) = R I(s)

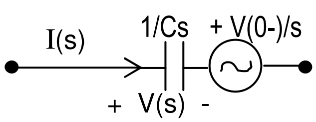


V(s) = s L I(s) - L i(0-)





$$V(s) = \frac{1}{C\,s} I(s) + \frac{v(0^-)}{s}$$



Q.29. Determine the elements of a ‘T’ – section which is equivalent to a π- section. **(7)**

Ans:
At any one frequency, a π network can be interchanged to a T network and vice-versa, provided certain relations are maintained.
Let Z_1, Z_2, Z_3 be the three elements of the T network and Z_A, Z_B, Z_C be the three elements
of the Π network as shown in Fig.3b.1 and Fig 3.b.2

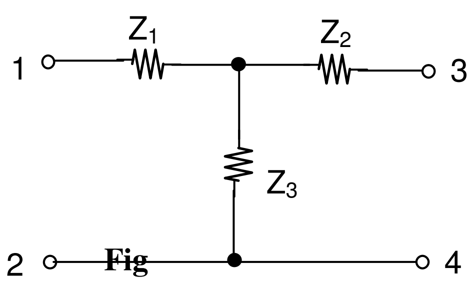


Fig 3.b.1

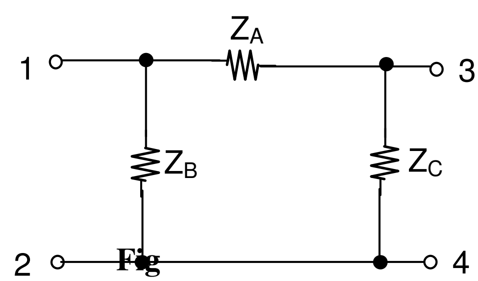


Fig 3.b.2

The impedance between the terminal 1 and terminal 3 is

$$Z_1 + Z_2 = \frac{(Z_B + Z_C)Z_A}{Z_B + Z_C + Z_A} \cdots\cdots\cdots\cdots eq.1$$

The impedance between the terminal 3 and terminal 4 is

$$Z_2 + Z_3 = \frac{(Z_A + Z_B)Z_C}{Z_B + Z_C + Z_A} \cdots\cdots\cdots\cdots eq.2$$

The impedance between the terminal 1 and terminal 2 is

$$Z_1 + Z_3 = \frac{(Z_A + Z_C)Z_B}{Z_B + Z_C + Z_A} \cdots\cdots\cdots\cdots eq.3$$

Adding eq.1, eq.2 and subtracting eq.3

$$Z_2 = \frac{Z_A Z_C}{Z_B + Z_C + Z_A} \cdots\cdots\cdots\cdots eq.4$$

Adding eq.2, eq.3 and subtracting eq.1

$$Z_3 = \frac{Z_B Z_C}{Z_B + Z_C + Z_A} \cdots\cdots\cdots\cdots eq.5$$

Adding eq.3, eq.1 and subtracting eq.2

$$Z_1 = \frac{Z_B Z_A}{Z_B + Z_C + Z_A} \cdots\cdots\cdots\cdots eq.6$$

Q.30. Discuss the characteristics of a filter. **(8)**

Ans:
Ideal filters should have the following characteristics:

- Filters should transmit pass band frequencies without any attenuation.
- Filters should provide infinite attenuation and hence, completely suppress all frequencies in the attenuation band.
- Characteristic impedance of the filter should match with the circuit to which it is connected throughout the pass band, which prevents the reflection loss. Since the power is to be transmitted in the pass band, the characteristic impedance Z_O of the filter within the pass band should be real and imaginary outside the pass band (i.e., within stop band) as the power has to be suppressed.
- The transition region between the stop and pass band is very small. The critical frequencies where the filter passes from pass band to a stop band are called the cut off frequencies. The cut off frequency is denoted by the letter f_c and is also termed as nominal frequency because the practical filter does not cut off abruptly at that point. Since Z_O is real in the pass band and imaginary in an attenuation band, f_c is the frequency at which Z_O changes from being real to being imaginary. This is the ideal requirement. Practically, it is not possible to realize such an abrupt change of impedance at f_c.

Q.31. State and prove Maximum power transfer theorem. **(8)**

Ans:
Maximum power transfer theorem states that for a generator with internal impedance ($Z_S = R_S + j\,X_S$), the maximum power will be obtained from it if the impedance ($Z_L = R_L + j\,X_L$), connected across the output is the complex conjugate of the source impedance.

Given $Z_S = R_S + j X_S$ and $Z_L = R_L + j X_L$

The power P in the load is $I_L^2 R_L$, where I_L is the current flowing in the circuit, which is given by,

$$I = \frac{V}{Z_S + Z_L} = \frac{V}{R_S + jX_S + R_L + jX_L} = \frac{V}{(R_S + R_L) + j(X_S + X_L)}$$

$\therefore$ Power to the load is $P = I_L^2 R_L$

$$P = \frac{V^2}{(R_S + R_L)^2 + (X_S + X_L)^2} \times R_L \qquad \text{-- (1)}$$

for maximum power, we vary X_L such that

$$\Rightarrow \frac{-2V^2 R_L (X_S + X_L)}{\left[(R_S + R_L)^2 + (X_S + X_L)^2\right]^2} = 0$$

$$\Rightarrow (X_S + X_L) = 0$$

$$\text{i.e. } X_S = -X_L$$

This implies the reactance of the load impedance is of the opposite sign to that of the source impedance. Under this condition $X_L + X_S = 0$

The maximum power is

$$P = \frac{V^2 R_L}{(R_S + R_L)^2}$$

For maximum power transfer, now let us vary R_L such that

$$\frac{dP}{dR_L} = 0$$

$$\Rightarrow \frac{V^2 (R_S + R_L)^2 - 2V^2 R_L (R_S + R_L)}{\left[R_S + R_L\right]^4} = 0$$

$$\Rightarrow V^2 (R_S + R_L) = 2V^2 R_L$$

$$\text{i.e. } R_S = R_L$$

$\therefore$ The necessary and sufficient condition for maximum power transfer from a voltage source, with source impedance $Z_S = R_S + j X_S$ to a load $Z_L = R_L + j X_L$ is that the load impedance should be a complex conjugate of that source impedance. $R_L = R_S$, $X_L = -X_S$

The value of the power transferred will be

$$P = \frac{V^2 R_L}{\left[R_S + R_L\right]^2} = \frac{V^2 R_L}{\left[2R_L\right]^2} = \frac{V^2 R_L}{4R_L^2}$$

Q.32. Differentiate between attenuator and amplifier. List the practical applications of attenuators. **(6)**

Ans:

An amplifier is used to increase the signal level by a give amount, while an attenuator is used to reduce the signal level by a given amount. An amplifier consists of active elements like transistors. An attenuator is a four terminal resistive network connected between the source and load to provide a desired attenuation of the signal. An attenuator can be either symmetrical or asymmetrical in form. It also can be either a fixed type or a variable type. A fixed attenuator is known as pad.

Applications of Attenuators:

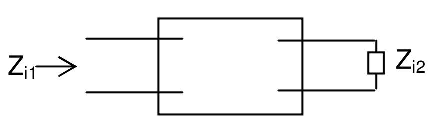
(i) Resistive attenuators are used as volume controls in broadcasting stations.

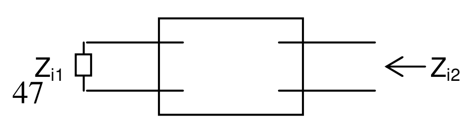
(ii) Variable attenuators are used in laboratories, when it is necessary to obtain small value of voltage or current for testing purposes.

(iii) Resistive attenuators can also be used for matching between circuits of different resistive impedances when insertion losses can be tolerated.

Q.33. Explain the terms Image impedance and Insertion loss. **(6)**

Ans:
Image impedance is that impedance, which when connected across the appropriate pair of terminals of the network, the other is presented by the other pair of terminals. If the driving point impedance at the input port with impedance Z_{i2} is Z_{i1} and if the driving point impedance at the output port with impedance Z_{i1} is Z_{i2}, then Z_{i1} and Z_{i2} are the image impedances of the two-port network.





Insertion loss: If a network or a line is inserted between a generator and its load, in general, there is a reduction in the power received in the load, and the load current will decrease. The loss produced by the insertion of the network or line is known as the insertion loss. If the load current without the network is I_1 and the load current with the network inserted is I_2, then the insertion loss is given by

$$Insertion\ Loss = 20\log_{10}\left|\frac{I_1}{I_2}\right| dB$$

$$= \log_e\left|\frac{I_1}{I_2}\right| nepers$$

The value of the insertion loss depends on the values of the source and the load.

Q.34. Explain the basis for construction of Smith chart. Illustrate as to how it can be used as an admittance chart. **(8)**

Ans:
The use of circle diagram is cumbersome, i.e. S and βℓ circles are not concentric, interpolation is difficult and only a limited range impedance values can be obtained in a chart of reasonable size. The resistive component, R and reactive component, X of an impedance are represented in a rectangular form while R and X of an impedance are represented in circular form in the Smith charts. Smith charts can be used as impedance charts and admittance charts.
If the normalized admittance is $Y = Y/Y_O = g - jb$

- Any complex admittance can be shown by a single point, (the point of intersection R/Y_O circle and $j\ X/Z_O$) on the smith chart. Since the inductive resistance is negative susceptance, it lies in the region below the horizontal axis, and since capacitive reactance is positive susceptance, it lies in the region above the horizontal axis.
- The points of voltage maxima lie in the region 0 to 1 on the horizontal axis, since the conductance is equal to 1/S at such points. The points of voltage minima lie in the region 1 to 0 on the horizontal axis, since the conductance is equal to S at such points.
- The movement in the clockwise corresponds to travelling from the load towards generator and movement in the anti-clockwise corresponds to travelling from the generator towards load.
- Open circuited end will be point A and short circuited end will be B

Q.35. What is resonance? Why is it required in certain electronic circuits? Explain in detail **(6)**

Ans:
An a.c. circuit is said to be in resonance when the applied voltage and the circuit current are in phase. Thus at resonance the equivalent complex impedance of the circuit consists of only the resistance, the power factor of the circuit being unity. Resonant circuits are formed by the combination of inductance and capacitance, which may be connected in series or in parallel giving rise to series resonant and parallel resonant circuits, respectively. In other words property of cancellation of reactance when inductive and capacitive reactance are in series, or cancellation of susceptance when in parallel is called resonance. Such cancellation leads to operation of reactive circuit leading only to resistive circuit under unity power factor conditions, or with current and voltage in phase.
There are two types of resonance, series resonance and parallel resonance. Parallel resonance is normally referred to as anti-resonance. In a series resonant circuit, the impedance is purely resistive, the current is maximum at the resonant frequency and the current decreases on both sides of the resonant frequency. In a parallel resonant circuit, the impedance is maximum and purely resistive at the resonant frequency. The impedance decreases on both sides of the resonant frequency. It is easy to select frequencies around the resonant frequency and reject the other frequencies. The resonant circuits are also known as tuned circuits in particular parallel resonant circuit is also known as tank circuit. The resonant circuits or tuned circuits are used in electronic circuits to select a particular radio frequency signal for amplification.

Q.36. Determine the relationship between y-parameters and ABCD parameter for 2-Port networks. By using these relations determine y-parameters of circuit given in Fig.7 and then deduce its ABCD parameters. **(7)**

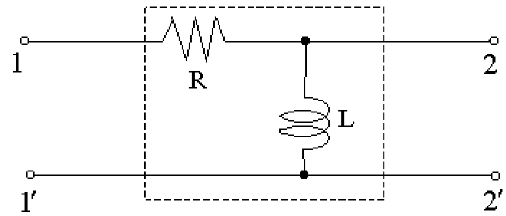


Fig.7

Ans:
For a Two port Network, the Y-Parameter equations are given by

$I_1 = Y_{11} V_1 + Y_{12}V_2$ (1)

$I_2 = Y_{21} V_1 + Y_{22}V_2$ (2)

The ABCD parameter equations are given by

$V_1 = AV_2 - B I_2$ (3)

$I_1 = CV_2 - D I_2$ (4)

From Equations 1 & 2

$Y_{21}V_1 = I_2 - Y_{22}V_2$

$$V_1 = I_2 (\frac{1}{Y_{21}}) - (\frac{Y_{22}}{Y_{21}})V_2$$

$$V_1 = (-\frac{Y_{22}}{Y_{21}})V_2 - (-\frac{1}{Y_{21}})I_2 \qquad --(5)$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 = Y_{11}\left[\left(\frac{-Y_{22}}{Y_{21}}\right)V_2 - \left(-\frac{1}{Y_{21}}\right)I_2\right] + Y_{12}V_2$$

$$I_1 = V_2\left[\frac{-Y_{11}Y_{22} + Y_{12}Y_{21}}{Y_{21}}\right] - (\frac{-Y_{11}}{Y_{21}})I_2 \qquad --(6)$$

Comparing equations (3) & (5), (4) & (6)

$$A = \frac{-Y_{22}}{Y_{21}}, \qquad B = \frac{-1}{Y_{21}}$$

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}, \qquad D = \frac{-Y_{11}}{Y_{21}}$$

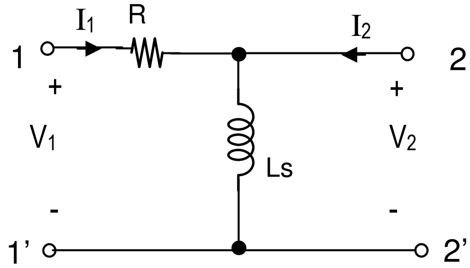


Fig 7.a.2

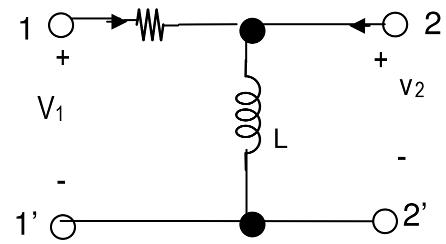


Fig 7.a.1

In s domain the equivalent circuit is shown in Fig 7.a.2

When port 2 is short circuited as in Fig 7.a.3, $V_2 = 0$

$V_1 = I_1R$

$$\Rightarrow Y_{11} = \frac{I_1}{V_1} = \frac{1}{R}$$

$$I_1 = -I_2$$

$$\Rightarrow Y_{21} = \frac{I_2}{V_1} = \frac{-I_1}{V_1} = -\frac{1}{R}$$

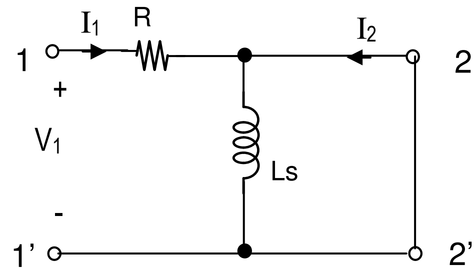


Fig 7.a.3

(To find Y_{11} and Y_{21})

When Port 1 is short circuited as in Fig 7.a.4, $V_1 = 0$

$$I_1 = -I_2 \times \frac{Ls}{R + Ls}$$

$$\text{Current in the Ls branch} = I_2 \times \frac{R}{R + Ls}$$

Voltage across Ls branch is

$$V_2 = Ls\left(\frac{R}{R+Ls}\right)\times I_2$$

$$\therefore Y_{22} = \frac{I_2}{V_2} = \frac{R+Ls}{RLs}$$

$$V_2 = I_2\left(\frac{LsR}{R+Ls}\right) = -I_1\left(\frac{R+Ls}{Ls}\right)\left(\frac{RLs}{R+Ls}\right)$$

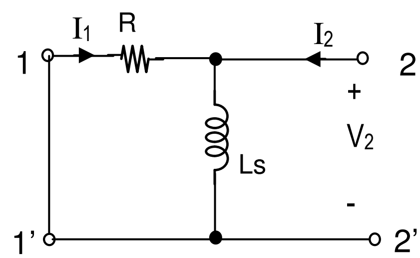


Fig **7.a.4**

(To find Y_{22} and Y_{12})

$$V_2 = -I_1(R)$$

$$\therefore Y_{12} = \frac{I_1}{V_2} = -\frac{1}{R}$$

$$A = \frac{-Y_{22}}{Y_{21}} = \frac{-(R+Ls)/RLs}{(-1/R)} = \frac{R+Ls}{Ls}$$

$$B = \frac{-1}{Y_{21}} = \frac{-1}{(-1/R)} = R$$

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}} = \frac{(-1/R)(-1/R) - (1/R)(R+Ls)/RLs}{(-1/R)} = \frac{-1}{R} + \frac{R+Ls}{RLs} = \frac{1}{Ls}$$

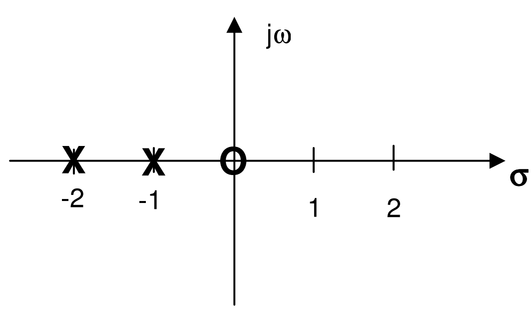
$$D = \frac{-Y_{11}}{Y_{21}} = \frac{(-1/R)}{(-1/R)} = 1$$

Q.37. What is the significance of poles and zeros in network functions. What is the criteria of stability of a network? For the transform current $I(s)=\frac{2s}{(s+1)(s+2)}$, plot its poles and zeros in s-plane and hence obtain the time domain response. **(7)**

Ans:
Poles and zeros provide useful information about the network functions.
(1) In impedance functions, a pole of the function implies a zero current for a finite voltage i.e. on open circuit, while a zero of the function implies no voltage for a finite current i.e. a short circuit.
(2) In admittance functions, a pole of the functions implies a zero voltage for a finite current, i.e. short circuit, while a zero of the function implies zero current for a finite value of voltage i.e. an open circuit.
(3) The poles determine the variation of the response while zero determine the magnitude the coefficients in the partial fraction expansion and hence determine the magnitude of the response.
Any active network or any general system is said to be stable if the transfer function has its poles confined to the left half of the s-plane.
A system will be stable if its polynomial roots have negative real parts.
The transform current is given by

$$I(s)=\frac{2s}{(s+1)(s+2)}$$

From the function it is clear that the function has poles at – 1 and –2 and a zero at the origin. The plot of poles and zeros in shown below:



(s+1) ad (s+2) are factors in the denominators, the time domain response is given by
$i(t)=K_1\,e^{-t}+K_2\,e^{-2t}$

To find the constants K_1 and K_2

From the pole zero plot
M_{01} = 1 and $\phi_{01} = 180^0$
Q_{21} = 1 and $\theta_{21} = 0^0$

$$K_1 = F\frac{M_{01}e^{j\phi_{01}}}{Q_{21}e^{j\theta_{21}}} = 2\frac{e^{j180^\circ}}{e^{j0^\circ}} = 2e^{j180^\circ} = -2$$

Where ϕ_{01} and ϕ_{02}, θ_{21} and θ_{12} are the angles of the lines joining the given pole to other finite zeros and poles.
Where M_{01} and M_{02} are the distances of the same poles from each of the zeros.
Q_{21} and Q_{12} are the distances of given poles from each of the other finite poles.
Similarly,
$M_{02} = 2$ and $\phi_{02} = 180^0$
$Q_{12} = 1$ and $\theta_{12} = 180^0$

$$K_2 = F\frac{M_{02}e^{j\phi_{02}}}{Q_{12}e^{j\theta_{12}}} = 2\frac{2\times e^{j180^\circ}}{1\times e^{j180^\circ}} = 4$$

Substituting the values of K_1 and K_2, the time domain response of the current is given by

$$i(t) = -2e^{-t} + 4e^{-2t}$$

Q.38. Determine the condition for resonance for the parallel circuit as shown in Fig.8. Determine it's
(i) resonant frequency ω_0 **(3)**
(ii) impedance z (j ω) at ω_0 **(3)**
(iii) half power bandwidth **(4)**
(iv) quality factor of the circuit. **(4)**

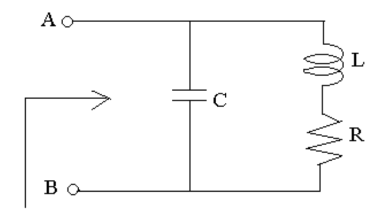


Fig. 8

Ans:
Consider an anti-resonant RLC circuit as shown in Fig. 9

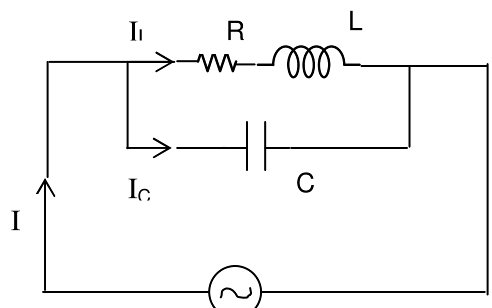


Fig 9.

The admittance

$$Y_L = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$Y_C = j\omega C$$

$$Y = Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

At resonance, the susceptance is zero.

$$\therefore \omega_0 C \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0 \quad \Rightarrow \omega_0 C \frac{\omega_0 L}{R^2 + \omega_0^2 L^2}$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\therefore \omega_0^2 = \frac{-R^2}{L^2} + \frac{1}{LC}$$

$$\therefore \omega_0 = \sqrt{\frac{-R^2}{L^2} + \frac{1}{LC}} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R is negligible

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}} \qquad \therefore f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(ii) At resonance, the susceptance is zero

$$\Rightarrow Y_o = \frac{R}{R^2 + \omega_o^2 L^2}$$

We know that

$$R^2 + \omega_0^2 L^2 = \frac{L}{C} \qquad \Rightarrow Y_o = \frac{RC}{L}$$

The impedance at resonance is $Z_O = \dfrac{L}{RC}$

(iii) Half power bandwidth: It is the band of frequencies, which lie on either side of the resonant frequency where the impedance falls to1/√2 of its value at resonance.
The impedance near resonance is

$$= \frac{(R + j\omega L)\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{(1 + \dfrac{j\omega L}{R})\dfrac{1}{j\omega C}}{1 + \dfrac{j\omega L}{R} - \dfrac{j}{\omega C}} = \frac{\dfrac{L}{RC} + \dfrac{1}{j\omega C}}{1 + \dfrac{j\omega L}{R}(1 - \dfrac{1}{\omega^2 LC})}$$

$$Z = \frac{\dfrac{L}{RC}(1 + \dfrac{R}{j\omega L})}{1 + \dfrac{j\omega L}{R}(1 - \dfrac{1}{\omega^2 LC})} \qquad \text{Let } \delta = \frac{\omega - \omega_o}{\omega_o} \Rightarrow \frac{\omega}{\omega_o} = 1 + \delta$$

$$\frac{\omega L}{R} = \frac{\omega_o}{\omega_o} \times \frac{\omega L}{R} = Q_o(1 + \delta)$$

$$\frac{1}{\omega^2 LC} = \frac{\omega_o^2}{\omega^2} \times \frac{1}{\omega_o^2 LC}$$

When Q_o is very large, R_L is very less then

$$\frac{1}{\omega^2 LC} = \frac{\omega_o^2}{\omega^2} = (1 + \delta)^2 \quad \text{as} \quad \omega_o^2 LC = 1$$

The impedance of the parallel resonant circuit is given by

$$Z = \frac{\dfrac{L}{RC}\left[1 - j\dfrac{1}{Q_o(1 + \delta)}\right]}{1 + jQ_o(1 + \delta)\left[1 - \dfrac{1}{(1 + \delta)^2}\right]} = \frac{Z_o\left[1 - j\dfrac{1}{Q_o(1 + \delta)}\right]}{1 + jQ_o(1 + \delta)\left[\dfrac{1 + \delta^2 + 2\delta - 1}{(1 + \delta)^2}\right]}$$

Where Z_o is the impedance at parallel resonance

$$Z = \frac{Z_o\left[1 - j\dfrac{1}{Q_o(1+\delta)}\right]}{1 + jQ_o\delta\left[\dfrac{\delta+2}{1+\delta}\right]}$$

If Z is very near to Z_o, $\delta \to 0$ as $\omega \to \omega_o$ then

$$Z = \frac{Z_o\left[1 - j\dfrac{1}{Q_o}\right]}{1 + jQ_o 2\delta}$$

As Q_o is very large

$$Z = \frac{Z_o}{1 + jQ_o 2\delta}$$

At half power points

$$Z = \frac{Z_o}{\sqrt{2}} \qquad \Rightarrow \left|\frac{Z}{Z_o}\right| = \frac{1}{\sqrt{1+(Q_o 2\delta)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sqrt{1+(Q_o 2\delta)^2} = \sqrt{2}$$

$$\Rightarrow Q_o 2\delta = \pm 1 \text{ or } \delta = \frac{1}{2Q_o}$$

Since $\delta = \dfrac{\omega - \omega_o}{\omega_o} = \dfrac{f - f_o}{f_o} = \pm\dfrac{1}{2Q_o}$

$$f_1 - f_o = -\frac{f_o}{2Q_o}, \qquad f_2 - f_o = +\frac{f_o}{2Q_o}$$

Bandwidth $f_2 - f_1 = \dfrac{f_o}{Q_o}$

(iv) The quality factor, Q of the circuit is given by

$$Q = \frac{\omega_o L}{R} = \frac{L}{R} \times \frac{1}{\sqrt{LC}} = \frac{1}{R} \times \sqrt{\frac{L}{C}}$$

Q.39. For the case of distributed parameters, determine the expressions for:
(i) Characteristic impedance (z_0)
(ii) Propagation constant (γ)
(iii) Attenuation and phase constants $(\alpha \text{ and } \beta)$ **(7)**

Ans:

(i) Consider a transmission line of length (Δx) units.

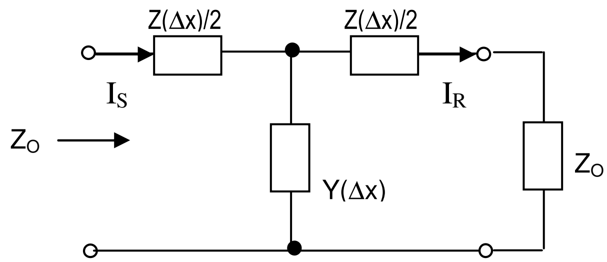


Fig 10.a

Where Z = R + jωL (series impedance per unit length)
and Y = G + jωC (admittance per unit length)
When the line is terminated in Z_O,

$$Z_o = \frac{Z}{2}(\Delta x) + \frac{\left[\frac{Z}{2}(\Delta x) + Z_o\right]\frac{1}{y(\Delta x)}}{\left[\frac{Z}{2}(\Delta x) + Z_o + \frac{1}{y(\Delta x)}\right]}$$

$$Z_o = \frac{\frac{Z}{2}(\Delta x)\left[\frac{Z}{2}(\Delta x) + Z_o + \frac{1}{y(\Delta x)}\right] + \left[\frac{Z}{2}(\Delta x) + Z_o\right]\frac{1}{y(\Delta x)}}{\left[\frac{Z}{2}(\Delta x) + Z_o + \frac{1}{y(\Delta x)}\right]}$$

$$Z_o\left[\frac{Z}{2}(\Delta x) + Z_o + \frac{1}{Y(\Delta x)}\right] = \frac{Z}{2}(\Delta x)\left[\frac{Z}{2}(\Delta x) + Z_o + \frac{1}{Y(\Delta x)}\right] + \left[\frac{Z}{2}(\Delta x) + Z_o\right]\frac{1}{Y(\Delta x)}$$

$$\Rightarrow Z_o[Z_o] = \frac{Z}{2}(\Delta x)\left[\frac{Z}{2}(\Delta x) + \frac{1}{Y(\Delta x)}\right] + \left[\frac{Z}{2}(\Delta x)\right]\frac{1}{Y(\Delta x)}$$

$$\Rightarrow Z_o^2 = \frac{Z^2}{4}(\Delta x)^2 + \frac{Z(\Delta x)}{Y(\Delta x)} \qquad \Rightarrow Z_o = \sqrt{\frac{Z^2}{4}(\Delta x)^2 + \frac{Z}{Y}} \qquad \text{as } \Delta x \to 0$$

$$Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$$I_R = I_S\left[\frac{\frac{1}{Y}}{\frac{1}{Y} + \frac{Z}{2} + Z_O}\right]$$

$$\therefore e^{\gamma} = \frac{I_S}{I_R} = 1 + \frac{ZY}{2} + Z_O Y$$

(ii) Propagation constant (γ)

For a line of length Δx,

$$e^{\gamma\Delta x} = 1 + \frac{Z\Delta x \cdot Y\Delta x}{2} + Z_O Y\Delta x$$

$$= 1 + \frac{Z \cdot Y (\Delta x)^2}{2} + \sqrt{\frac{Z}{Y}} \cdot Y\Delta x$$

$$= 1 + \sqrt{Z \cdot Y}\,\Delta x + \frac{Z \cdot Y (\Delta x)^2}{2} \qquad Eq\ (1)$$

From the theory of exponential series

$$e^{\gamma\Delta x} = 1 + \gamma\Delta x + \frac{\gamma^2 (\Delta x)^2}{\underline{|2}} + - - -$$

as $\Delta x \to 0$ *terms containing* $(\Delta x)^3$ *and higher are neglected* .

$$\therefore e^{\gamma\Delta x} = 1 + \gamma\Delta x + \frac{\gamma^2 (\Delta x)^2}{2} \qquad Eq\ (2)$$

on comparing $Eq\,(1)$ *and* $Eq\,(2)$

$$\gamma = \sqrt{Z \cdot Y} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

(iii) Attenuation and phase constants $(\alpha \text{ and } \beta)$

We know that

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$(\alpha + j\beta)^2 = (R + j\omega L)(G + j\omega C)$$

$$\Rightarrow \alpha^2 - \beta^2 + 2j\alpha\beta = RG - \omega^2 LC + j\omega(LG + RC)$$

Equating the real parts

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

$$(\alpha + j\beta) = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$|\alpha + j\beta| = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} = \alpha^2 + \beta^2$$

$$\Rightarrow \alpha^2 + \beta^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}$$

Adding equations (3) and (4)

$$2\alpha^2 = RG - \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}$$

$$\alpha = \sqrt{\frac{1}{2}\left[RG - \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}\right]}$$

Subtracting equation (3) from (4)

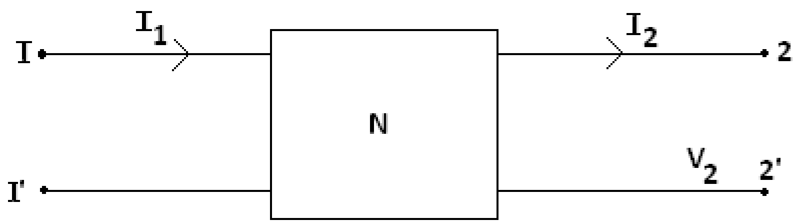
$$2\beta^2 = -RG + \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}$$

$$\beta = \sqrt{\frac{1}{2}\left[\omega^2 LC - RG + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}\right]}$$

Q.40. Define ABCD parameter

Ans:

The voltage and current of the input port are expressed in terms of the voltage and current of the output port. The equations are given by



2 Port network

$V_1 = AV_2 - BI_2$ ---- eq 6.a.1

$I_1 = CV_2 - DI_2$ ---- eq 6.a.2

When the output terminal is open circuited, $I_2 = 0$

$V_1 = AV_2$ $\qquad I_1 = CV_2$

$$A = \frac{V_1}{V_2} \qquad C = \frac{I_1}{V_2}$$

When the output terminal is short circuited, $V_2 = 0$

$V_1 = - BI_2$ $\qquad I_1 = - DI_2$

$$B = \frac{V_1}{-I_2} \qquad D = \frac{I_1}{-I_2}$$

A network is said to be reciprocal if the ratio of the response variable to the excitation variable remains identical even if the positions of the response and the excitation are interchanged.

A network is said to be symmetrical if the input and output ports can be interchanged without altering the port voltages and currents.

With excitation V_1 at input port and shorting the output

$V_1 = - BI_2$

$$\frac{I_2}{V_1} = -\frac{1}{B} \quad \text{--------- eq 6.a.3}$$

With interchange of excitation and response, the voltage source V_2 is at the output port while the short circuit current I_1 is obtained at the input port.

$0 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$$I_2 = \frac{AV_2}{B} \text{ and}$$

$$I_1 = CV_2 - \frac{ADV_2}{B} = V_2\left[C - \frac{AD}{B}\right]$$

$$\frac{I_1}{V_2} = \frac{BC - AD}{B} \quad \text{---------- eq 6.a.4}$$

When $V_1 = V_2$, the left hand side of eq.6.a.3 and eq.6.a.4 will be identical provided AD - BC = 1

Thus the condition of reciprocity is given by

AD - BC = 1

From the concept of symmetry for Z-parameter network,

$$Z_{11} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{A}{C} \qquad \text{at } I_2 = 0$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{D}{C} \qquad \text{at } I_1 = 0$$

For a symmetrical for Z-parameter network $Z_{11}=Z_{22}$
This implies for the ABCD parameters

$$\frac{A}{C} = \frac{D}{C} \qquad \textbf{OR}$$

A = D is a condition for symmetry.

Q.41. State the types of distortions in a transmission line. Derive the conditions to eliminate the two types of distortions. **(8)**

Ans:
Distortion is said to occur when the frequencies are attenuated by different amounts or different frequency components of a complex voltage wave experience different amounts of phase shifts. Distortions in transmission lines are of two types:
Frequency distortion: when various frequency components of the signal are attenuated by different amounts then frequency distortion is said to occur. When the attenuation constant α is not a function of frequency, there is no frequency distortion.

$$\alpha = \sqrt{\frac{1}{2}(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}$$

To eliminate frequency distortion

$$\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} = \omega^2 LC + K$$

$$\Rightarrow (R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2) = \omega^4 L^2 C^2 + K^2 + 2K\omega^2 LC$$

$$\Rightarrow R^2 G^2 + \omega^4 L^2 C^2 + \omega^2 (L^2 G^2 + R^2 C^2) = \omega^4 L^2 C^2 + K^2 + 2K\omega^2 LC$$

Comparing the coefficients of ω^2 and the constant terms
K = RG and
Comparing the coefficients of ω^2 and the constant terms
K = RG and
$L^2G^2 + R^2C^2 = 2KLC$
$\therefore L^2G^2 + R^2C^2 = 2RGLC$
$\therefore (LG - RC)^2 = 0$
$\therefore LG = RC$

$$\frac{R}{L} = \frac{G}{C}$$

Delay distortion: When various frequency components arrive at different times (delay is not constant) then delay distortion or phase distortion is said to occur. When the

phase velocity is independent of frequency or phase constant β is a constant multiplied by ω, there is no delay distortion or phase distortion.

$$\beta = \sqrt{\frac{1}{2}(\omega^2 LC - RG) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}$$

To eliminate delay distortion

$\beta = \omega K$

$$\Rightarrow 2\varpi^2 K^2 - \varpi^2 LC + RG = \sqrt{(R^2 + \varpi^2 L^2)(G^2 + \varpi^2 C^2)}$$

$$\Rightarrow (\varpi^2(2K^2 - LC) + RG)^2 = (R^2 + \varpi^2 L^2)(G^2 + \varpi^2 C^2)$$

$$\Rightarrow \varpi^4(2K^2 - LC)^2 + R^2G^2 + 2\varpi^2(2K^2 - LC)RG = R^2G^2 + \varpi^4 L^2C^2 + \varpi^2(L^2G^2 + R^2C^2)$$

Comparing the coefficients of ω^4 and ω^2

$(2K^2 - LC)^2 = L^2C^2$

$4K^4 + L^2C^2 - 4K^2 LC = L^2C^2$

$2K^2(K^2 - LC) = 0$

K = 0 or

$K = \sqrt{LC}$

$L^2G^2 + R^2C^2 = 2RG(2K^2 - LC)$

$L^2G^2 + R^2C^2 = 2RG(2LC - LC)$

$\therefore L^2G^2 + R^2C^2 = 2RGLC$

$\therefore (LG - RC)^2 = 0$

$\therefore LG = RC$

$$\frac{R}{L} = \frac{G}{C}$$

$\therefore$ The condition to eliminate both the frequency and delay distortions is

$$\frac{R}{L} = \frac{G}{C}$$

Q.42. Derive the design equations of an asymmetrical lattice attenuator to have a characteristic impedance of $R_0\Omega$ and attenuation of N in nepers. **(8)**

Ans:

The open circuit impedance of the lattice network looking from the input terminals a-b is

$$R_{oc} = \frac{(R_1 + R_2)(R_1 + R_2)}{R_1 + R_2 + R_1 + R_2} = \frac{(R_1 + R_2)}{2}$$

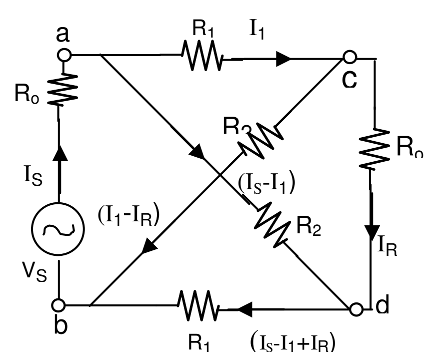


Fig 9.b.1

The short circuit impedance of the lattice network looking from the input terminals a-b is

$$R_{sc} = \frac{R_1R_2}{R_1 + R_2} + \frac{R_1R_2}{(R_1 + R_2)} = \frac{2R_1R_2}{(R_1 + R_2)}$$

The characteristic impedance of the two-port network is given by

$$R_o = \sqrt{R_{oc}R_{sc}} = \sqrt{\frac{2R_1R_2}{(R_1 + R_2)} \times \frac{(R_1 + R_2)}{2}}$$

$$R_o = \sqrt{R_1R_2}$$

Applying kirchoff's voltage law for the loop *acdb* of Fig 9.b.1

$$I_1 R_1 + I_R R_o + (I_s - I_1 + I_R) R_1 = V_s = I_s R_o$$
$$I_s R_o = I_R R_o + I_s R_1 + I_R R_1$$
$$I_s (R_o - R_1) = I_R (R_o + R_1)$$
$$\frac{I_s}{I_R} = \frac{(R_o + R_1)}{(R_o - R_1)} = N$$

$\therefore$ Attenuation

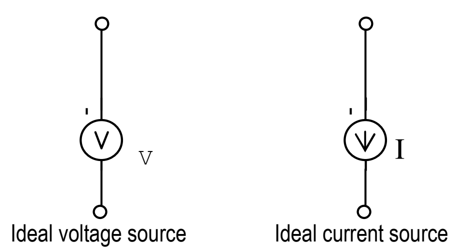
$$N = \frac{(R_o + R_1)}{(R_o - R_1)}$$
$$\Rightarrow N(R_o - R_1) = (R_o + R_1)$$
$$\Rightarrow R_1 (N + 1) = R_o (N - 1)$$
$$\Rightarrow R_1 = R_o \left[\frac{N - 1}{N + 1} \right]$$

Since

$$R_o = \sqrt{R_1 R_2}$$
$$R_2 = \frac{R_o^{\ 2}}{R_1} = \frac{R_o^{\ 2} (N + 1)}{R_o (N - 1)}$$
$$R_2 = R_o \left[\frac{N + 1}{N - 1} \right]$$

Q.43. Define an ideal voltage source and an ideal current source. **(2+2)**

Ans:
An ideal voltage source is one in which the internal resistance is zero, the voltage across the source is equal to the terminal voltage, and is independent of the load current. An ideal current source is one in which the internal conductance is zero, the resistance being in parallel with the source, and is independent of the load resistance.



Q.44. Define input impedance of a transmission line. Derive an expression for the input impedance of a line and show that Z_{in} for a lossless line is

$$Z_{in} = Z_o \frac{Z_R + jZ_o \tan\frac{2\pi\ell}{\lambda}}{Z_o + jZ_R \tan\frac{2\pi\ell}{\lambda}}.$$ **(14)**

Ans:
Input impedance of transmission line is defined as the impedance measured across the input terminals. If V_s is the sending end voltage and I_s is the sending end current then Input impedance, Z_{in} is given by

$$Z_{in} = \frac{V_s}{I_s}$$

Consider a transmission line of length ℓ terminated in an impedance Z_R. Let V_R be the voltage cross Z_R and I_R be the current flowing through it.
The voltage, V and current, I at a point distance x from the sending end of a transmission line are given by

$$V = V_R \cosh\gamma(l-x) + I_R Z_o \sinh\gamma(l-x)$$

$$I = I_R \cosh\gamma(l-x) + \frac{V_R}{Z_o}\sinh\gamma(l-x)$$

At the sending end, x = 0, V = V_s and I = I_s

$$V_s = V_R \cosh\gamma l + I_R Z_o \sinh\gamma l$$

$$I_s = I_R \cosh\gamma l + \frac{V_R}{Z_o}\sinh\gamma l$$

$$\therefore Z_{in} = \frac{V_s}{I_s} = \frac{V_R \cosh\gamma l + I_R Z_o \sinh\gamma l}{I_R \cosh\gamma l + \frac{V_R}{Z_o}\sinh\gamma l}$$

$$Z_{in} = Z_o \frac{\frac{V_R}{I_R}\cosh\gamma l + Z_o \sinh\gamma l}{\frac{V_R}{I_R}\sinh\gamma l + Z_o \cosh\gamma l}$$

$$Z_{in} = Z_o \frac{Z_R + Z_o \tanh\gamma l}{Z_R \tanh\gamma l + Z_o} \qquad \because Z_R = \frac{V_R}{I_R}$$

For a lossless line $\alpha = 0$, $\gamma = j\beta$

$$Z_{in} = Z_o \frac{Z_R + Z_o \tanh j\beta l}{Z_o + Z_R \tanh j\beta l}$$

But tanh jβ = j tan β

$$Z_{in} = Z_o \frac{Z_R + jZ_o \tan \beta l}{Z_o + jZ_R \tan \beta l}$$

$$Since \quad \beta = \frac{2\pi}{\lambda}$$

$$\therefore Z_{in} = Z_o \frac{Z_R + jZ_o \tan \frac{2\pi}{\lambda} l}{Z_o + jZ_R \tan \frac{2\pi}{\lambda} l}$$

Q.45. Explain how a quarter wave transformer is used for impedance matching. **(6)**

Ans:

A transmission line of length equal to one-fourth of the wavelength of the fundamental frequency of the wave propagating through it is called quarter wave transformer. Quarter wave transformer is used for impedance matching particularly at high frequencies. The input impedance (Z_{in}) of a uniform line is given by

$$Z_{in} = Z_o \left[\frac{Z_R \cosh(\gamma l) + Z_o \sinh(\gamma l)}{Z_o \cosh(\gamma l) + Z_R \sinh(\gamma l)} \right]$$

Where Z_R is the terminating impedance, Z_o is the characteristic impedance of the line, □ is the propagation constant of the line and l is the length of the line.
At high frequencies the line is lossless, i.e. $\alpha = 0$ and $\gamma = j\beta$

$$Z_{in} = Z_o \left[\frac{Z_R \cosh(j\beta l) + Z_o \sinh(j\beta l)}{Z_o \cosh(j\beta l) + Z_R \sinh(j\beta l)} \right]$$

For a line of quarter wavelength

$$l = \frac{\lambda}{4}, \beta = \frac{2\pi}{\lambda}$$

$$\Rightarrow \beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_o \left[\frac{Z_R \cos\frac{\pi}{2} + jZ_o \sin\frac{\pi}{2}}{Z_o \cos\frac{\pi}{2} + jZ_R \sin\frac{\pi}{2}} \right] = Z_o \left[\frac{jZ_o}{jZ_R} \right] = \frac{Z_o^{\ 2}}{Z_R}$$

$$Z_o = \sqrt{Z_{in} \times Z_R}$$

The impedance at the input of a quarter wave transformer depends on the load impedance and characteristic impedance of the transmission line. When Z_o varies, the input impedance of the quarter wave transformer also changes accordingly. The load is matching to the characteristic impedance of the transmission line. When Z_R is high, the input impedance is low and vice-versa, Z_o being constant. If the load is inductive the input impedance is capacitive and vice-versa. The quarter wave transformer has a length λ/4 at only one frequency and then is highly frequency dependant.

Q.46. Derive the expression of resonant frequency for a parallel R-L-C circuit in terms of Q, R, L and C. **(8)**

Ans:
Consider an anti-resonant RLC circuit as shown in Fig. 9.a.i
When the capacitor is perfect and there is no leakage and dielectric loss.
i.e. $R_C = 0$ and let $R_L = R$ as shown in Fig 9.a.ii

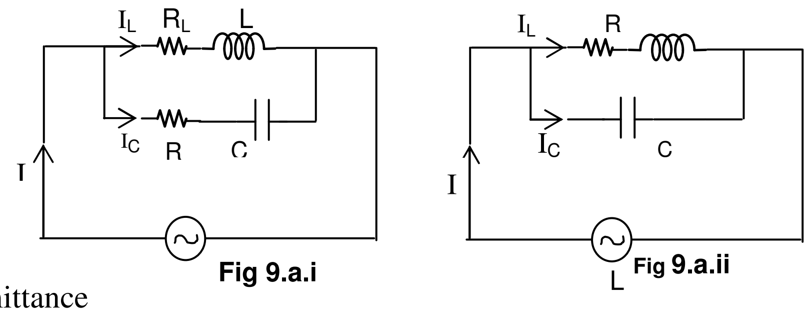


Fig 9.a.i Fig 9.a.ii

The admittance

$$Y_L = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$Y_C = j\omega C$$

$$Y = Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

At resonance, the susceptance is zero.

$$\therefore \omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0{}^2 L^2} = 0 \qquad \Rightarrow \omega_0 C = \frac{\omega_0 L}{R^2 + \omega_0{}^2 L^2}$$

$$R^2 + \omega_0{}^2 L^2 = \frac{L}{C}$$

$$\therefore \omega_0{}^2 = \frac{-R^2}{L^2} + \frac{1}{LC}$$

$$\therefore \omega_0 = \sqrt{\frac{-R^2}{L^2} + \frac{1}{LC}} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R is negligible $\therefore \omega_0 = \frac{1}{\sqrt{LC}} \;\therefore f_0 = \frac{1}{2\pi\sqrt{LC}}$

The quality factor, Q of the circuit is given by

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\Rightarrow \omega_o = \frac{RQ}{L} \quad or \quad \omega_o = \frac{1}{QCR}$$

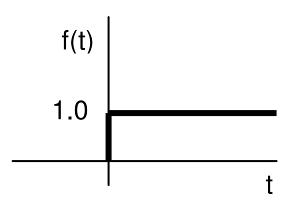
Q.47. Write short notes on:-
(i) Smith chart and its applications.
(ii) Poles and zeros of a network function. **(10)**

Ans:
(i) Smith chart and its applications.
The Smith chart, developed by P.H.Smith in 1938-39 is plotted within a circle of unit radius of reflection coefficient that is it is a circular chart drawn with an outside radius that represents unit magnitude of reflection coefficient (K = 1). The magnitude of K is represented by the radius from the centre. The lines representing constant values of r and x (i.e., resistance and reactance) are superimposed on this chart. The lines of constants r and x form two orthogonal families of circles. The smith chart plots K in polar co-ordinates and has lines corresponding to (Z_R/Z_O) i.e., normalized Z. the advantages of Smith Chart are that the lines are less crowded in the low impedance region and also that the usual values of K and Z are contained in the finite area. The values of Z are much easier to read when plotted.
Smith Charts are used for:
- ❖ Smith chart as admittance diagram.
- ❖ Converting impedance into admittance.
- ❖ Determination of input impedance.
- ❖ Determination of load impedance.
- ❖ Determination of input impedance and admittance of short-circuited line.
- ❖ Determination of input impedance and admittance of on open-circuited line.

(ii) Poles and zeros of a network junction.
Poles and zeros provide useful information about the network functions.
(1) In impedance functions, a pole of the function implies a zero current for a finite
(2) In admittance functions voltage i.e. on open circuit, while a zero of the function implies no voltage for a finite current i.e. a short circuit., a pole of the functions implies a zero voltage for a finite current, i.e. short circuit, while a zero of the function implies zero current for a finite value of voltage i.e. an open circuit.
(3) The poles determine the variation of the response while zero determines the magnitude of the coefficients in the partial fraction expansion and hence determine the magnitude of the response.
Any active network or any general system is said to be stable if the transfer function has its poles confined to the left half of the s-plane.
A system will be stable if its polynomial roots have negative real parts.

Q.48. Define unit step, Sinusoidal, Co sinusoidal function. Derive the Laplace transforms for these function. **(8)**

Ans:
The unit step function is defined as
u(t) = 0 $t \le 0$
         1    $t > 0$
The laplace transform is given by



$$F(s) = Lu(t) = \int_0^\infty e^{-st}dt = \left[\frac{-1}{s}e^{-st}\right]_0^\infty = \frac{1}{s}$$

The Sinusoidal function is defined as

$$F(t) = \sin\alpha t = \frac{(e^{j\alpha t} + e^{-j\alpha t})}{2j}$$

The laplace transform is given by

$$F(s) = \frac{1}{2j}\int_0^\infty (e^{j\alpha t} - e^{-j\alpha t})e^{-st}dt = \frac{1}{2j}\int_0^\infty (e^{-(s-j\alpha)t} - e^{-(s+j\alpha)t})dt$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\alpha} - \frac{1}{s+j\alpha}\right]_0^\infty = \frac{\alpha}{s^2+\alpha^2}$$

The Co sinusoidal function is defined as

$$F(t) = \cos\alpha t = \frac{1}{\alpha}\frac{d}{dt}(\sin\alpha t)$$

The laplace transform is given by

$$F(s) = \frac{1}{\alpha}\left[sF(s) - f(0)\right]$$

$$= \frac{1}{\alpha}\left[s\frac{\alpha}{s^2+\alpha^2} - 0\right] = \frac{s}{s^2+\alpha^2}$$

Q.49. Why is loading of lines required? Explain the different methods of loading a line. **(6)**

Ans:

The transmission properties of the line are improved by satisfying the condition $\frac{L}{R} = \frac{C}{G}$, where, L is the inductance of the line, R is the resistance, C is the capacitance and G is the capacitance of the line. The above condition is satisfied either by increasing L or decreasing C. C cannot be reduced since it depends on the construction. The process of increasing the value of L to satisfy the condition $\frac{L}{R} = \frac{C}{G}$ so as to reduce attenuation and distortions of the line is known as "loading of the line".

It is done in two ways. (i) Continuous loading. (ii) Lumped loading.

- Continuous loading: Continuous loading is done by introducing the distributed inductance throughout the length of the line. Here one type of iron or some other material as mumetal is wound around the conductor to be loaded thus increasing the permeability of the surrounding medium. Here the attenuation increases uniformly with increase in frequency. It is used in submarine cables. This type of loading is costly.
- Lumped loading: Lumped loading is done by introducing the lumped inductances in series with the line at suitable intervals. A lumped loaded line behaves as a low pas filter. The lumped loading is usually provided in open wire lines and telephone cables. The a.c resistance of the loading coil varies with frequency due to hysteresis and eddy current losses and hence a transmission line is never free from distortions.

Q.50. State and prove the theorem connecting the attenuation constant, α and characteristic impedance Z_o of a filter. **(4)**

Ans:

Theorem connecting α and Z_o: If the filter is correctly terminated in its characteristic impedance Z_o, then over the range of frequencies, for which the characteristic impedance Z_o of a filter is purely resistive (real), the attenuation is zero and over the range of frequencies for which characteristic impedance Z_o of a filter is purely reactive, the attenuation is greater than zero. Proof: - The characteristic impedance Z_o is given by

$$Z_{0T} = \sqrt{\frac{Z^2{}_1}{4} + Z_1 Z_2}$$

The propagation constant γ is given by

$$\frac{I_S}{I_R} = e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_o}{Z_2} \text{ , where } \gamma = \alpha + j\beta$$

The attenuation constant α is given by

$$\alpha = 20\log_{10}\left|\frac{I_S}{I_R}\right|$$

Let $Z_1 = jX_1$, and $Z_2 = jX_2$

$$\therefore Z_{oT} = \sqrt{\frac{j^2 X_1{}^2}{4} + jX_1 X_2}$$

$$= j\sqrt{\frac{X_1{}^2}{4} + X_1 X_2}$$

Where X_1 and X_2 are real but positive or negative

$$\therefore \frac{I_S}{I_R} = 1 + \frac{X_1}{2X_2} - \frac{jZ_o}{X_2}$$

Depending upon, the sign of X_1 and X_2

a) $\frac{X_1{}^2}{4} + X_1 X_2$ is negative = - A

b) $\frac{X_1{}^2}{4} + X_1 X_2$ is positive = B

Case (a) Z_0 is purely real (purely resistive)

$$Z_0 = j\sqrt{-A} = \sqrt{A}$$

$$\frac{I_S}{I_R} = 1 + \frac{X_1}{2X_2} - j\frac{\sqrt{A}}{X_2}$$

$$\left|\frac{I_S}{I_R}\right| = \sqrt{\left(1 + \frac{X_1}{2X_2}\right)^2 + \left(\frac{-X_1^2}{4X_2^2} - \frac{X_1 X_2}{X_2^2}\right)}$$

$$= \sqrt{1 + \frac{X_1}{X_2} + \frac{X_1^2}{4X_2^2} - \frac{X_1^2}{4X_2^2} - \frac{X_1}{X_2}}$$

$$\left|\frac{I_S}{I_R}\right| = 1$$

$\therefore \alpha = 20 \times \log_{10} 1 = 0$, If Z_o is real

Case (b) Z_0 is imaginary (purely reactive)

$$Z_0 = j\sqrt{\frac{X_1^2}{4} + X_1 X_2} = jB$$

$$\frac{I_S}{I_R} = 1 + \frac{X_1}{2X_2} - j\frac{Z_o}{X_2}$$

$$= 1 + \frac{X_1}{2X_2} + \frac{\sqrt{B}}{X_2}$$

$$\left|\frac{I_S}{I_R}\right| = 1 + \frac{X_1}{2X_2} + \sqrt{\frac{X_1^2}{4X_2^2} + \frac{X_1}{X_2}}$$

Which is real and greater than unity

$\therefore \alpha \neq 0$, If Z_o is purely reactive

Q.51. With the help of frequency response curves, give the classification of filters. **(4)**

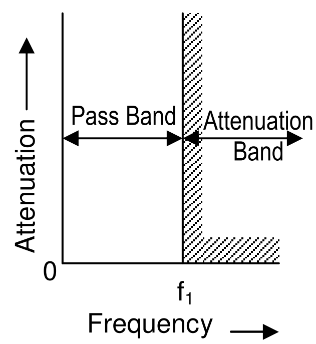
Ans:

According to their frequency response curves, the filters are classified as:

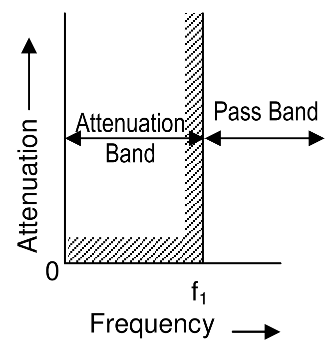
(i) Low Pass Filters. (ii) High Pass Filters.

(iii) Band Pass Filters. (iv) Band Stop Filters.

(i) Low Pass Filters: In a low pass filter, the pass band extends from zero to cut-off frequency f_1 in which region the attenuation is zero. The attenuation band extends from cut-off frequency f_1 to infinity and in this range the attenuation is large as shown in Fig 9.c.1.

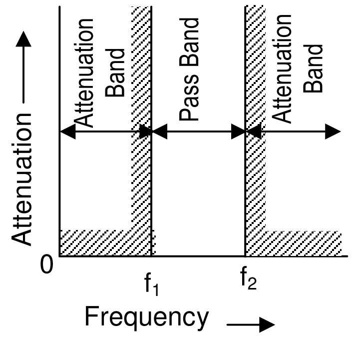


9.c.1 Low Pass Filter

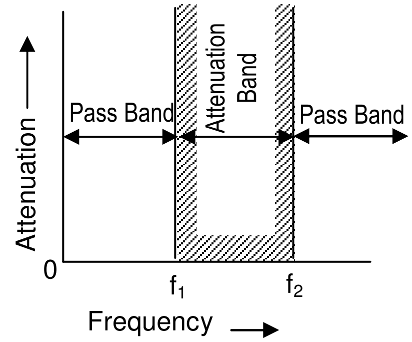


9.c.2 High Pass Filter

(ii) High Pass Filters: In a high pass filter, the pass band extends from cut-off frequency f_1 to infinity in which region the attenuation is zero. The attenuation band extends from zero to cut-off frequency f_1 and in this range the attenuation is large as shown in Fig 9.c.2.



9.c.3 Band Pass Filter



9.c.4 Band Stop Filter

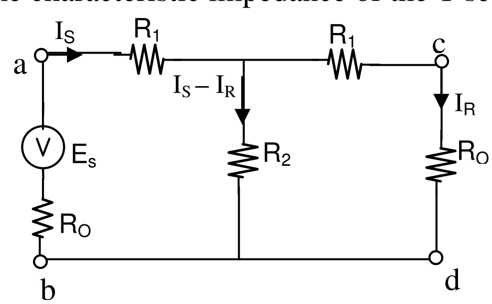
(iii) Band Pass Filters: In a band pass filter, the pass band extends from cut-off frequency f_1 to cut-off frequency f_2 in which region the attenuation is zero. The attenuation band extends from zero to cut-off frequency f_1 and cut-off frequency f_2 to infinity and in this range the attenuation is large as shown in Fig 9.c.3.

(iv) Band Stop Filters: In a band stop filter, the pass band extends from zero to cut-off frequency f_1 and cut-off frequency f_2 to infinity in which region the attenuation is zero. The attenuation band extends from cut-off frequency f_1 to cut-off frequency $f_2$and in this range the attenuation is large as shown in Fig 9.c.4.

Q.52. Derive an expression for design impedance of a symmetrical T attenuator. **(8)**

Ans:

The series arm impedances are given by R_1 and shunt arm impedance by R_2. A voltage source with internal impedance R_O is applied at the input port (a - b) and a resistor equivalent to the characteristic impedance of the T section is connected at port (c - d).



On applying kirchoff's voltage law to the second loop

$$(I_S - I_R)R_2 - I_R(R_1 + R_O) = 0$$

$$(I_S - I_R)R_2 = I_R(R_1 + R_O)$$

$$I_S R_2 = I_R(R_1 + R_2 + R_O)$$

$$\Rightarrow \frac{I_S}{I_R} = \frac{(R_1 + R_2 + R_O)}{R_2}$$

$$\therefore N = \frac{(R_1 + R_2 + R_O)}{R_2} \qquad \left[\because N = \frac{I_S}{I_R}\right] \qquad \text{---- Eq.1}$$

$$i.e \quad N - 1 = \frac{(R_1 + R_O)}{R_2} \qquad \Rightarrow R_O = R_2(N-1) - R_1$$

When the terminal (a-b) is open, the input impedance looking through the terminal a – b is given by

$$R_O = \mathrm{R}_1 + \frac{R_2(\mathrm{R}_1 + \mathrm{R}_O)}{R_2 + \mathrm{R}_1 + \mathrm{R}_O}$$

$$R_O = \frac{\mathrm{R}_1(R_2 + \mathrm{R}_1 + \mathrm{R}_O) + R_2(\mathrm{R}_1 + \mathrm{R}_O)}{R_2 + \mathrm{R}_1 + \mathrm{R}_O}$$

$$\mathrm{R}_O(R_2 + \mathrm{R}_1 + \mathrm{R}_O) = \mathrm{R}_1(R_2 + \mathrm{R}_1 + \mathrm{R}_O) + R_2(\mathrm{R}_1 + \mathrm{R}_O)$$

$$\mathrm{R}_O{}^2 + R_2\mathrm{R}_O + \mathrm{R}_1\mathrm{R}_O = \mathrm{R}_1 R_2 + \mathrm{R}_1{}^2 + \mathrm{R}_O\mathrm{R}_1 + \mathrm{R}_1 R_2 + \mathrm{R}_O R_2$$

$$\mathrm{R}_O{}^2 = 2\mathrm{R}_1 R_2 + \mathrm{R}_1{}^2$$

Q.53. Explain how the reactance and impedance of a high pass filter varies with frequency. **(8)**

Ans:

Let the total series arm impedance be $Z_1 = \dfrac{1}{j\omega C}$

And the shunt impedance be $Z_2 = j\omega L$

Hence $Z_1 Z_2 = j\omega L \times \dfrac{1}{j\omega C} = \dfrac{L}{C} = R_o^{\,2}$, $\mathrm{R_o}$ being a real quantity.

The characteristic impedance of a T section is given by

$$Z_{OT} = \sqrt{\left(\frac{Z_1}{2}\right)^2 + Z_1 Z_2}$$

The reactance frequency curve is shown below.

For a HPF,

$$Z_{OT} = \sqrt{-\frac{1}{4\omega^2 C^2} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}}\sqrt{1 - \frac{1}{4\omega^2 LC}}$$

$$= R_o\sqrt{1 - \frac{1}{4\omega^2 LC}}$$

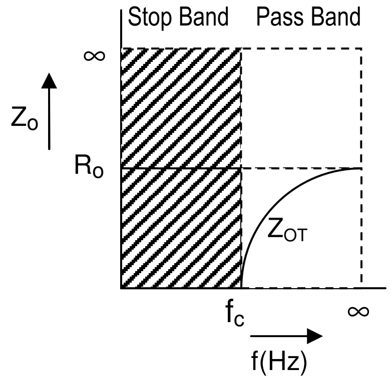
The cut off frequency is given by

$$4\omega_c^{\,2} LC = 1$$

$$\omega_c^{\,2} = \frac{1}{4LC} \qquad \Rightarrow \omega_c = \frac{1}{2\sqrt{LC}}$$

$$\therefore f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$\therefore Z_{OT} = R_o\sqrt{1 - \frac{1}{4\omega^2 LC}} = R_o\sqrt{1 - \frac{\omega_c^{\,2}}{\omega^2}} = R_o\sqrt{1 - \frac{f_c^{\,2}}{f^2}}$$



Z_O profile of HPF

Q.54. Differentiate between:
(i) Unilateral and Bilateral elements. Give examples.
(ii) Distributed and lumped elements. **(8)**

Ans:

(i) Bilateral elements: Network elements are said to be bilateral elements if the magnitude of the current remains the same even if the polarity of the applied voltage is changed. The bilateral elements offer the same impedances irrespective of the flow of current.

e.g. Resistors, Capacitors, and Inductors.

Unilateral elements: Network elements are said to be unilateral elements if the magnitude of the current passing through and element is affected, when the polarity of the applied voltage is changed. The unilateral elements offer varying impedances with variations in the flow of current.

e.g. Diodes, Transistors.

(ii) Lumped elements: When the circuit elements are lumped as single parameters like single resistance, inductance, capacitance then they are known as lumped elements.

Distributed elements: When the elements are distributed throughout the entire line, which are not physically separable then they are known as distributed elements

Q.55. Determine the input impedance of a transmission line when the far end is:
(i) Short circuited. (ii) Open circuited. **(6)**

Ans:

In general the input impedance of a transmission line, when the far end is terminated by any impedance Z_R, is given by

$$Z_{in} = Z_o\left[\frac{Z_R + jZ_o \tan \beta l}{Z_o + jZ_R \tan \beta l}\right]$$

(i) When the far end is open circuited, $Z_R = \infty$ and $I_R = 0$

$$Z_{in} = Z_o\left[\frac{1 + j\dfrac{Z_o}{Z_R}\tan \beta l}{\dfrac{Z_o}{Z_R} + j\tan \beta l}\right] = Z_o\left[\frac{1}{j\tan \beta l}\right] = -jZ_o \cot \beta l$$

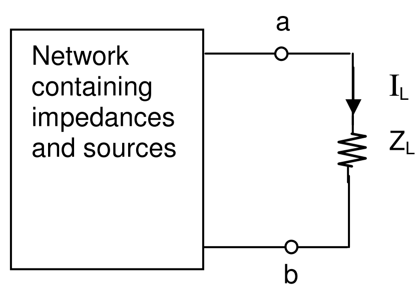
(ii) When the far end is short circuited, $Z_R = 0$ and $I_R = \infty$

$$Z_{in} = Z_o \left[\frac{0 + jZ_o \tan \beta l}{Z_o + 0} \right] = jZ_o \tan \beta l$$

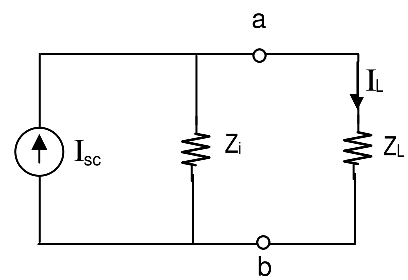
Q.56. State Norton's theorem. Derive the Thevenin's equivalent from a given Norton equivalent circuit. **(8)**

Ans:

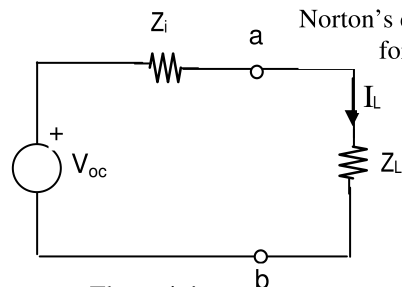
Norton's theorem states that the current in any load impedance Z_L connected to the two terminals of a network is the same as if this load impedance Z_L were connected to a current source (called Norton's equivalent current source) whose source current is the short circuit current at the terminals and whose internal impedance (in shunt with the current source) is the impedance of the network looking back into the terminals with all the sources replaced by impedances equal to their internal impedances.



Given Network



Norton's equivalent form



Thevenin's equivalent

From the Norton' equivalent circuit the load current is given by

$$I_L(N) = I_{SC} \frac{Z_i}{Z_i + Z_L} \qquad \text{---- Eq.1}$$

Where, I_{SC} is the short circuit current (Norton's equivalent current source)

Z_i is the Norton's equivalent impedance

and Z_L is the load impedance of the total network.

From the Thevenin's equivalent circuit, the load current is given by

$$I_L(Th) = \frac{V_{oc}}{Z_i + Z_L} \qquad \text{---- Eq.2}$$

Where, V_{OC} is the open circuit voltage (Thevenin's equivalent voltage source)

Z_i is the Thevenin's equivalent impedance

and Z_L is the load impedance of the total network.

On short circuiting the terminals a and b of the Thevenin's equivalent network

$$I_{SC} = \frac{V_{oc}}{Z_i}$$

$$\Rightarrow V_{oc} = I_{sc} \times Z_i \qquad \text{---- Eq.3}$$

From Eq.2 and Eq.3

$$I_L(Th) = \frac{I_{sc} \times Z_i}{Z_i + Z_L} \qquad \text{---- Eq.4}$$

Comparing Eq.1 and Eq.4

$$I_L(Th) = \frac{I_{sc} \times Z_i}{Z_i + Z_L} = I_L(N) \qquad \text{---- Eq.5}$$

From the above conditions it is evident that the Norton's equivalent can be replaced by a Thevenin's equivalent circuit governing the relations of Eq.3 and Eq.4.

Q.57. With the help of frequency curves, explain the effect of coefficient of coupling on the primary and the secondary currents. **(6)**

Ans:

When the coefficient of coupling is small, the effect of coupled impedances is negligible. The current versus frequency curve of the primary is similar to the resonance curve of the primary circuit alone and the current versus frequency curve of the secondary is similar to the resonance curve of the secondary circuit alone.

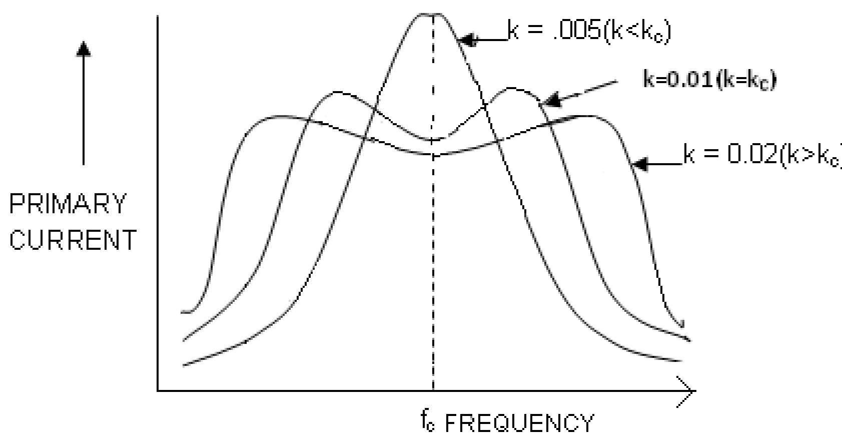


Fig 7.b.1- Primary current versus frequency curves for different values of coefficient of coupling

As the coefficient of coupling k increases, the primary current versus frequency curve broadens and the maximum value of the primary current reduces. With increase in k, the secondary current versus frequency curve broadens but the peak value of the increases and a stage reaches when the secondary current is maximum. For this value of k, the secondary current curve is broader with relatively flat top than the resonance curve of the secondary circuit alone. For the same value of k, the primary current has two peaks. At k = k_c (critical coefficient of coupling), the resistance component of impedance reflected from the secondary into primary is equal to the resistance of the primary.

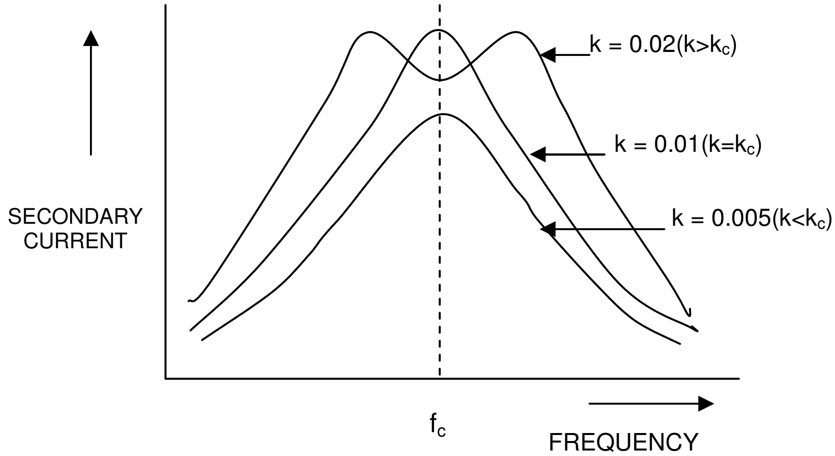


Fig 7.b.2 - Secondary current versus frequency curves for different values of coefficient of coupling

When $k > k_c$

(i) Both the primary and secondary currents have lower and lower values than at resonance.

(ii) Both the primary and secondary current curves will have more and more prominent double humps.

(iii)The peak spread increases.

Q.58. Define the terms voltage standing wave ratio (VSWR) and reflection coefficient. State and derive the relations between VSWR and reflection coefficient. **(2+2+4)**

Ans:

At the points of voltage maxima,

$|V_{max}| = |V_I| + |V_R|$

where V_I is the r.m.s value of the incident voltage.

where V_R is the r.m.s value of the reflected voltage.

Here the incident voltages and reflected voltages are in phase and add up.

At the points of voltage minima,

$|V_{min}| = |V_I| - |V_R|$

Here the incident voltages and reflected voltages are out of phase and will have opposite sign.

VSWR (Voltage Standing Wave Ratio) is defined as the ratio of maximum and minimum magnitudes of voltage on a line having standing waves.

$$VSWR = \frac{|V_{\max}|}{|V_{\min}|}$$

The voltage reflection coefficient, *k* is defined as the ratio of the reflected voltage to incident voltage.

$$k = \frac{|V_R|}{|V_I|}$$

$$VSWR(s) = \frac{|V_{\max}|}{|V_{\min}|} = \frac{|V_I| + |V_R|}{|V_I| - |V_R|} = \frac{1 + \left|\frac{V_R}{V_I}\right|}{1 - \left|\frac{V_R}{V_I}\right|}$$

$$\Rightarrow s = \frac{1+|k|}{1-|k|}$$

$$\therefore k = \frac{s-1}{s+1}$$

Q.59. What is an attenuator? Give two uses of an attenuator. With the help of a suitable example give the relation for the attenuation constant (N) in Nepers, for a symmetric T-network. **(2+2+4)**

Ans:

An attenuator is a four terminal resistive network connected between the source and load to provide a desired attenuation of the signal. An attenuator can be either symmetrical or asymmetrical in form. It also can be either a fixed type or a variable type. A fixed attenuator is known as pad.

Applications of Attenuators:

(i) Resistive attenuators are used as volume controls in broadcasting stations.

(ii) Variable attenuators are used in laboratories, when it is necessary to obtain small value of voltage or current for testing purposes.

(iii) Resistive attenuators can also be used for matching between circuits of different resistive impedances.

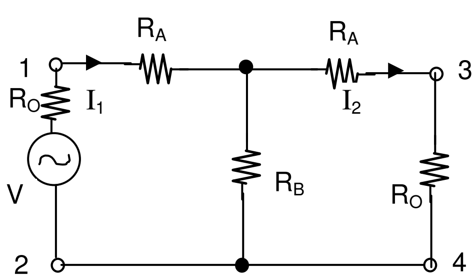


Fig 9.a

The characteristic impedance is resistive, R_o since resistive elements are only used in the attenuator. Consider a symmetric T attenuator a shown in Fig 9.a. The attenuator is driven at the input port by a voltage source V of internal impedance R_o and it feeds a resistor R_o at the output port. The T network consists of a divided series arm R_A and one central shunt arm R_B.

The output current I_2 is given by

$$I_2 = I_1 \times \frac{R_B}{R_A + R_B + R_O}$$

$$N = \frac{I_1}{I_2} = \frac{R_A + R_B + R_O}{R_B} = 1 + \frac{R_A}{R_B} + \frac{R_O}{R_B}$$

Where N is the attenuation in Nepers.

Q.60. Find the input impedance for the circuit shown in Fig.2. **(4)**

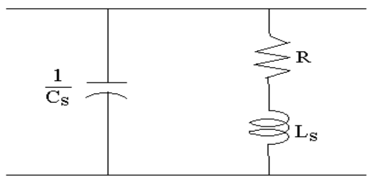


Fig.2

Ans:

$$Z_i(s) = \frac{(\frac{1}{Cs})(R+Ls)}{\frac{1}{Cs}+(R+Ls)} \quad = \frac{R+Ls}{1+RCs+LCs^2}$$

$$\Rightarrow Z_i(s) = \frac{L(\frac{R}{L}+s)}{LC(s^2+\frac{R}{L}s+\frac{1}{LC})}$$

$$\therefore \text{Input impedance } Z_i(s) = \frac{(\frac{R}{L}+s)}{C(s^2+\frac{R}{L}s+\frac{1}{LC})}$$

Q.61. State and prove Convolution Theorem. **(6)**

Ans:
The laplace transform of convolution of two time domain functions is given by convolution theorem. The convolution theorem states that the laplace transform of the convolution of $f_1(t)$ and $f_2(t)$ is the product of individual laplace transforms.
$L [f_1(t) * f_2(t)] = F_1(s) F_2(s)$.
Consider the two functions $f_1(t)$ and $f_2(t)$ which are zero for $t < 0$.
Convolution of two real functions is the multiplication of their functions. The convolution of $f_1(t)$ and $f_2(t)$ in time domain is normally denoted by $f_1(t) * f_2(t)$ and is given by

$$f_2(t) * f_1(t) = \int_0^t (\tau) . f_2(t-\tau) d\tau \qquad \text{and}$$

$$f_1(t) * f_2(t) = \int_0^t f_2(\tau) . f_1(t-\tau) d\tau \qquad \text{Where } \tau \text{ is a dummy variable part.}$$

By the definition of laplace transform

$$L[f_1(t) * f_2(t)] = L\left[\int_0^t .f_1(t-\tau) f_2(\tau) d\tau\right]$$

$$= \int_0^\infty \left[\int_0^t .f_1(t-\tau) f_2(\tau) d\tau\right] e^{-st} \; dt$$

Now $\int_0^t .f_1(t-\tau) f_2(\tau) d\tau$ may be written as

$$\int_0^{\infty} .\mathrm{f}_1(\mathrm{t}-\tau)u(t-\tau)\mathrm{f}_2(\tau)d\tau$$

Where u(t - τ) is a shifted step function because

$$u(t-\tau) = 1 \qquad \tau \le t$$
$$= 0 \qquad \tau > t$$

and the integrand is zero for values of τ > t

$$L[f_1(t) * f_2(t)] = \int_0^{\infty}\left[\int_0^{\infty} .\mathrm{f}_1(\mathrm{t}-\tau)u(t-\tau)\mathrm{f}_2(\tau)d\tau\right]e^{-st}\,dt$$

(t - τ) = x at t = 0, x = - τ

dt = dx at t = ∞, x = -∞

$$\therefore L[f_1(t) * f_2(t)] = \int_{-\tau}^{\infty}\left[\int_0^{\infty} .\mathrm{f}_1(\mathrm{x})u(x)\mathrm{f}_2(\tau)d\tau\right]e^{-s(x-\tau)}\,dx$$

$$= \int_{-\tau}^{\infty}\mathrm{f}_1(\mathrm{x})u(x)e^{-sx}\int_0^{\infty}\mathrm{f}_2(\tau)\,e^{-s\tau}d\tau dx$$

$$= \int_0^{\infty}\mathrm{f}_1(\mathrm{x})u(x)e^{-sx}dx\int_0^{\infty}\mathrm{f}_2(\tau)\,e^{-s\tau}d\tau$$

$= F_1(s)F_2(s)$ as u(x) = 0 for x < 0

Q.62. Explain how double tuned circuits are used in radio receivers. **(4)**

Ans:

Ideally a broadcast receiver should have uniform response to amplitude-modulated signals occupying a total bandwidth of 10KHz centered on the carrier frequency. Double tuned circuits with critical coupling are used as load impedances in the IF (intermediate frequency) amplifier stage of a super heterodyne receiver. The critically double tuned circuits have almost a flat topped response with small double humps. They also give high fidelity, i.e. equal reproduction of all audio modulating frequencies. The response drops rapidly with frequency at the edges of the 10KHz pass band i.e. it has high selectivity. The response curve is shown in Fig 7.c. the pass band extends from P_1 to P_2 having response equal to the center frequency.

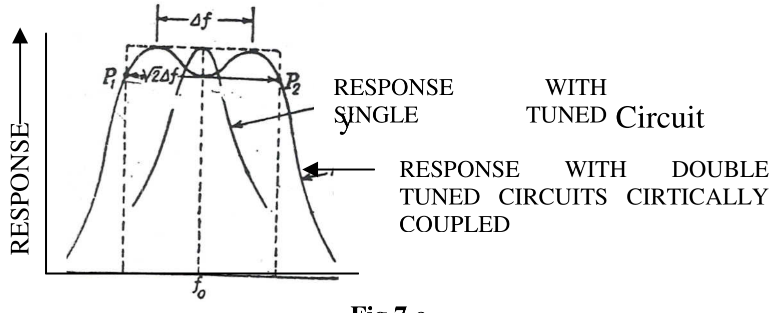


Fig 7.c

Response curve of an amplifier with single tuned and double tuned

The bandwidth between P_1 and P_2 is equal to $\sqrt{2}\Delta f$ where Δf is the bandwidth between the current peaks. The slope of the response curves depends on the quality factor (Q) of the circuit. Higher Q, the curves are steeper. Therefore high Q circuits are used for better selectivity. Double tuned circuits are used for good selectivity.

Q.63. Derive an expression for a condition of minimum attenuation in a transmission line. **(8)**

Ans:
In a continuous long line, the signal received may be too low to serve any useful purpose. Hence communication lines should have minimum attenuation. The attenuation constant α is given by

$$\alpha = \sqrt{\frac{1}{2}\left[\left\{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right\}+(RG-\omega^2LC)\right]} \quad \text{--- Eq.1}$$

Consider R, G and C to be constant and L to be variable, the optimum value of L to provide minimum attenuation is given by differentiating the attenuation constant with respect to L and equating $\frac{d\alpha}{dL}=0$.

Squaring and differentiating Eq.1 with respect to L

$$2\alpha\frac{d\alpha}{dL} = \frac{1}{2}\times\frac{2\omega^2L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} - \omega^2C \quad \text{--- Eq.2}$$

Equating $\frac{d\alpha}{dL}=0$

$$\frac{d\alpha}{dL} = \frac{\omega^2L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} - \omega^2C = 0$$

or, $\frac{\omega^2L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} = \omega^2C$

or , $L(G^2+\omega^2C^2) = C\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$

or, $L^2(G^2+\omega^2C^2)^2 = C^2(R^2+\omega^2L^2)(G^2+\omega^2C^2)$

or, $L^2G^2+\omega^2L^2C^2 = C^2R^2+\omega^2L^2C^2$

or, $\frac{L}{C}=\frac{R}{G}$

$\therefore$ The condition for minimum attenuation in a transmission line is $\frac{L}{C}=\frac{R}{G}$

Q.64. Determine the elements of a π-section, which is equivalent to a given T-section. **(6)**

Ans:
At any one frequency, a π network can be interchanged to a T network and vice-versa, provided certain relations are maintained.
Let Z_1, Z_2, Z_3 be the three elements of the T network and Z_A, Z_B, Z_C be the three elements of the π network as shown in Fig.4.c.1 and Fig 4.c.2
The impedance between the terminal 1 and terminal 3 is

$$Z_1 + Z_2 = \frac{(Z_B + Z_C)Z_A}{Z_B + Z_C + Z_A} \quad \text{----- Eq.1}$$

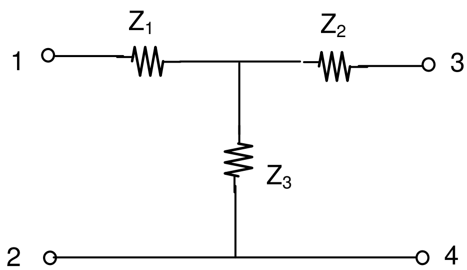


Fig 4.c.1

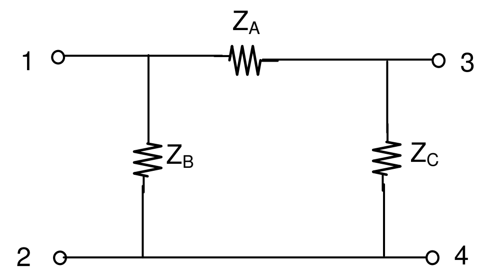


Fig 4.c.2

The impedance between the terminal 3 and terminal 4 is

$$Z_2 + Z_3 = \frac{(Z_A + Z_B)Z_C}{Z_B + Z_C + Z_A} \quad \text{---- Eq.2}$$

The impedance between the terminal 1 and terminal 2 is

$$Z_1 + Z_3 = \frac{(Z_A + Z_C)Z_B}{Z_B + Z_C + Z_A} \quad \text{---- Eq.3}$$

Adding eq.1, eq.2 and subtracting eq.3

$$Z_2 = \frac{Z_A Z_C}{Z_B + Z_C + Z_A} \quad \text{---- Eq.4}$$

Adding eq.2, eq.3 and subtracting eq.1

$$Z_3 = \frac{Z_B Z_C}{Z_B + Z_C + Z_A} \quad \text{---- Eq.5}$$

Adding eq.3, eq.1 and subtracting eq.2

$$Z_1 = \frac{Z_B Z_A}{Z_B + Z_C + Z_A} \quad \text{---- Eq.6}$$

Consider $Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = \Sigma Z_1Z_2$

From eq.4, eq.5, eq.6 we get

$$\sum Z_1 Z_2 = \frac{Z_B{}^2 Z_A Z_C + Z_B Z_A{}^2 Z_C + Z_B Z_A Z_C{}^2}{(Z_B + Z_C + Z_A)^2}$$

$$\sum Z_1 Z_2 = \frac{Z_B Z_A Z_C (Z_B + Z_C + Z_A)}{(Z_B + Z_C + Z_A)^2}$$

$$\sum Z_1 Z_2 = \frac{Z_A Z_B Z_C}{Z_B + Z_C + Z_A} = \frac{Z_A Z_B Z_C}{\sum Z_A}$$

From eq .6

$$Z_1 = \frac{Z_B Z_A}{\sum Z_A}$$

$$\sum Z_1 Z_2 = \frac{Z_B Z_A}{\sum Z_A} \times Z_C = Z_1 \times Z_C$$

$$\Rightarrow Z_C = \frac{\sum Z_1 Z_2}{Z_1}$$

From eq .5

$$Z_3 = \frac{Z_B Z_C}{\sum Z_C}$$

$$\sum Z_1 Z_2 = \frac{Z_B Z_C}{\sum Z_C} \times Z_A = Z_3 \times Z_A$$

$$\Rightarrow Z_A = \frac{\sum Z_1 Z_2}{Z_3}$$

From eq.4

$$Z_2 = \frac{Z_A Z_C}{\sum Z_B}$$

$$\sum Z_1 Z_2 = \frac{Z_A Z_C}{\sum Z_B} \times Z_B = Z_2 \times Z_B$$

$$\Rightarrow Z_B = \frac{\sum Z_1 Z_2}{Z_2}$$

Q.65. Explain reflection coefficient and VSWR of a transmission line. **(8)**

Ans:

VSWR (Voltage Standing Wave Ratio) is defined as the ratio of maximum and minimum magnitudes of voltage on a line having standing waves.

At the points of voltage maxima,

$|V_{max}| = |V_I| + |V_R|$

where V_I is the r.m.s value of the incident voltage.

where V_R is the r.m.s value of the reflected voltage.

Here the incident voltages and reflected voltages are in phase and addup.

At the points of voltage minima,

$|V_{min}| = |V_I| - |V_R|$

Here the incident voltages and reflected voltages are out of phase and will have opposite sign.

$$VSWR = \frac{|V_{\max}|}{|V_{\min}|}$$

The voltage reflection coefficient, k is defined as the ratio of the reflected voltage to incident voltage.

$$K = \frac{|V_R|}{|V_I|}$$

$$VSWR(S) = \frac{|V_{\max}|}{|V_{\min}|} = \frac{|V_I| + |V_R|}{|V_I| - |V_R|} = \frac{1 + \left|\frac{V_R}{V_I}\right|}{1 - \left|\frac{V_R}{V_I}\right|}$$

$$\Rightarrow S = \frac{1+|K|}{1-|K|}$$

$$K = \frac{S-1}{S+1}$$

Q.66. Explain stub matching in a transmission line. **(8)**

Ans:
When there are no reflected waves, the energy is transmitted efficiently along the transmission line. This occurs only when the terminating impedance is equal to the characteristic impedance of the line, which does not exist practically. Therefore impedance matching is required. If the load impedance is complex, one of the ways of matching is to tune out the reactance and then match it to a quarter wave transformer. The input impedance of open or short circuited lossless line is purely reactive. Such a section is connected across the line at a convenient point and cancels the reactive part of the impedance at this point looking towards the load. Such sections are called impedance matching stubs. The stubs can be of any length but usually it is kept within quarter wavelength so that the stub is practically lossless at high frequencies. A short circuited stub of length less than $\lambda/4$ offers inductive reactance at the input while an open circuited stub of length less than $\lambda/4$ offers capacitive reactance at the input. The advantages of stub matching are:

- Length of the line remains unaltered.
- Characteristic impedance of the line remains constant.

At higher frequencies, the stub can be made adjustable to suit variety of loads and to operate over a wide range of frequencies.

Q.67. Explain Z parameters and also draw an equivalent circuit of the Z parameter model of the two port network. **(8)**

Ans:
In a Z parameter model, the voltage of the input port and the voltage of the output port are expressed in terms the current of the input port and the current of the output port. The equations are given by

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

When the output terminal is open circuited, $I_2 = 0$, we can determine Z_{11} and Z_{21}, where Z_{11} is the input impedance expressed in ohms and Z_{21} is the forward transfer impedance.

$$\left.\begin{aligned} V_1 &= Z_{11}I_1 \\ V_2 &= Z_{21}I_1 \end{aligned}\right|_{when\ I_2=0}$$

$Z_{11} = \dfrac{V_1}{I_1}$ ohms and $Z_{21} = \dfrac{V_2}{I_1}$ ohms

When the input terminal is open circuited, $I_1 = 0$, we can determine Z_{12} and Z_{22}, Where Z_{12} is the reverse transfer impedance and Z_{22} is the output impedance expressed in ohms.

$$\left.\begin{aligned} V_1 &= Z_{12}I_2 \\ V_2 &= Z_{22}I_2 \end{aligned}\right|_{when\ I_1=0}$$

$Z_{12} = \frac{V_1}{I_2}$ Ohms and $Z_{22} = \frac{V_2}{I_2}$ Ohms

The equivalent circuit of the Z parameter representation is shown in Fig 3.b
Where $Z_{12}I_2$ is the controlled voltage source and $Z_{21}I_1$ is the controlled voltage source.

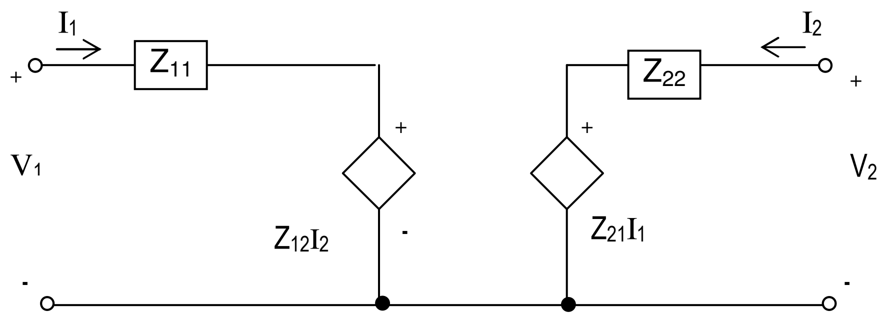


Fig 3.b

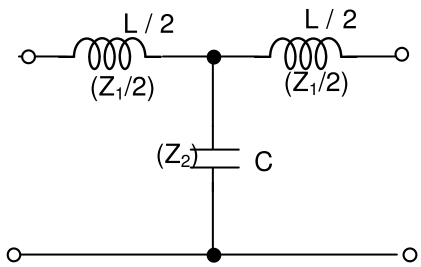
Q. 68. What is an attenuator? Define the terms Decibel and Neper. Derive the relation between the two. **(2+3+3)**

Ans:
An attenuator is a four terminal resistive network connected between the source and load to provide a desired attenuation of the signal. An attenuator can be either symmetrical or asymmetrical in form. It also can be either a fixed type or a variable type. A fixed attenuator is known as pad.
The attenuation in decibel (dB) is given by
1 dB = 20 x $\log_{10}$(N)
where $N = \frac{I_S}{I_R} = \frac{V_S}{V_R}$
The attenuation in Neper (Nep) is given by
1 Nep = $\log_e$(N)
The relation between decibel and neper is
1 dB = 20 x $\log_{10}$(N)
= 20 x $\log_e$(N) x $\log_{10}$(e)
= 20 x $\log_e$(N) x 0.434
= 8.686 $\log_e$(N)
$\therefore$ Attenuation in decibel = 8.686 x attenuation in Neper.
$\therefore$ Attenuation in Neper = 0.1151 x attenuation in decibel.

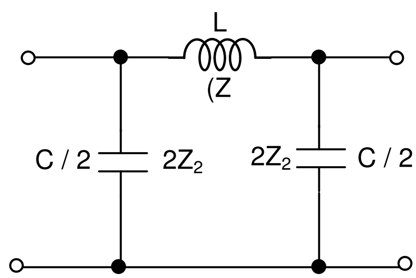
Q. 69. Write short notes on any **TWO** of the following:
(i) Low-pass filter and its approximation/design.
(ii) T and π - attenuators.
(iii) Single stub matching in transmission line.
(iv) H-parameters, its relations with z-parameters and y-parameters. **(2x7=14)**

Ans:

i) Consider a constant – K filter, in which the series and shunt impedances, Z_1 and Z_2 are connected by the relation
$Z_1 Z_2 = R_0^2$
Where R_0 is a real constant independent of frequency. R_0 is often termed as design impedance or nominal impedance of the constant – K filter.



T-section



Π-section

($Z_1 = j\omega L$, $Z_2 = 1/j\omega C$)
($Z_1 = j\omega L$, $Z_2 = 1/j\omega C$)
Let $Z_1 = j\omega L$ and $Z_2 = 1/j\omega C$

$$Z_1 Z_2 = j\omega L \times \frac{1}{j\omega C} = \frac{L}{C} = R_0^{\ 2}$$

$$\therefore R_0 = \sqrt{\frac{L}{C}} \text{ --------- } (eq - 3a.1)$$

At cutoff-frequency f_c

$$\frac{\omega_c^{\ 2} LC}{4} = 1$$

$$f_c = \frac{1}{\pi\sqrt{LC}} \text{ --------- } (eq - 3a.2)$$

Given the values of R_0 and ω_c, using the eq 3a.1 and 3a.2 the values of network elements L and C are given by the equations

$$L = \frac{R_o}{\pi f_c} \qquad C = \frac{1}{\pi R_o f_c}$$

(ii) T and Π attenuators:

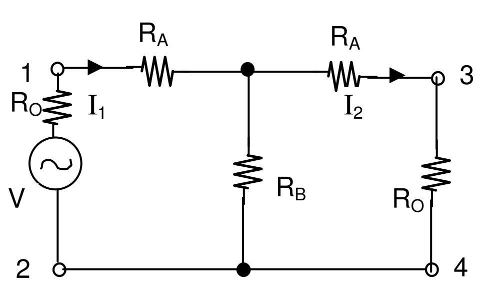


Fig 11.ii.1
(T – attenuator)

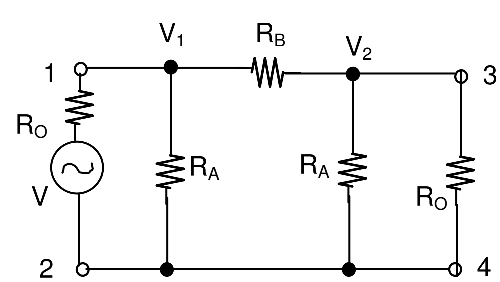


Fig 11.ii.2
(π - attenuator)

Attenuators are of two types. They are symmetrical and asymmetrical attenuators. Symmetrical attenuators are placed between two equal impedances of value equal to the characteristic impedance of the attenuator. The characteristic impedance is resistive, R_o since resistive elements are only used in the attenuator. Depending on the placement of the elements, T and Π configurations are shown in Fig 11.ii.1 and Fig 11.ii.2. The attenuator is driven at the input port by a voltage source V of internal impedance R_o and it feeds a resistor R_o at the output port.
The T network consists of a divided series arm and one central shunt arm.
The output current I_2 is given by

$$I_2 = I_1 \times \frac{R_B}{R_A + R_B + R_O}$$

$$N = \frac{I_1}{I_2} = \frac{R_A + R_B + R_O}{R_B} = 1 + \frac{R_A}{R_B} + \frac{R_O}{R_B}$$

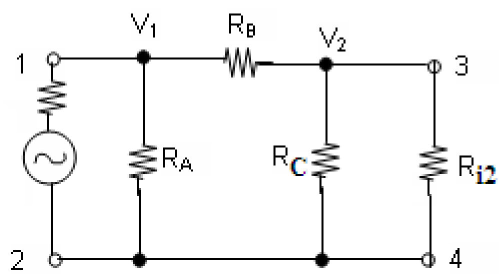


Fig. 11.ii.4

Where N is the attenuation in Nepers.
The Π network consists of a series arm and two shunt arms.
The output voltage V_2 is given by

$$V_2 = V_1 \times \frac{R_A \parallel R_O}{R_B + (R_A \parallel R_O)} = V_1 \times \frac{\frac{R_A R_O}{R_A + R_O}}{R_B + (\frac{R_A R_O}{R_A + R_O})}$$

$$N = \frac{V_1}{V_2} = \frac{R_B + (\frac{R_A R_O}{R_A + R_O})}{\frac{R_A R_O}{R_A + R_O}} = 1 + \frac{R_B}{\frac{R_A R_O}{R_A + R_O}} = 1 + \frac{R_B (R_A + R_O)}{R_A R_O}$$

$$N = 1 + \frac{R_B}{R_A} + \frac{R_B}{R_O}$$

Asymmetrical attenuators are placed between two unequal impedances of value equal to the image impedances of the network. The image impedances are resistive, R_{i1} and R_{i2} since resistive elements are only used in the attenuator. Depending on the placement of the elements, asymmetrical T and are shown in Fig 11.ii.3 and Fig 11.ii.4

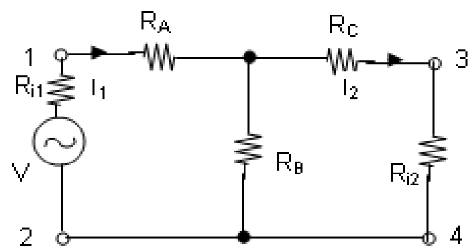


Fig 11.ii.3

The attenuation N, in Nepers is given by

$$N = \frac{I_1}{I_2}\sqrt{\frac{R_{i1}}{R_{i2}}} = \frac{V_1}{V_2}\sqrt{\frac{R_{i2}}{R_{i1}}}$$

(iii) Stub matching: When there are no reflected waves, the energy is transmitted efficiently along the transmission line. This occurs only when the terminating impedance is equal to the characteristic impedance of the line, which does not exist practically. Therefore impedance matching is required. If the load impedance is complex, one of the ways of matching is to tune out the reactance and then match it to a quarter wave transformer. The input impedance of open or short circuited lossless line is purely reactive. Such a section is connected across the line at a convenient point and cancels the reactive part of the impedance at this point looking towards the load. Such sections are called impedance matching stubs. The stubs can be of any length but usually it is kept within quarter wavelength so that the stub is practically lossless at high frequencies. A short circuited stub of length less than $\lambda/4$ offers inductive reactance at the input while an open circuited stub of length less than $\lambda/4$ offers capacitive reactance at the input. The advantages of stub matching are:

- Length of the line remains unaltered.
- Characteristic impedance of the line remains constant.
- At higher frequencies, the stub can be made adjustable to suit variety of loads and to operate over a wide range of frequencies.

(iv) In a hybrid parameter model, the voltage of the input port and the current of the output port are expressed in terms the current of the input port and the voltage of the output port. The equations are given by

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

When the output terminal is short circuited, $V_2 = 0$

$V_1 = h_{11}I_1$

$I_2 = h_{21}I_1$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

Where h_{11} is the input impedance expressed in ohms and h_{21} is the forward current gain.

When the input terminal is open circuited, $I_1 = 0$

$V_1 = h_{12}V_2$

$I_2 = h_{22}V_2$

$$h_{21}=\left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22}=\left.\frac{I_2}{I_1}\right|_{I_1=0}$$

Where h_{12} is the reverse voltage gain and h_{22} is the output admittance expressed in mhos.

The Z parameter equations are given by

$V_1 = Z_{11}I_1 + Z_{12} I_2$

$V_2 = Z_{21}I_1 + Z_{22} I_2$

The Y parameter equations are given by

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

The h parameters in terms of Z parameters are given by

$$h_{11} = \frac{\Delta Z}{Z_{22}} \qquad h_{12} = \frac{Z_{12}}{Z_{22}} \qquad h_{21} = \frac{-Z_{21}}{Z_{22}} \qquad h_{22} = \frac{1}{Z_{22}}$$

Where $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

The h parameters in terms of Y parameters are given by

$$h_{11} = \frac{1}{Y_{11}} \qquad h_{21} = \frac{Y_{21}}{Y_{11}} \qquad h_{12} = \frac{-Y_{12}}{Y_{11}} \qquad h_{22} = \frac{\Delta Y}{Y_{11}}$$

Where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$