TYPICAL QUESTIONS & ANSWERS

PART – I

OBJECTIVE TYPES QUESTIONS

Each Question carries 2 marks.

Choose correct or the best alternative in the following:

Q.1 Two concentric spherical shells carry equal and opposite uniformly distributed charges over their surfaces as shown in Fig.1.
Electric field on the surface of inner shell will be

(A) zero.

(B) $\dfrac{Q}{4\Pi\in_0 R^2}$.

(C) $\dfrac{Q}{8\Pi\in_0 R^2}$.

(D) $\dfrac{Q}{16\Pi\in_0 R^2}$.

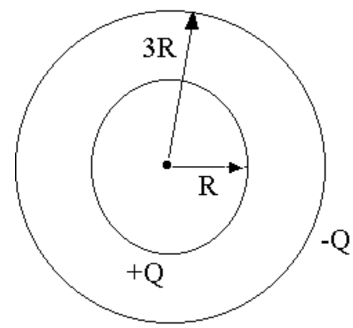


Fig.1

Ans: A

Q.2 The magnetic field intensity (in A/m) at the centre of a circular coil of diameter 1 metre and carrying current of 2 A is

(A) 8. **(B)** 4.
(C) 3. **(D)** 2.

Ans: A

Q.3 The polarization of a dielectric material is given by

(A) $\vec{P} = \in_r \vec{E}$. **(B)** $\vec{P} = (\in_r - 1)\vec{E}$.

(C) $\vec{P} = \vec{E}\in_0 (\in_r - 1)$. **(D)** $\vec{P} = (\in_r - 1)\in_0$.

Ans: B

Q.4 In a travelling electromagnetic wave, E and H vector fields are

(A) perpendicular in space .
(B) parallel in space.
(C) E is in the direction of wave travel.
(D) H is in the direction of wave travel.

Ans: C

Q.5 For a broad side linear array which of the following is not correct

(A) the maximum radiation occurs perpendicular to the line of the array at $\phi = 90^\circ$.
(B) the progressive phase shift (α) between elements is zero.
(C) width of principal lobe is less than that of an end fire array.
(D) the maximum radiation occurs along the line of array at $\phi = 0^\circ$.

Ans: D

Q.6 Two point charges Q_1 and Q_2 are located at A and B on a straight line as shown in the Fig.1. The electric field will be zero at a point

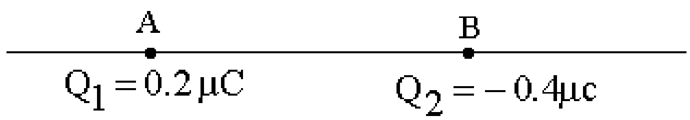


Fig.1

(A) between A and B. **(B)** to the left of A.
(C) to the right of B. **(D)** perpendicular to AB.

Ans: B

Q.7 The ratio of conduction current density to the displacement current density is

(A) $\frac{\sigma}{j\omega\in}$ **(B)** $\frac{j\sigma}{\omega\in}$
(C) $\frac{\sigma\omega}{j\in}$ **(D)** $\frac{\sigma\in}{j\omega}$

Where symbols have their usual meaning.

Ans: A

Q.8 A wave is incident normally on a good conductor. If the frequency of a plane electromagnetic wave increases four times, the skin depth, will

(A) increase by a factor of 2. **(B)** decrease by a factor of 4.
(C) remain the same. **(D)** decrease by a factor of 2.

Ans: D

Q.9 Electric field of a travelling wave is given by $100\cos\left(10^9 t - 4x\right)$. The velocity and the wavelength are

(A) $3\times10^8\ {}^{m}\!/_{sec}$ and 4 m respectively.
(B) $2.5\times10^8\ {}^{m}\!/_{sec}$ and ${}^{\pi}\!/_{2}$ m respectively.
(C) $4\times10^9\ {}^{m}\!/_{sec}$ and 8π m respectively.
(D) $10^9\ {}^{m}\!/_{sec}$ and ${}^{100}\!/_{4}$ m respectively.

Ans: B

Q.10 In a dielectric-conductor boundary (interface), the tangential component of electric field is

(A) E_t **(B)** $2E_t$
(C) zero **(D)** infinity

Ans: C

Q.11 Which of the following matching is incorrect:

(A) Ampere's circuital law $\rightarrow \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$.

(B) Displacement current density $\rightarrow \vec{J} = \frac{\partial \vec{D}}{\partial t}$.

(C) Poisson's equation $\rightarrow \nabla^2 V = 0$.

(D) Continuity equation $\rightarrow \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$.

Ans: C

Q.12 For a transmission line terminated in its characteristic impedance, which of the following statement is incorrect:

(A) It is a smooth line.
(B) The energy distribution between magnetic and electric field is not equal.
(C) Standing wave does not exist.
(D) Efficiency of transmission of power is maximum.

Ans: B

Q.13 For a line of characteristic impedance, Z_O terminated in a load, Z_R such that $Z_R > Z_O$, the Voltage Standing Wave Ratio (VSWR) is given by

(A) Z_R / Z_O. **(B)** Z_O.
(C) Z_R. **(D)** Z_O / Z_R.

Ans: A

Q.14 The lower cut-off frequency of a rectangular wave guide with inside dimensions (3×4.5 cm) operating at 10 GHz is

(A) 10 GHz. **(B)** 9 GHz.
(C) $\frac{10}{9}$ GHz. **(D)** $\frac{10}{3}$ GHz.

Ans: D

Q.15 The directive gain cannot be stated as

(A) the ratio of the radiation intensity in that direction to the average radiated power.
(B) the function of angles.

(C) the directivity of an antenna when directive gain is maximum.
(D) independent of angles.

Ans: D

Q.16 Electric field intensity due to line charge of infinite length is.

(A) $\frac{\rho_L}{2\,\pi\varepsilon R}$. **(B)** $\frac{\rho_L}{4\,\pi\varepsilon R}$.
(C) $\frac{2\rho_L}{\pi\varepsilon R}$. **(D)** $\frac{2\rho_L}{4\pi\varepsilon R}$.

Ans: A

Q.17 With respect to equipotential surface pick the odd one out.
(A) Potential is same every where
(B) Work done in moving charge from one point to another is zero
(C) Potential is different every where
(D) No current flows on this surface

Ans: C

Q.18 Energy stored in a magnetic field is

(A) $W=\frac{1}{2}\sqrt{\frac{\mu H}{4}}$. **(B)** $W=\mu\sqrt{\frac{H^2}{2}}$.
(C) $W=\frac{\mu H^2}{2}$. **(D)** $W=\frac{\mu\sqrt{H}}{2}$.

Ans: C

Q.19 The intrinsic impedance of free space is
(A) 75 ohm. **(B)** 73 ohm.
(C) 120 π ohm. **(D)** 377ohm.

Ans: D

Q.20 During night which layer does not exist?
(A) D layer **(B)** F_1 layer
(C) F_2 layer **(D)** E layer

Ans: A

Q.21 The characteristic impedance is given by

(A) $Z_0=\frac{\sqrt{Zoc}}{Zsc}$ **(B)** $\frac{\sqrt{Zsc}}{Zoc}$
(C) $\sqrt{Zsc\;Zoc}$ **(D)** $(Zsc\cdot Z_{oc})$

Ans: C

Q.22 Transverse electric wave traveling in z- direction satisfies

(A) $E_z = 0;\ H_z = 0$ **(B)** $E_z = 0;\ H_z \neq 0$
(C) $E_z \neq 0;\ H_z = 0$ **(D)** $E_z \neq 0;\ H_z \neq 0$.

Ans: B

Q.23 Radiation resistance of a $\lambda/2$ dipole is

(A) 73 Ω. **(B)** 75 ohm
(C) 120 π ohm **(D)** 377 ohm

Ans: A

Q.24 Poisson's equation in CGS Gaussian system is

(A) $\nabla^2 V = -\dfrac{\rho}{\epsilon_o}$ **(B)** $\nabla^2 V = -4\pi\sigma$
(C) $\nabla^2 V = -4\pi\rho$ **(D)** $\nabla^2 V = 0$

Ans: C

Q.25 The electric potential due to linear quadrapole varies inversely with

(A) r **(B)** r^2
(C) r^3 **(D)** r^4

Ans: C

Q.26 In an electromagnetic wave, the phase difference between electric and magnetic field vectors $\vec{E}$ and $\vec{B}$ is

(A) zero **(B)** $\pi/2$
(C) π **(D)** $\pi/4$

Ans: A

Q.27 If the electrostatic potential is given by $\phi = \phi_o\left(x^2 + y^2 + z^2\right)$ where ϕ_o is constant, then the charge density giving rise to the above potential would be

(A) zero **(B)** $-6\phi_o \epsilon_o$
(C) $-2\phi_o \epsilon_o$ **(D)** $-\dfrac{\epsilon\, \phi_o}{\epsilon_o}$

Ans: B

Q.28 Find the amplitude of the electric field in the parallel beam of light of intensity $2.0\, W/m^2$.

(A) $28.8\, N/c$ **(B)** $15.6\, N/c$
(C) $38.8\, N/c$ **(D)** $26.6\, N/c$

Ans: C

Q.29 The material is described by the following electrical parameters at a frequency of 10 GHz, $\sigma = 10^6 \text{ mho/m}$, $\mu = \mu_o$ and $\frac{\sigma}{\sigma_c} = 10$, the material at this frequency is considered to be

(A) a good conductor
(B) neither a good conductor nor a good dielectric
(C) a good dielectric
(D) a good magnetic material

Ans: A

Q.30 Consider a transmission line of characteristic impedance 50 ohms and the line is terminated at one end by +j50 ohms, the VSWR produced in the transmission line will be

(A) +1 **(B)** zero
(C) infinity **(D)** -1

Ans: C

Q.31 A very small thin wire of length $\frac{\lambda}{100}$ has a radiation resistance of

(A) 0 Ω **(B)** 0.08Ω
(C) 7.9 Ω **(D)** 790Ω

Ans: B

Q.32 Which one of the following conditions will not gurantee a distortionless transmission line

(A) R = 0 = G
(B) RC = LG
(C) very low frequency range (R>> ωL, G >> ωC)
(D) very high frequency range (R<< ωL, G << ωC)

Ans: C

Q.33 The dominant mode of rectangular wave guide is

(A) TE_{11} **(B)** TM_{11}
(C) TE_{01} **(D)** TE_{10}

Ans: D

Q.34 In a certain medium $\vec{E} = 10 \cos(10^8 t - 3y) \ \overrightarrow{a_x}$ V/m. What type of medium is it?

(A) Free space **(B)** Lossy dielectric
(C) Lossless dielectric **(D)** Perfect conductor

Ans: C

Q.35 Which of the following statements is not true of waves in general?

(A) It may be a function of time only
(B) It may be sinusoidal or cosinusoidal

(C) It may be a function of time and space
(D) For practical reasons, it must be finite in extent.

Ans: A

Q.36 Plane y=0 carries a uniform current of 30 $\overrightarrow{a_z}$ mA/m. At (1, 10, –2), the magnetic field intensity is

(A) $-15\ \overrightarrow{a_x}$ mA/m　　**(B)** $15\ \overrightarrow{a_x}$ mA/m
(C) $477.5\ \overrightarrow{a_y}$ μA/m　　**(D)** $18.85\ \overrightarrow{a_y}$ nA/m

Ans: A

Q.37 A loop is rotating about y-axis in a magnetic field $\vec{B} = B_0 \sin \omega t\ \overrightarrow{a_x}$ Wb/m^2. The voltage induced in the loop is due to

(A) Motional emf
(B) Transformer emf
(C) A combination of motional & transformer emf
(D) None of the above

Ans: C

Q.38 A parallel plate capacitor consists of two metal plates of area *A* separated by a distance *d* and has a capacitance *C*. If another metal plate of area *A* is held parallel to either plate of the capacitor at distance *d*/2 from either plate, the new capacitance will be

(A) *C*/2　　(B) *C*
(C) 2*C*　　**(D)** 4*C*

Ans: A

Q.39 If $\nabla \cdot \vec{D} = \epsilon\, \nabla \cdot \vec{E}$ and $\nabla \cdot \vec{J} = \sigma \nabla \cdot \vec{E}$ in a given material, the material is said to be

(A) Linear　　**(B)** Homogeneous
(C) Isotropic　　**(D)** Linear & Homogeneous

Ans: D

Q.40 Point charges 30 nC, –20 nC and 10 nC are located at (–1, 0, 2), (0, 0,0) and (1, 5, –1) respectively. The total flux leaving a cube of side 6m centered at the origin is

(A) –20 nC　　**(B)** 10 nC
(C) 20 nC　　**(D)** 30 nC

Ans: B

Q.41 Lorentz force law is

(A) $\vec{F} = Q\vec{E}$　　**(B)** $\vec{F} = \vec{V} * \vec{B}$
(C) $\vec{F} = Q(\vec{E} + \vec{V} * \vec{B})$　　**(D)** $\vec{F} = Q(\vec{V} * \vec{B})$

Ans: C

Q.42 The equation $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$ is the generalization of

(A) Amperes Law **(B)** Faraday Law
(C) Gauss's Law **(D)** Biot-Saverts Law

Ans: B

Q.43 For a transmission line terminated by a load, the reflection co-efficient magnitude $|\Gamma|$ and the voltage standing wave ration S are related as:

(A) $S = 1/(1+|\Gamma|)$ **(B)** $S = 1/(1-|\Gamma|)$
(C) $S = (1-|\Gamma|)/(1+|\Gamma|)$ **(D)** $S = (1+|\Gamma|)/(1-|\Gamma|)$

Ans: D

Q.44 Unit of relative permeability is
(A) Henry **(B)** Henry/meter
(C) Henry/ meter^2 **(D)** It is dimensionless

Ans: D

Q.45 Reciprocal of reluctance is
(A) Henry/meter **(B)** Henry
(C) meter/Henry **(D)** Henry^{-1}

Ans: B

Q.46 For a rectangular wave guide, 2.5cm x 1.2cm, dominant cut off wavelength is
(A) 5 cm **(B)** 2.5 cm
(C) 2.4 cm **(D)** 3.7 cm

Ans: A

Q.47 For a line of characteristic impedance, Z_0 terminated in a load Z_R such that $Z_R = Z_0/3$, the reflection coefficient is

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$
(C) $-\frac{1}{3}$ **(D)** $-\frac{1}{2}$

Ans: D

Q.48 Plane z=10m carries charge $20 nC/m^2$. The electric field intensity at the origin is

(A) $-10\hat{i}_z \ v/m$ **(B)** $-18\pi\hat{i}_z \ v/m$

(C) $-72\pi\hat{i}_z$ v / m (D) $-360\pi\hat{i}_z$ v / m

Ans: D

Q.49 A positive charged pendulum is oscillating in a uniform electric field (Fig. 1). Its time period as compared to that when it was unchanged

(A) will increase
(B) will decrease
(C) will not change
(D) will first increase and then decrease

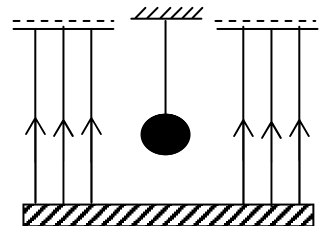


Fig.1

Ans: B

Q.50 Field due to infinitely long line charge along z-axis varies with

(A) ϕ **(B)** z
(C) ρ **(D)** both ϕ and z

Ans: C

Q.51 Which one of the following is correct?

(A) $\overline{\nabla} \cdot \overline{E} = \rho_v$ **(B)** $\overline{\nabla} \cdot \overline{E} = \rho_v / \in_o$
(C) $\overline{\nabla} \cdot \overline{E} = -\rho_v$ **(D)** $\overline{\nabla} \cdot \overline{E} = \in_o \rho_v$

Ans: B

Q.52 When a magnetic flux cuts across 200 turns at the rate of 2 Wb/s, the induced voltage is

(A) 400 V **(B)** 100 V
(C) 600 V **(D)** 0 V

Ans: A

Q.53 When an EM wave is incident on a dielectric, it is

(A) fully transmitted
(B) fully reflected
(C) partially transmitted and partially reflected
(D) none of these.

Ans: C

Q.54 If a line is terminated in an open circuit, the VSWR is

(A) 0 **(B)** 1
(C) ∞ **(D)** -1

Ans: C

Q.55 A hollow rectangular waveguide acts as a

(A) High pass filter **(B)** Low pass filter
(C) Band pass filter **(D)** Low frequency radiator

Ans: A

Q.56 For a 300 Ω antenna operating with 5A of current, the radiated power is

(A) 7500 W **(B)** 750 W
(C) 75 W **(D)** 7500 mW

Ans: A

Q.57 If a current element is z-directed, vector magnetic potential is

(A) x-directed **(B)** y-directed
(C) θ-directed **(D)** z-directed

Ans: D

Q.58 Divergence theorem is applicable for

(A) static field only
(B) time varying fields only
(C) both static and time varying fields
(D) electric fields only

Ans: C

Q.59 Depth of penetration in free space is

(A) α **(B)** $1/\alpha$
(C) 0 **(D)** ∞

Ans: B

Q.60 When the separation between two charges increases, the electric potential energy of charges

(A) increases. **(B)** decreases.
(C) remains the same. **(D)** may increase or decreases.

Ans: B

Q.61 Which one of the following conditions will not guarantee a distortionless Transmission Line?

(A) R=0, G=0 **(B)** LG=RC
(C) $R >> \omega L$, $G >> \omega C$ **(D)** $R << \omega L$, $G << \omega C$

Ans: C

Q.62 A uniform plane wave in air is incident normally on an infinitely thick slab. If the refractive index of the glass slab is 1.5, then the percentage of the incident power that is reflected from the air-glass interface is

(A) 0% (B) 4%
(C) 20% (D) 10%

Ans: B

Q.63 Some unknown material has a conductivity of $10^6\,\text{mho/m}$, and a permeability of $4\pi\times10^{-7}\,\text{H/m}$. The skin depth for the material at 1 GHz is

(A) 15.9 μm (B) 20.9 μm
(C) 25.9 μm (D) 30.9 μm

Ans: A

Q.64 An electromagnetic wave is incident obliquely at the surface of a dielectric medium $2(\mu_2, \in_2)$ from dielectric medium $1(\mu_1, \in_1)$. The angle of incidence and the critical angle are θ_i and θ_c respectively. The phenomenon of total reflection occurs when

(A) $\in_1 > \in_2$ and $\theta_i < \theta_c$ (B) $\in_1 < \in_2$ and $\theta_i > \theta_c$
(C) $\in_1 < \in_2$ and $\theta_i < \theta_c$ (D) $\in_1 > \in_2$ and $\theta_i > \theta_c$

Ans: D

Q.65 Poisson's equation is

(A) $\nabla^2 V = \frac{-\rho}{\in}$ (B) $\nabla^2 V = -4\pi\sigma$
(C) $\nabla^2 V = -4\pi\rho$ (D) $\nabla^2 V = 0$

Ans: A

Q.66 The radio wave is incident on layer of ionosphere at an angle of 30° with the vertical. If the critical frequency is 1.2 MHz, the maximum usable frequency (MUF) is

(A) 1.2 MHz (B) 2.4 MHz
(C) 0.6 MHz (D) 1.386 MHz

Ans: D

Q.67 A transmission line with a characteristic impedance Z_1 is connected to a transmission line with characteristic impedance Z_2. If the system is being driven by a generator connected to the first line, then the overall transmission coefficient will be

(A) $\frac{2Z_1}{Z_1+Z_2}$ (B) $\frac{Z_1}{Z_1+Z_2}$
(C) $\frac{2Z_2}{Z_1+Z_2}$ (D) $\frac{Z_2}{Z_1+Z_2}$

Ans: A

Q.68 A rectangular waveguide has dimension $1.0\,\text{cm}\times0.5\,\text{cm}$, its cutoff frequency for the dominant mode is

(A) 5 GHz **(B)** 10 GHz
(C) 15 GHz **(D)** 20 GHz

Ans: C

Q.69 Poynting vector gives

(A) rate of energy flow.
(B) direction of polarization.
(C) intensity of electric field.
(D) intensity of magnetic field.

Ans:A

PART – II

NUMERICALS

Q.1 Using Gauss's theorem, show that a symmetrical spherical charge distribution is equivalent to a concentrated point charge at the centre of the sphere as far as external fields are concerned. **(6)**

Ans: Refer section **2.21 & 2.22** Pg No 109 of "Electromagnetic Field and Waves" by K D Prasad.

Q.2 A spherical volume of radius R has a volume charge density given by $\rho = Kr$, where r is the radial distance and K is a constant. Develop expressions for $\vec{E}$ and V and sketch their variation with respect to $r(0 \le r \le \infty)$. **(8)**

Ans: For r < R : Gauss's law $\Rightarrow$

$\in_o \int Eds$ = charge enclosed by sphere of radius r.

$$\text{Or } \in_o E_r 4\pi r^2 = \int \rho dv$$

$$= \int kr.4\pi r^2 dr$$

$$= 4\pi k \int r^3 dr$$

$$= \pi k r^4$$

$$\text{Or } E_r = \frac{kr^2}{4\in_o} i_r \quad \text{for } r < R$$

$$\text{At } r = R, \quad E_R = \frac{kR^2}{4\in_o} i_r \quad \text{max value at } r = R$$

$$\text{For } r \ge R; \ \in_0 E_r 4\pi r^2 = \int_0^R \rho dv$$

$$= 4\pi k \int_0^R r^3 dr = \pi K R^4$$

$$\text{Or } E_r = \frac{KR^4}{4\in_0 r^2} i_r \quad \text{for } r \ge R.$$

Q.3 Give an example in which the current in a wire enclosed by a closed path is not a uniquely defined. Is it correct to apply Ampere's circuital law for the static case in such a situation? Explain. **(6)**

Ans: Refer section 4.2, Page No 306 of "Electromagnetic Field and Waves" by K D Prasad.

Q.4 The electric field $\vec{E}$ in free space is given as $E = E_m \cos(\omega t - \beta z)\hat{i}_x$.

Determine $\vec{D}, \vec{B}$ and $\vec{H}$. Sketch E and H at t =0. **(8)**

Ans: $\underline{D} = \in_0 \underline{E} = \in_0 E_m \cos(\omega t - \beta z)\underline{i}_x$

$$\underline{\nabla} \times \underline{E} = -\frac{\partial B}{\partial t} = \begin{bmatrix} \underline{i}_x & \underline{i}_y & \underline{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_m \cos(\omega t - \beta z) & 0 & 0 \end{bmatrix}$$

Or $-\frac{\partial B}{\partial t} = \frac{\partial}{\partial z}\left[E_m \cos(\omega t - \beta z)\right]\underline{i}_y$

$= +\beta E_m \sin(\omega t - \beta z)\underline{i}_y$

Integrating, in view of its being a static field, the constant of integration is treated as zero.

$-B = -\frac{\beta E_m}{\omega}\cos(\omega t - \beta z)\underline{i}_y$

Or $B = \frac{\beta E_m}{\omega}\cos(\omega t - \beta z)\underline{i}_y$

$H = \frac{B}{\mu_0} = \frac{\beta E_m}{\omega \mu_0}\cos(\omega t - \beta z)\underline{i}_y$.

Q.5 What is the physical interpretation of Gauss's law for the magnetic field? How gauss' law for the magnetic field in differential form can be derived from it integral form? **(3 + 5)**

Ans: Refer section 8.5, Page No 251 of Engineering Electromagnetics by William H Hagt, 6th edition.

Q.6 The one-dimensional Laplace's equation is given as $\frac{d^2V}{dx^2} = 0$. The boundary conditions are V = 9 at x = 1 and V = 0 at x = 10. Find the potential and also show the variation of V with respect to x. **(6)**

Ans: $\frac{d^2V}{dx^2} = 0$ (Laplace's eqn)

$\Rightarrow \frac{dv}{dx} = a$

$\Rightarrow V = ax + b$ ------------------ (A)

(where a & b are content of integration and are to be determined by boundary conditions)

$$\left.\begin{aligned} &\text{At } x = x_1, \quad V_1 = ax_1 + b \\ &\text{And At } x = x_2, \quad V_2 = ax_2 + b \end{aligned}\right\}$$ ------------------ (i)

$\Rightarrow \quad a(x_1 - x_2) = V_1 - V_2$

$\Rightarrow a = \frac{V_1 - V_2}{x_1 - x_2}$ ------------------ (ii)

$$b = V_1 - ax_1 = \frac{V_2 x_1 - V_1 x_2}{x_1 - x_2} \qquad \text{------------------ (iii)}$$

Substituting values a & b in eqn (A)

$$V = \frac{V_1(x - x_2) - V_2(x - x_1)}{x_1 - x_2}.$$

Q.7 State and explain Biot-Savart's Law relating the magnetic field produced at a point due to current in a small elemental wire. **(6)**

Ans:Biot - Savart's Law:

Consider a differential current element of a filamentary conductor carrying a current I as shown in Fig.7.3. The law of Biot-Savart then states that at any point *P* the magnitude of the magnetic field intensity produced by the differential element is proportional to the product of the current, the magnitude of the differential length, and the sine of the angle lying between the filament and a line connecting the filament to the point *P* where the field is desired. The magnitude of the magnetic field intensity is inversely proportional to the square of the distance from the differential element to the pint *P*. The direction of the magnetic field intensity is normal to the plane containing the differential filament and the line drawn from the filament to the point *P*. Of the two possible normals, that one is to be chosen which is in the direction of progress of a right handed screw turned from *dL*, through the smaller angle to the line from the filament to *P*.

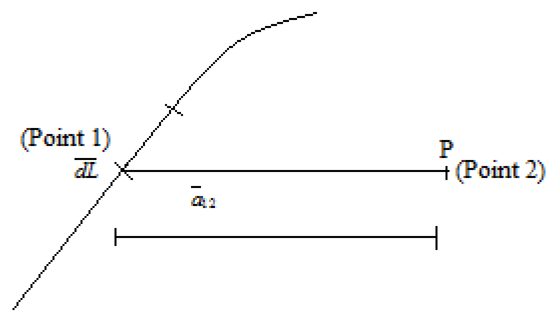


Fig.7.3

The Biot - Savart's Law may be written using vector notation as

$$\overline{dH} = \frac{I\,\overline{dL} \times \overline{a}_{12}}{4\pi d^2} \qquad \text{------------------ (7.1)}$$

The unit of $\overline{H}$ is ampere *I* m. In terms of surface current density ($\overline{K}$) and volume current density ($\overline{J}$) the expression for magnetic field intensity are

$$\overline{dH} = \frac{\overline{K} dS \times \overline{a}_{12}}{4\pi d^2} \qquad \text{------------------ (7.2)}$$

And

$$\overline{dH} = \frac{\overline{J} dv \times \overline{a}_{12}}{4\pi d^2} \qquad \text{------------------ (7.3)}$$

Where point 2 is that point at which magnetic field intensity is desired, point *I* is the point at which differential current element is located and *d* is the distance between points 1 and 2.

Q.8 What is a uniform plane wave? Why is the study of uniform plane waves important? Discuss the parameters ω, β and v_p associated with sinusoidally time-varying uniform plane waves. **(2 + 2 +4)**

Ans: A uniform plane wave is defined as a wave whose value remains constant throughout a plane which is transverse to the direction of propagation of the wave. For example, if the wave is travelling in $\bar{a}_z$ direction, then the plane z = constant will be perpendicular to the direction of propagation. In the plane z = constant, the variables are *x* and *y*. Hence for this uniform plane wave, the electric field intensity and magnetic field intensity are independent of *x* and *y* coordinate and are the functions of *z* coordinates only.

UNIFORM PLANE WAVE OR TRANSVERSE ELECTRO-MAGNETIC WAVE:

Transformation of a time varying quantity into phasor:

Let E_x be the time varying quantity which is either a function of sine or cosine. Then the phasor of E_x is written as E_{xx} and is obtained by dropping Re and suppressing $e^{j\omega t}$. If

$$E_x = E_0 \cos(\omega t + \psi) = E_0 \operatorname{Re}[\cos(\omega t + \psi) + j\sin(\omega t + \psi)]$$
$$= E_0 \operatorname{Re}[e^{j(\omega t + \psi)}]$$
$$= E_0 \operatorname{Re}[e^{j\omega t + j\psi)}]$$

Then,

$$E_{xx} = E_0 e^{j\psi}$$

Where Re means the real part of the following quantity is to be taken. Further it can be proved that the phasor of $\frac{dE_x}{dt}$ is given by multiplying the phasor of E_x by $j\omega$, i.e. phasor of $\left(\frac{dE_x}{dt}\right) = j\omega E_{xx}$.

Transformation of phasor into a time varying quantity.

To transfer a phasor into a time varying quantity, multiply that phasor by $e^{j\omega t}$ and then consider the real part (Re) of that product. If $E_{xx} = E_o e^{j\psi}$, then

$$E_x = \operatorname{Re}\left[E_{xx} e^{j\omega t}\right] = \operatorname{Re}\left[E_o e^{j\omega t} e^{j\psi}\right] = \operatorname{Re}\left[E_o e^{j(\omega t + \psi)}\right]$$
$$= E_o \cos(\omega t + \psi)$$

Maxwell's equations in phasor form.

The four Maxwell's equations in point form or differential form are

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu . \frac{\partial \bar{H}}{\partial t}$$
$$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}$$
$$\bar{\nabla} . \bar{D} = \rho v$$
$$\bar{\nabla} . \bar{B} = 0$$

These equations in phasor form may be written as

$$\bar{\nabla} \times \bar{E}_s = -j\omega\mu . \bar{H}_s \qquad \text{---------------} \; (10.1\ a)$$
$$\bar{\nabla} \times \bar{H}_s = (\sigma + j\omega\varepsilon)\bar{E}_s \qquad \text{---------------} \; (10.1\ b)$$
$$\bar{\nabla} . \bar{D}_s = \rho v \qquad \text{---------------} \; (10.1\ c)$$
$$\bar{\nabla} . \bar{B}_s = 0 \qquad \text{---------------} \; (10.1\ d)$$

Wave equation for electric field intensity.

Using the vector identity

$$\bar{\nabla} \times \bar{\nabla} \times \bar{E}_s = \bar{\nabla}(\bar{\nabla} . \bar{E}_s) - \bar{\nabla}^2 \bar{E}_s$$

LHS = $\overline{\nabla}\times\overline{\nabla}\times\overline{E}_s$

$= \overline{\nabla}\times\left(-j\omega\mu\overline{H}_s\right)$ [∵ Using Eq 10.1a]

$= -j\omega\mu\overline{\nabla}\times\overline{H}_s$

$= -j\omega\mu(\sigma + j\omega\varepsilon)\overline{E}_s$ [∵ Using Eq. 10.1b]

RHS = $\overline{\nabla}\left(\overline{\nabla}.\overline{E}_s\right) - \overline{\nabla}^{-2}\overline{E}_s = \overline{\nabla}\left(\overline{\nabla}.\dfrac{\overline{D}_s}{\varepsilon}\right) - \overline{\nabla}^{-2}\overline{E}_s$

$= \dfrac{1}{\varepsilon}\overline{\nabla}(\rho v) - \overline{\nabla}^{-2}\overline{E}_s$ [∵ Using Eq. 10.1c]

Assuming ρv is equal to zero, we get

RHS = $-\overline{\nabla}^{-2}\overline{E}_s$

$\therefore\ -\overline{\nabla}^{-2}\overline{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\overline{E}_s$, or

$\overline{\nabla}^{-2}\overline{E}_s = j\omega\mu(\sigma + j\omega\varepsilon)\overline{E}_s$

The quantity $j\omega\mu(\sigma + j\omega\varepsilon)$ is denoted by γ^2. Hence

$\overline{\nabla}^{-2}\overline{E}_s = \gamma^2\overline{E}_s$ -------------------- (10.2a)

This is the wave equation for electric field intensity. Similarly it can be proved that, the wave equation for magnetic field intensity is

$\overline{\nabla}^{-2}\overline{H}_s = \gamma^2\overline{H}_s$ -------------------- (10.2b)

Propagation constant, Attenuation constant and Phase constant

In the wave equation

$\overline{\nabla}^{-2}\overline{E}_s = \gamma^2\overline{E}_s$

The quantity γ is known as propagation constant and is defined by the equation

$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

Or $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

The units of propagation constant is per m. It is a complex number. The real part is denoted by α and is known as attenuation constant. The unit is N_p / m where N_p stands for neper. The imaginary part is denoted by β

Q.9 . Distinguish between internal inductance and external inductance. Discuss the concept of flux linkage pertinent to the determination of the internal inductance. **(3 + 3)**

Ans: Refer section 9.10 Page No 313 of Engineering electromagnetics by William H Hayt.

Q.10 Explain the following:

(i) Characteristic impedance.
(ii) Distortionless line.
(iii) Voltage Standing Wave Ratio (VSWR).
(iv) Reflection coefficient. **(8)**

Ans: Refer section 8.**13**, 8.14, 8.27.1, 8.28 Page No 646 of "Electromagnetic Field and Waves" by K D Prasad.

Q.11 Explain the impedance transformation property of a quarter wave transmission line. **(7)**

Ans: Refer section 8.29.2, Page No 646 of "Electromagnetic Field and Waves" by K D Prasad.

Q.12 Calculate the characteristic impedance Z_0, propagation constant γ and the line constants of an open wire loss less line of 50 Km long operating at f = 700 Hz if

$$Z_{0C} = 286\angle -40^\circ\,\Omega$$

$$Z_{SC} = 1520\angle 16^\circ\,\Omega$$

(7)

Ans: $Z_o = \sqrt{Z_{oc} \times Z_{sc}} = \sqrt{286\angle -40 \times 1520\angle 16}$

$= 659.3\angle -12^0$

$Y = \dfrac{659.3}{700}$.

Q.13. What is dominant mode? Which one of the rectangular waveguide modes is the dominant mode? How do the dimensions of a rectangular cavity resonator determine the frequencies of oscillation of the resonator? **(4+6)**

Ans: Refer section 9.14 and 9.7 of "Electromagnetic Field and Waves" by K D Prasad.

Q.14 A rectangular waveguide measures 3×4.5cm internally and has a 10 GHz signal propagated in it. Calculate the cut off frequency (λ_c) and the guide wavelength (λ_g).

(4)

Ans: $\lambda_c = \dfrac{2}{\sqrt{\left(\dfrac{m}{a}\right)^2 + \left(\dfrac{n}{b}\right)^2}}$

TE_{10} mode m = 1, n = 0

$\lambda_c = \dfrac{2a}{m} = 2\times 0.045 = 0.090m$

$\lambda_g = \dfrac{\lambda}{\sqrt{1-\left(\dfrac{\lambda}{\lambda_c}\right)^2}}$ where $\lambda = \dfrac{3\times 10^8}{10\times 10^9} = 0.03m$

$\lambda_g = 0.0318m$.

Q.15 What is a Hertzian dipole? Discuss the time variations of the current and charges associated with the Hertzian dipole. Also discuss the characteristics of the electromagnetic field due to the Hertzian dipole. **(2 + 3 + 3)**

Ans: Refer section 10.42, Page No 850 of "Electromagnetic Field and Waves" by K D Prasad.

Q.16 Deduce the radiation resistance and the directivity for a half wave dipole? **(6)**

Ans: Refer section 10.47, Page No 855 of "Electromagnetic Field and Waves" by K D Prasad.

Q.17 Distinguish between broadside and end fire radiation patterns with suitable sketches. What is an array factor? Provide a physical explanation for the array factor. **(4 + 4)**

Ans: Refer section 10.3, Page No 781 of "Electromagnetic Field and Waves" by K D Prasad.

Q.18 Estimate the maximum usable frequency (MUF) for a critical frequency of 10 MHz and an angle of incidence of 45°. **(6)**

Ans: $MUF = fc\sec\theta$

$= 10MHz \times \sec 45^0$

$= 10\sqrt{2}MHz$

Q.19 What are the boundary conditions for static electric fields in the general form at the interface between two different dielectric media? Explain. **(8)**

Ans: At the boundary between two dielectric media, the tangential and the normal component electric displacement have to be analysed.

(i) Tangential component:

Let E_1 & E_2 are the permittivities for two dielectric media.

E_{t_1} & E_{t_2} are the tangential components in media I & media II.

A rectangular closed path abcd has been considered.

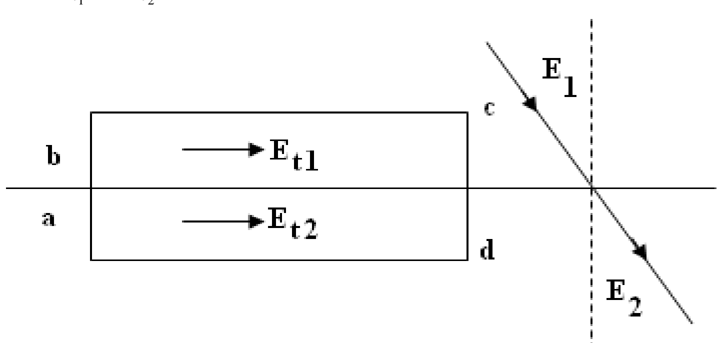
For the closed path

$$\oint E.dl = \int_a^b E.dl + \int_b^c E.dl + \int_c^d E.dl + \int_d^a E.dl = 0$$

Let ab = cd $\Rightarrow$ 0, at the boundary

So $\oint E.dl = 0 + E_{t_1}cd + 0 - E_{t_1}cd = 0$

So $E_{t_1} = E_{t_2}$



(ii) Normal components of electric displacement

A small cylindrical pill box of height Δh and crosssectional area Δs has been considered. We know that

$\oint_s D.ds = Q$ enclosed

Or $\int_{top} D.ds + \int_{bottom} D.ds + \int_{sides} D.ds = Q$ (enclosed)

Let $\Delta h \to 0$ at the boundary and ρ_s be the surface charge density enclosed at the surface of boundary.

Also, D_{n_1} & D_{n_2} are the normal components of D.

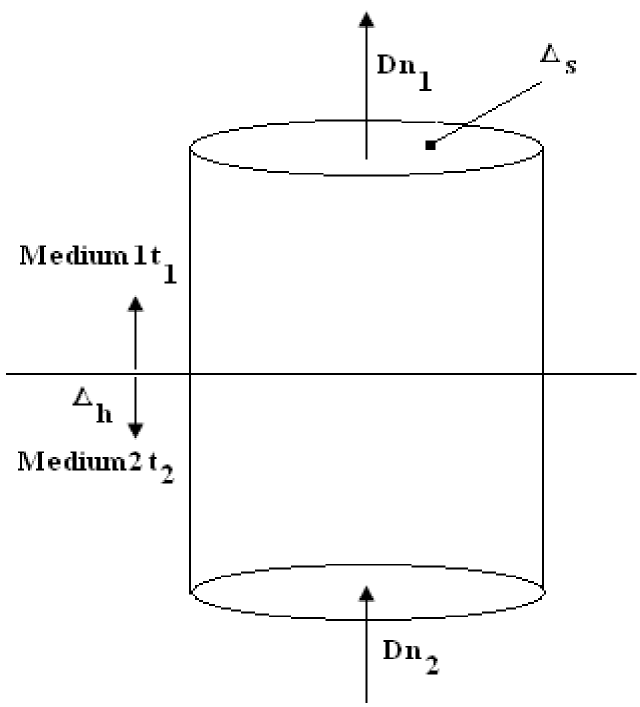
So, $D_{n_1}\Delta s - D_{n_2}\Delta s + 0 = \rho_s \Delta s \qquad \because \quad Q = \rho_s \Delta s$

Or $D_{n_1} - D_{n_2} = \rho_s$

For free surface charge density, $\rho_s = 0$

$D_{n_1} = D_{n_2}$, for $\rho_s = 0$.

Fig(ii)



Q.20 Using Gauss's law in integral form, obtain the electric field due to following charge distribution in spherical coordinates:

Charge density, $$\rho(r,\theta,\varphi) = \begin{cases} K/r^2 & \text{for } 0 < r < R \\ 0 & \text{for } R < r < \infty \end{cases}$$

Where, R is the radius of spherical volume,
K is constant & r is radial distance. **(8)**

Ans:<u>For r < R,</u> $dQ_r = \rho dv$

$$= \frac{K}{r^2} 4\pi r^2 dr$$

$$= 4\pi K dr$$

So, total charge enclosed within radius r (r < R)

$$Q_r = \int dQ_r = \int 4\pi K dr = 4\pi K r \qquad \text{-------------------------} \ (1)$$

From Gauss's law, (r < R)

$$\in_0 \int E.ds = \int \rho dv$$

$$\in_0 E 4\pi r^2 = 4\pi K r$$

Or, $E = \dfrac{K}{\in_0 r} \underline{i_r}$, for r < R

<u>At r = R</u>,

$$\in_0 \int_0^R E.ds = \int \rho dv$$

$$= \int_0^R \frac{K}{r^2} 4\pi r^2 dv$$

Or, $\in_0 E 4\pi r^2 = 4\pi K R$

$E = \dfrac{K}{\in_0 R} \underline{i_r}$, for r = R

<u>For r > R</u>

$$\in_0 \int_0^r E ds = \int_0^R \rho dv = \int_0^R \frac{K}{r^2} 4\pi r^2 dr$$

Or, $\in_0 E 4\pi r^2 = 4\pi K R$

$E = \dfrac{K}{\in_0} \dfrac{R}{r^2} \underline{i_r}$ for r > R.

Q.21 State Poisson's equation. How is it derived? Using Laplace's equation, for a parallel plate capacitor with the plate surfaces normal to X-axis, find the potential at any point between the plates. Given, $V = V_1$ at $x = x_1$ and $V = V_2$ at $x = x_2$. **(2+4+4)**

Ans: From Maxwell's first equation i.e. Gauss's Law in differential form, we have

$\vec{\nabla}.\vec{D} = \rho$ $\qquad$ $\vec{D} = \in \vec{E}$

Using $\vec{E} = -\vec{\nabla} V$ $\qquad$ Where $\in$ is the permittivity of the medium.

We obtain

$$-\nabla.\in \nabla V = \rho$$

Using vector identity

$$\nabla.\phi A = \phi \nabla.A + A.\nabla \phi$$

We get

$$\in \vec{\nabla}.\vec{\nabla} V + \vec{\nabla} \in .\vec{\nabla} V = -\rho$$

Or $\in \nabla^2 V + \nabla \in .\nabla V = -\rho$

Or $\nabla^2 V = -\dfrac{\rho}{\in}$ $\qquad$ as $\nabla \in = 0$ for uniform $\in$.

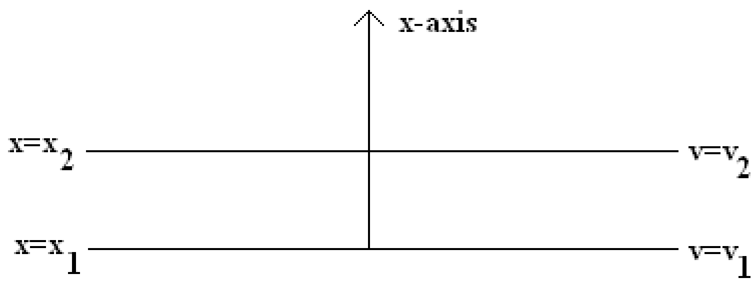
This is Poisson's equation which governs the relationship between the volume charge density ρ in a region of uniform permittivity $\in$ to the electric scalar potential V in that region.

In carlesian coordinate. Poisson's equation becomes

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=-\frac{\rho}{\in}$$

If V is a function of x only then we have

$$\frac{\partial^2 V}{\partial x^2}=-\frac{\rho}{\in}$$



Laplace equation for a parallel plate capacitor is

$$\frac{\partial^2 V}{\partial x^2}=0$$

$$\frac{d^2 V}{dx^2}=0$$

On integration,

$$\frac{\partial V}{\partial x}=a$$

And again after integration

$$V=ax+b \qquad \text{------------------ (1)}$$

Where a & b are constants of integration to be determined by boundary condition.

Boundary conditions are

$V=V_1$ at $x=x_1$

$V=V_2$ at $x=x_2$

We then have

$$\left.\begin{aligned}V_1&=ax_1+b\\V_2&=ax_2+b\end{aligned}\right\} \qquad \text{------------------ (2)}$$

Equation (2) $\Rightarrow a=\dfrac{V_1-V_2}{x_1-x_2}$ and $b=\dfrac{V_2x_1-V_1x_2}{x_1-x_2}$

Putting the value of a and b in equation (1), we obtain

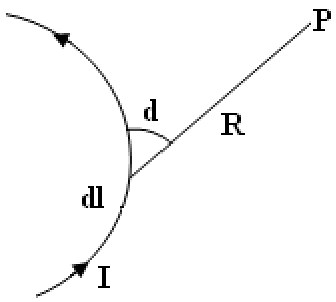
$$V=\frac{V_1(x-x_2)-V_2(x-x_1)}{x_1-x_2}$$

Which gives potential at any point x between the plates i.e. $x_1<x<x_2$.

Q.22 State and explain Biot-Savart's law. **(2+4)**

Ans: The Biot-Savart Law states that the magnetic flux density due to a current element Idl(current I passing from an infinite-simal small length of conductor, dl) in free space is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\overrightarrow{dl}\times\hat{i}_R}{R^2}$$



Where μ_0 is the permeability of free space which is equal to $4\pi\times10^{-7}$.
R = distance from the element to the point P in meters.
$\hat{i}_R$ = a unit vector directed from element to P.
The above equation can also be expressed as

$$dB = \frac{\mu_0 Idl}{4\pi R^2}\sin\alpha.\hat{i}_\phi$$

Where α is the angle between the current element and the line joining it to p measured in the same order $\left(\overrightarrow{dl}\ to\ \hat{i}_R\right)$. The direction of $\overrightarrow{dB}$ is perpendicular to the plane ($\overrightarrow{dl}\times\overrightarrow{i_R}$) containing the element and the line joining the element to P as given by cross product ($\overrightarrow{dl}\times\hat{i}_R$).

For α = 0, i.e. along a line in alignment with the element, no field ($\overrightarrow{dB}=0$) is produced and for $\alpha = 90^0$ i.e. along a line perpendicular to the element. Also the field is inversely proportional to R^2.

Q.23 What is Poynting vector? How is the Poynting theorem derived from Maxwell's curl equations? Explain Poynting theorem. **(2+8)**

Ans: The quantity $\vec{E}\times\vec{H}$ as the power flow density vector associated with the electromagnetic field. It is known as the poynting vector after the name of J. H. Poynting and is denoted by the symbol $\vec{P}$. Thus,

$$\vec{P} = \vec{E}\times\vec{H}$$

It is instantaneous poynting vector, since $\vec{E}$ and $\vec{H}$ are instantaneous field vectors.

Poynting theorem:

From vector identity

$$\vec{\nabla}.\left(\vec{E}\times\vec{H}\right) = \vec{H}.\vec{\nabla}\times\vec{E} - \vec{E}.\vec{\nabla}\times\vec{H} \qquad \text{------------------------ (1)}$$

And Maxwell's curl equations,

$$\left.\begin{aligned}\vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} = -\mu\frac{\partial\vec{H}}{\partial t}\\ \vec{\nabla}\times\vec{H} &= \vec{J}_c + \frac{\partial\vec{D}}{\partial t} = \sigma\vec{E} + \in\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\} \qquad \text{------------------------ (2)}$$

Using Maxwell's curl equations (2) in equation (1), we have

$$\vec{\nabla}.\left(\vec{E}\times\vec{H}\right)=\vec{H}.\left(-\mu\frac{\partial\vec{H}}{\partial t}\right)-\vec{E}.\left(\sigma\vec{E}+\in\frac{\partial\vec{E}}{\partial t}\right)$$

$$=-\mu\vec{H}.\frac{\partial\vec{H}}{\partial t}-\sigma\vec{E}.\vec{E}-\in\vec{E}.\frac{\partial\vec{E}}{\partial t}$$

$$=-\mu\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{H}.\vec{H}\right)-\sigma\vec{E}.\vec{E}-\in\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{E}.\vec{E}\right)$$

$$=-\sigma E^2-\frac{\partial}{\partial t}\left(\frac{1}{2}\in E^2\right)-\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right) \quad \text{----------------------- (3)}$$

Substituting $\vec{P}$ for $\vec{E}\times\vec{H}$ and taking the volume integral of both sides of equation (3) over the volume V, we obtain

$$\int_v(\nabla.P)dv=-\int_v\sigma E^2dv-\int_v\frac{\partial}{\partial t}\left(\frac{1}{2}\in E^2\right)dv-\int_v\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right)dv$$

Using divergence theorem on left hand side we have

$$\oint_s\vec{P}.d\vec{s}=-\int_v\sigma E^2dv-\frac{\partial}{\partial t}\int_v\frac{1}{2}\in E^2dv-\frac{\partial}{\partial t}\int_v\frac{1}{2}\mu H^2dv \quad \text{-------------------- (4)}$$

Where s is the surface bounding the volume v. The equation (4) is called the pointing theorem in which

The power dissipation density, $P_d=\sigma E^2$

The electric stored energy density, $w_e=\frac{1}{2}\in E^2$

The magnetic stored energy density, $w_m=\frac{1}{2}\mu H^2$

And, the power flowing out of the closed surface s i.e. LHS of eqn (4) = $\oint_s P.ds$.

Q.24 The conduction current density in a lossy dielectric is given as $J_C=0.02\,\mathrm{Sin}\,10^9t\ \ \mathrm{A/m^2}$, find the displacement current density if $\sigma=10^3\,\mathrm{mho/m}$, $\in_r=6.5$ & $\in_o=8.854\times10^{-12}$.

Ans: **(6)**

$$J_c=\sigma E=0.02\sin10^9t$$
$$=10^3E$$

So, $E=2\times10^{-5}\sin10^9t$

The displacement current density is given by

$$J_d=\frac{\partial D}{\partial t}=\frac{\partial}{\partial t}\left(\in_0\in_r\ 2\times10^{-5}\sin10^9t\right)$$
$$=8.854\times10^{-12}\times6.5\times2\times10^{-5}\times10^9\cos10^9t$$

Thus, $J_d=1.15\times10^{-6}\cos10^9t$ A/m.

Q.25 Derive general expressions for reflection coefficient and transmission coefficient for $\vec{E}$ and $\vec{H}$ fields when an electromagnetic wave is incident normally on the boundary separating two different perfectly dielectric media. **(8)**

Ans: When a plane electromagnetic wave is incident normally on the surface of a perfect dielectric, part of energy is transmitted and part of it is reflected
A perfect dielectric is one with two conductivity so there is no loss of power in propagation through the dielectric.
In the figure suffix 'i', 'r' & 't' signify the "incident", "reflected" and "transmitted" components respectively for E and H. Intrinsic impedances

For medium 1, $\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$

& for medium 2, $\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$

The relationships for electric and magnetic fields are

$E_i = \eta_1 H_i$ ------------------ (1)

$E_r = -\eta_1 H_r$ ------------------ (2)

(energy direction changed, so –ve sign)

And $E_t = \eta_2 H_t$ ------------------ (3)

The continuity of components of E & H require that

$$\left.\begin{aligned} E_i + E_r &= E_t \\ H_i + H_r &= H_t \end{aligned}\right\} \quad \text{------------------ (4)}$$

(1) & (2) $\Rightarrow H_i + H_r = \frac{1}{\eta_1}(E_i - E_r) = H_t = \frac{1}{\eta_2}(E_i + E_r)$

Or $\eta_2(E_i - E_r) = \eta_1(E_i + E_r)$ ------------------ (5)

Using eqn (4) & (5), the reflection coefficient, $R = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

And transmission coefficient, $T = \frac{E_t}{E_i} = \frac{E_i + E_r}{E_i}$

$$= 1 + \frac{E_r}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Or $T = \frac{2\eta_2}{\eta_1 + \eta_2}$

Similarly, we can get reflection and transmission coefficients for H as

$R = \frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$

And $T = \frac{H_t}{H_i} = \frac{\eta_1 E_t}{\eta_2 E_i} = \frac{2\eta_1}{\eta_1 + \eta_2}$

Q.26 Consider the volume current density distribution in cylindrical coordinates as

$$\begin{aligned} J(r, \phi, z) &= 0, \quad 0 < r < a \\ &= J_o\left(\frac{r}{a}\right)\hat{i}_z, \quad a < r < b \\ &= 0, \quad b < r < \infty \end{aligned}$$

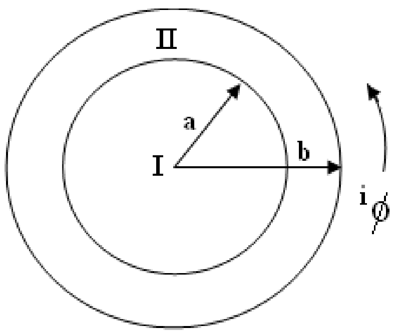
Where a and b are inner and outer radii of the cylinder. Find the magnetic field intensity in various regions. **(8)**

Ans: For region: 0 < r < a, J(r, ¢, z) = 0
Using Ampere's law,

$$\oint \vec{H}.\overrightarrow{dl} = \int \vec{J}.\overrightarrow{ds} = \int_{r=0}^{a}\int_{\phi=0}^{2\pi} J r dr d\phi i_z = 0$$

Or H.2πr = 0
Or H = 0 for region I (0 < r < a)

Region II, a < r < b, $J = J_0\left(\frac{r}{a}\right)\hat{i}_z$



Applying Ampere's Law,

$$\oint H.dl = \int_{\phi=0}^{2\pi}\int_{r=a}^{r} J_0 \frac{r}{a} r dr d\phi = \frac{J_0}{a}\frac{r^3}{3}\Big|_a^r \phi\Big|_0^{2\pi}$$

Or, $H.\cancel{2\pi}r = \frac{\cancel{2\pi}}{3a} J_0\left(r^3 - a^3\right)$

Or, $H = \frac{J_0}{3ar}\left(r^3 - a^3\right)\hat{i}_\phi$

At r = b, $H = \frac{J_0}{3ab}\left(b^3 - a^3\right)\hat{i}_\phi$

Region III, b < r < ∞, J = 0

We have, $\oint H.dl = \int_{\phi=0}^{2\pi}\int_{r=a}^{r=b} \frac{J_0 r}{a} r dr d\phi$

Or, $H = \frac{J_0}{3ar}\left(b^3 - a^3\right)\hat{i}_\phi$.

Q.27 Calculate the electric field due to a line charge considering it a special Gaussian surface. **(5)**

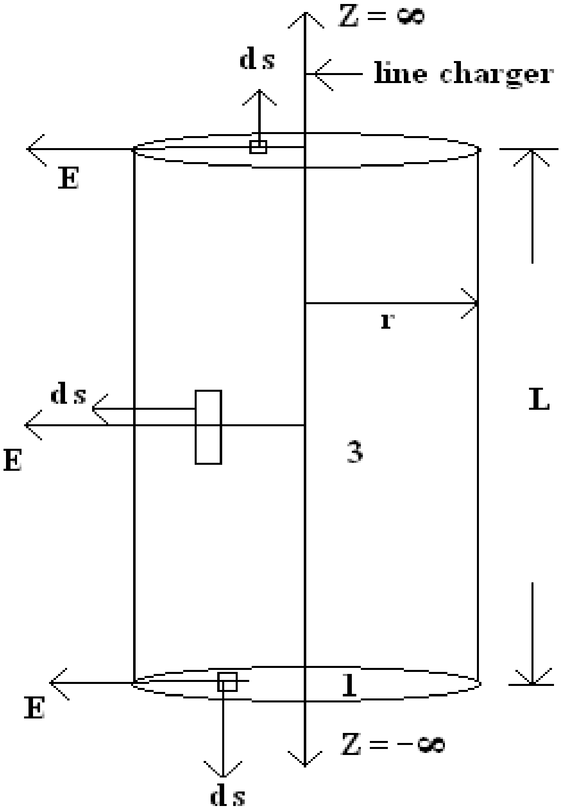
Ans: Assuming line charge along z-axis Electric field E or flux density D can only have an r component can only depend on r.
A closed right circular cylinder or radius r, whose axis is z-axis, has been assumed for the purpose of Gussian surface.
From Gauss's Law,

$$Q = \int_{s=1} D.ds + \int_{s=2} D.ds + \int_{s=3} D.ds$$

$$= 0 + 0 + \int_{s=3} \in E.ds$$

[as on the surface 1 & 2, E & ds are far so E.ds = 0.]

$$= \in E2\pi rL \qquad \text{------------------ (1)}$$



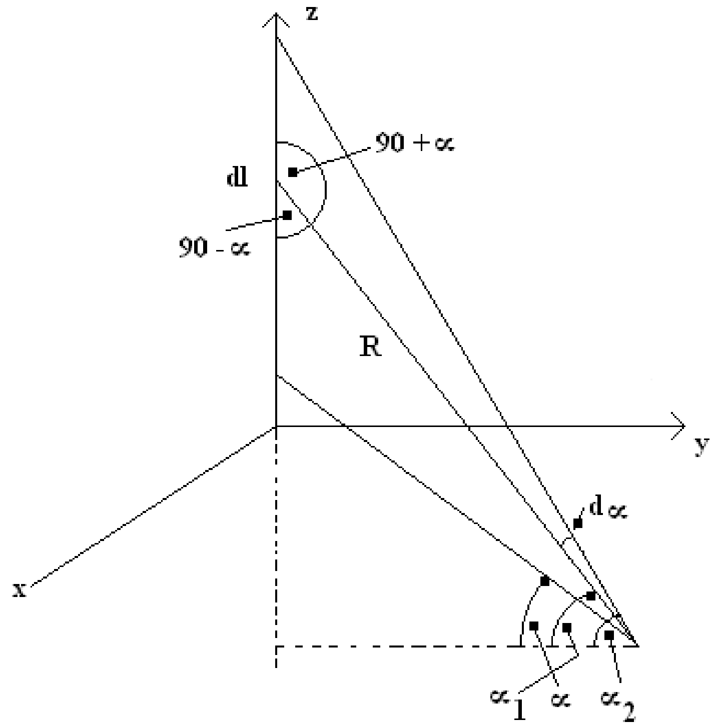
If ρ is the charge per unit length and the length of Gaussian Cylindrical surface is, L,

$$\rho = \frac{Q}{L}$$

$$(1) \Rightarrow \rho L = \in E2\pi rL \Rightarrow E = \frac{\rho}{2\pi r \in} \underline{i}_r$$

Q.28 Find out the magnetic field intensity at any point due to a current carrying conductor of finite length using the Biot-Savart law. **(5)**

Ans: $dH = \frac{Idl \times \underline{i}_R}{4\pi R^2} = \frac{Idl \sin(90+\alpha)}{4\pi R^2} \underline{i}_\phi$



Now $Rd\alpha = dl\cos\alpha$

$$dl = \frac{Rd\alpha}{\cos\alpha}$$

So $dH = \dfrac{IR\dfrac{d\alpha}{\cos\alpha}.\cos\alpha}{4\pi R^2}.\underline{i_\phi}$

$$= \frac{Id\alpha}{4\pi R} = \frac{Id\alpha}{4\pi\left(\frac{r}{\cos\alpha}\right)}.\underline{i_\phi}$$

Or, $H = \dfrac{1}{4\pi R}\displaystyle\int_{\alpha_1}^{\alpha_2}\cos\alpha d\alpha.\underline{i_\phi}$

Or, $H = \dfrac{1}{4\pi R}(\sin\alpha_2 - \sin\alpha_1).\underline{i_\phi}$.

Q.29 Find the characteristic impedance, propagation constant and velocity of propagation for a transmission line having the following parameters:

R = 84 ohm/Km, $G = 10^{-6}$ mho/Km, L=0.01 Henry / Km,
C=0.061 μF /Km and frequency = 1000 Hz. **(6)**

Ans: Characteristic impedance,

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Where $Z = R + j\omega L = 84 + j2\pi 1000 \times 0.01$

$$= 84 + j62.83 = 104.9\angle 36.8^0 \text{ ohm/km}$$

And $Y = G + j\omega C = 10^{-6} + j2\pi \times 1000 \times 0.061 \times 10^{-6}$

$= 10^{-6} + j383.27 \times 10^{-6}$

$= 383.27 \times 10^{-6} \angle 89.85^0$ mho/km

$$\therefore \quad Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{104.9\angle 36.8^0}{383.27 \times 10^{-6} \angle 89.85^0}}$$

$= 523.16\angle -26.53^0$ ohm.

Propagation Constant

$\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$

$= \sqrt{104.9\angle 36.8^0 \times 383.27 \times 10^{-6} \angle 89.85}$

$= 0.201\angle 63.33^0$

$= 0.09 + j0.18$ per km $= \alpha + j\beta$.

So $\beta = 0.18$ rad/km

$\therefore$ Velocity of propagation,

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 1000}{0.18 \times 10^{-3}} = 34.9 \times 10^{-6} \text{ m/sec.}$$

Q.30 How does dissipation-less transmission lines act as tuned circuit elements? Explain. **(9)**

Ans: At high frequencies (300 MHz to 3 GHz), a transmission line can be used as circuit element (like a capacitor or an inductor). At these frequencies the physical length of the line is convenient to use.

For lossless line,

$\gamma = j\beta$ and $Z_0 = R_0$

And $\tan h\gamma l = j \tan \beta l$.

The input impedance of the line Z_{in} with load Z_L becomes

$$Z_{in} = R_0 \left[\frac{Z_L + JR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l} \right] \quad \text{---------------------- (1)}$$

(i) Open circuited line $(Z_L \to \infty)$

Equation (1) $\Rightarrow Z_{in} = -j\dfrac{R_0}{\tan \beta l} = -jR_0 \cot \beta l$ ---------------- (2)

$= jX_0$ (purely reactive)

So, the input impedance of an open circuited lossless line is purely reactive, i.e. the line can be either capacitive or indirective depending on the value of βl.

Now $(X_0 = -R_0 \cot \beta l)$ vs l is plotted as given below:

X_0 can vary from $-\infty$ to $+\infty$.

For a very short length of the line $\beta l << 1$

i.e. $\tan \beta l = \beta l$.

$$\therefore \quad Z_{in} = jX_0 = -j\frac{R_0}{\beta l} = -j\frac{\sqrt{L/C}}{\omega\sqrt{LC}l} = -j\frac{1}{\omega Cl}$$

Which is capacitive reactance.

(ii) Short circuited line ($Z_L = 0$)

Equation (1) $\Rightarrow Z_{in} = jR_0 \tan \beta l$

$= jX_0$ (which is purely reactive)

The value of tan βl can vary from $-\infty$ to $+\infty$.

Similarly Z_{in} is either purely inductive or purely capacitive depending upon the value of βl.

Now $X_s (= R_0 \tan \beta l)$ vs l is plotted as given below:

When the length of a short circuited line is very short, $\beta l << 1$ we have

$$Z_{in} = jX_s = jR_0 \beta l = j\sqrt{\frac{L}{C}}\omega\sqrt{LC}l = j\omega Ll$$

Which is inductive reactance.

For first quarter wave length, a short-circuited line acts as an inductance, where as an open-circuited line appears as a capacitance. These reactances reverse each quarter wave length.

The similarity of performance of open or short-circuited lines to that of series-resonant or anti resonant circuits may be noted by comparing the above curves (for open & short circuited lines) with the curves of resonant circuits which suggest the use of lines as reactive circuit elements or as tuned circuits. For example, the input of $\lambda/4$ short circuited line appears similar to that of a parallel resonant circuit & $\lambda/4$ open circuited as series resonant circuit.

Q.31 What do you mean by matched transmission line? What are the advantages of impedance matching on high frequency lines? **(7)**

Ans:A transmission line is matched when the load impedance is equal to the characteristic impedance of the line and no reflection of waves takes place.

When a finite transmission is terminated with its own characteristic impedance, the voltage and current distributions on the line are exactly the same as through the line has been extended to infinity, i.e. a line of finite length, terminated in a load equivalent to its characteristic impedance, appears to the sending-end generator as an infinite line, i.e. a finite line terminated in Z_0 and an infinite line are indistinguishable by measurements at the source.

For such lines, $Z_{in} = Z_0$

And reflection coefficient, k = 0

V S W R = 1.

There is no reflected wave, the incident power is fully absorbed by the load. The maximum power transfer with minimum loss is possible when a transmission line is matched to the load. Also efficiency of transmission is greater when there is no reflected wave.

Q.32 What is a linear array? Define a uniform array? Show that the width of the principal lobe of a uniform end fire array is greater than that for a uniform broad side array of the same length. **(2+3+5)**

Ans: An array is linear when the elements of the array are spaced equally along a straight line as shown in the figure.

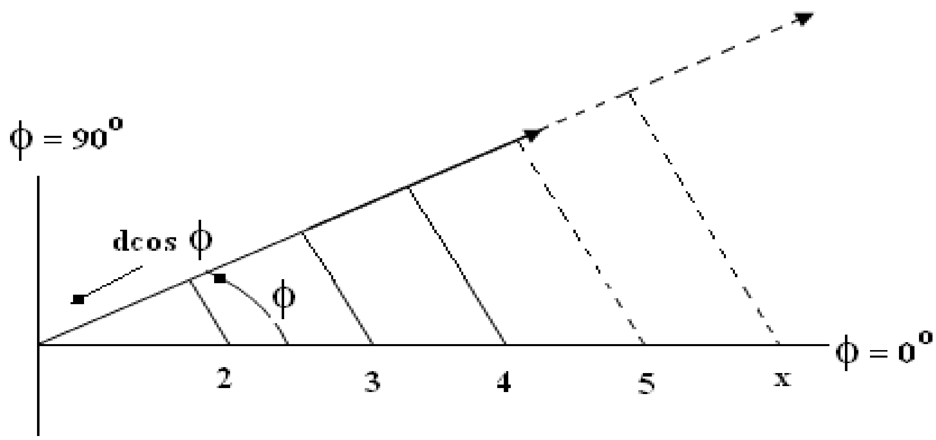
In a uniform linear array the elements are fed with currents of equal magnitude and having a uniform progressive phase shift along the line. The pattern of such an array is obtained by adding vectorially the field strengths due to each of elements.

$E_T = E_0(1 + e^{j\psi} + e^{j3\psi} + ... + e^{j(n-1)\psi})$ --------------------- (1)

$= E_0 \sum_{n=1}^{N} e^{j(n-1)\psi}$, where, $\psi = \beta d \cos\phi + \alpha$

d = spacing between sources

α = the progressive phase shift between consecutive element (sources). i.e. α is the angle by which the current in any elements lead the current in the preceding element.



Multiplying equation (1) by $e^{j\psi}$ yields

$E_T e^{j\psi} = E_0\left(e^{j\psi} + e^{j2\psi} + e^{j3\psi} + ... + e^{jn\psi}\right)$ ------------------- (2)

Subtracting equation (2) from equation (1), we have

$E_T\left(1 - e^{j\psi}\right) = \left(1 - e^{jn\psi}\right)E_0$

$$E_T = E_0\left(\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right) = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)} \angle(n-1)\psi/2$$

Keeping the centre of the array as reference for phase,(i.e. $(n-1)\psi/2$, phase angle = 0) & for isotropic sources, as the principal maximum occurs at $\psi = 0$.

$$E_T = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)} \quad \text{------------------- (3)}$$

For principal Maximum, $\psi = 0 \Rightarrow \cos\phi = -\dfrac{\alpha}{\beta d}$.

In broadside array, the direction of principal lobe

$\phi = 90^0$ i.e. $\alpha = 0$

In end fire array, the direction of principal lobe is

$\phi = 0^0$ i.e. $\alpha = -\beta d$

For Minimum (nulls of patterns):

Equation (3) becomes zero at $\frac{n\psi}{2} = m\pi$ m = 1, 2, 3, ...

For Secondary Maximum:

Secondary maximum occurs approximately midway between nulls, when numerator of equation (3) becomes maximum i.e.

When $\frac{n\psi}{2} = (2k+1)\Pi/2$, k = 1, 2, 3, ...

I Secondary Maximum occurs when n (k = 1) i.e.

$$\frac{n\psi}{2} = \frac{3\pi}{2}$$

$$(3) \Rightarrow \frac{E_T}{E_0} = \frac{1}{\sin\left(\frac{3\pi}{2n}\right)} = \frac{2n}{3\pi} \quad \text{(for large n)} \quad \text{----------------- (4)}$$

Amplitude of principal maximum,

$$\frac{E_T}{E_0} = n \quad \text{--------------------------- (5)}$$

(4) & (5) $\Rightarrow$ so the first secondary maximum is

$\frac{2}{3\pi} = 0.212$ times the principal maximum.

<u>Bandwidth of the principal lobe:</u>

The principal lobe is measured between the first nulls. It is twice the angle between the principal maximum and the first null. This is got by putting

$\frac{n\psi'}{2} = \pi$ or, $\psi' = \frac{2\pi}{n}$ (condition for Min)

In Broadside array:

Since α = 0, $\cos\phi = \frac{\psi}{\beta d}$ and principal maximum occurs at $\phi = \pi/2$. The first null occurs at an angle $\left(\pi/2 + \Delta\phi\right)$ where

$$\cos\left(\pi/2 + \Delta\phi\right) = \frac{\psi'}{\beta d} = \frac{2\pi\,\lambda}{n \times 2\pi \times d} = \frac{\lambda}{nd} = \sin\Delta\phi$$

For small $\Delta\phi$, $\Delta\phi = \frac{\lambda}{nd}$ and the width ($2\Delta\phi$) of the principal lobe is approximately twice the reciprocal of the array length in wave lengths, i.e. the width of principal lobe, $2\Delta\phi = \frac{2\lambda}{nd}$. ------------------------ (6)

<u>In endfire array:</u>

$\psi = \beta d(\cos\phi - 1)$, since $\sin\alpha = -\beta d$.

The principal maximum is at $\phi = 0$ and the first null at $\phi_1 = \Delta\phi$, $\psi_1 = \beta d(\cos\phi_1 - 1) = -\frac{2\pi}{n}$

Or $\cos\Delta\phi - 1 = -\frac{\lambda}{nd}$

For small $\Delta\phi$, $\frac{(\Delta\phi)^2}{2} = \frac{\lambda}{nd}$

$$\cos\Delta\phi - 1$$
$$= (1 - \sin^2\Delta\phi)^{1/2} - 1$$
$$= -\frac{(\Delta\phi)^2}{2}$$

Or, $\Delta\phi = \sqrt{\dfrac{2\lambda}{nd}}$

Thus, the width of principal lobe,

$$2\Delta\phi = 2\sqrt{\frac{2\lambda}{nd}} \qquad \text{---------------------} \ (7)$$

Equation (6) & (7) inculcates that the width of principal lobe of a uniform end fire array is greater than that for a uniform broad side array of same length.

Q.33 Find the radiation resistance of a dipole antenna $\lambda/10$ long. Also, find the antenna efficiency if the loss resistance is 1 ohm. **(6)**

Ans: We know that

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

$$= 80\pi^2\left(\frac{1}{10}\right)^2 = 7.9\Omega, \text{ since } l = \lambda/10$$

If there is any heat loss in the antenna due to finite conductivity of the dipole or to losses in the associated dielectric structure. So the terminal resistance,

$R = R_{loss} + R_{rad}\Omega$ where $R_{loss} \rightarrow$ loss resistance, Ω

$R_{rad} \rightarrow$ radiation resistance, Ω

In this case of $\lambda/10$ dipole, $R_{loss} = 1\Omega$

Then the terminal resistance, $R = 1.0 + 7.9 = 8.9\Omega$

The antenna effieciency,

$$= \frac{Power\ radiated}{Power\ input} = \frac{R_{rad}}{R_{rad} + R_{loss}}$$

$$= \frac{7.9}{8.9} = 89\%$$

i.e. A longer dipole with larger radiation resistance would be more efficient provided R_{loss} remains small.

Q.34 State and explain the different types of propagations possible between a transmitter and receiver. Name the various layers of ionosphere and indicate their approximate height. Which ionospheric layers disappear at night? **(9)**

Ans: The energy radiated from a transmitting antenna may reach the receiving antenna over any of many possible paths as given below –

Waves that arrive at receiving antenna after reflection or scattering in ionosphere are known as sky waves which are ionospherically reflected/scattered in the troposphere (upto 10 km) are termed as tropospheric waves. Waves propagated near the earth's surface are called ground waves. Ground waves can be divided into space wave and surface wave. The space wave is made up of the direct waves from transmitted to receiver where as the ground waves are the reflection waves from the earth's surface & then reach to the receiver.

Ground or surface wave propagation :

Ground or surface wave propagation operates mainly in the three frequency bands, VLF, LF and MF or 3 KHz to 3 MHz. The range over which the ground wave can be used with realistic transmitter powers is determined by the attenuation of the fields and as attenuation increases with frequency, VLF propagation is virtually world wide.

Space wave propagation :

Space wave propagation is extensively used in VHF and UHF for line of sight transmission. The propagation is affected by refraction by troposphere, diffraction by obstacles along the propagation path, scattering by urban environment and fading.

Ionospheric propagation:

The ionospheric is a region of the upper atmosphere extending from approximately 50 km to more than 1000 km above earth's surface. In this regions the continent gases are ionised mostly because of ultraviolet radiation from the sun which results in the production of positive ions (almost immobile) and electrons. The electrons are free to move under the influence of fields of a wave incident on the medium. The electron motion produces a current that influences the wave propagation. The ionospheric region is described as D-layer, E-layer, F_1-layer and F_2-layers in which the ionization changes with the hour of the day, the season and the sunspot cycle.

(i) D-Layer: it has low free electron density ($10^9/m^3$) compared to its molecular density of $10^{20}/m^3$. It occurs during daylight hours at a height of 60-90 km and responds very quickly due to movement of sun. It has almost no effect on bending of high frequency radio waves, but produces considerable attenuation at lower sky wave frequencies in medium wave band. D-layer disappears in the night.

(ii) E-layer: the E-layer is located at 90-130 km above the ground level with free electron density nearly $10^{11}/m^3$. It causes some bending and attenuation mainly at lower sky wave frequencies.

(iii) F-layer: the F-layer is in many respects the most useful part of the ionosphere. The electron density has peak value between 250 and 350 km. The F-layer is located above 130 km (upto 400 km). During day times, F-layer splits into F_1 & F_2 layer.

Q.35 Find the following:

(i) the possible transmission modes in a hollow rectangular waveguide of inner dimension 3.44 × 7.22 cm at an operating frequency of 3000 MHz.

(ii) the corresponding values of phase velocity, group velocity and phase constant.

(7)

Ans: Free space wave length, $\lambda_0 = \frac{c}{f} = \frac{3\times10^8}{3000\times10^6}$ metre

$= 10$ cm

Also, we know that the cut off wave length

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Possible modes:

(i) For TE_{00} mode i.e. m = 0, n = 0, $\lambda_c = \infty$, (i.e. $\lambda_c > \lambda_0$) and hence there will be no propagation.

(ii) For TE_{10} mode i.e. m = 1, n = 0 and $\lambda_c = 2a = 2\times7.22 = 14.44$ cms. Hence this mode will propagate because $\lambda_c > \lambda_0$.

(iii) For TE_{01} mode i.e. m =0, n = 1, and $\lambda_c = 2b = 2\times3.44 = 6.88$ cm. Hence this mode will not propagate because $\lambda_c < \lambda_0$.

Obviously the higher TE mode will not propagate for $\lambda_c < \lambda_0$ for other values of m & n. Also for TM_{mn} mode the lowest value of m & n is unity hence no TM mode is possible at the frequency.

Since the guide wave length $\lambda_g = \dfrac{\lambda_0}{\sqrt{1-\left(\dfrac{\lambda_c}{\lambda_0}\right)^2}} = \dfrac{0.1}{\sqrt{1-\left(\dfrac{0.1}{0.144}\right)^2}}$ = 0.141 metres.

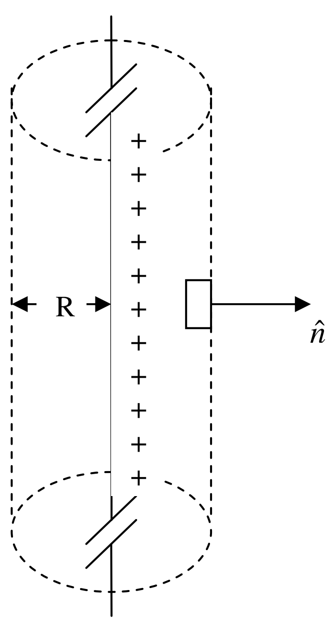
We get phase velocity $v_p = \left(\dfrac{\lambda_g}{\lambda_0}\right)c = \dfrac{0.141}{0.100}\times3\times10^8 = 4.23\times10^8$ m/sec.

And group velocity, $v_g = \left(\dfrac{\lambda_0}{\lambda_g}\right)c = \dfrac{0.100}{0.141}\times3\times10^8 = 2.28\times10^8$ m/sec.

The phase constant, $\beta = \dfrac{2\pi}{\lambda_g} = \dfrac{2\times3.14}{0.141} = 44.7$.

Q.36 Using Gauss theorem, derive an expression for the electric field intensity due to a line charge of infinite length at distance R. **(8)**

Ans:



Figure

$$d\Psi = \overrightarrow{D}.\ \overrightarrow{ds}$$

$$= D\,ds\ \hat{r}.\hat{n} \quad (\hat{r}\ \&\ \hat{n}\ are\ parallel)$$

$$= D\,ds$$

where $\hat{r}$ is unit vector in the direction of flux density D and $\hat{n}$ in the direction of normal drawn to the surface.
Total flux from lateral surface.

$$\psi_L = \int_s d\psi_L$$

$$= \int_s Dds \cos 0 \left(\hat{r} \,\&\, \hat{n}\ are\ parallel\right)$$

$$= D\int_s ds$$

= D (2 πRl)

Total flux from Top surface

$$\psi_T = \int_s d\Psi_T$$

$$= \int_s Dds \cos 0 \left(\hat{r} \,\&\, \hat{n}\ are\ parallel\right)$$

$$\psi_T = 0$$

Total flux from bottom surface

$$\Psi_B = \int_s d\Psi_B$$

$$= \int_s Dds \cos\theta \left(\theta = 90^0, because\ \hat{r} \,\&\, \hat{n}\ are \perp^{er}\ to\ each\ other\right)$$

ψ_B = O
Therefore total flux coming one of the cylinder is given by.
$\psi = \psi_L + \psi_T + \psi_B$
$\psi = D\ 2\ \pi\ Rl$
By Gauss law ψ = Q, total charge enclosed
Q = D 2 π Rl

$\rho_L l = D\ 2\ \pi\ Rl \qquad \rho = \dfrac{Q}{l}$

$\rho_L l = \varepsilon E 2\pi Rl \left(\vec{D} = \varepsilon\vec{E}\right)$

$$E = \frac{\rho_L}{2\pi\varepsilon R}\hat{r}$$

where $\hat{r}$ is a unit vector normal to the line charge.

Q.37 A positive charge density of Q_v C/m^3 occupies a solid sphere. At a point in the interior at a distance 'r' from the center a small probe charge of +q is inserted. What is the force acting on the probe charge? **(8)**

Ans: $Q^1 = Q_v\ X\ {}^4\!/_3\,\pi\, r^3$

Total flux over the surface = charge enclosed.

$$\oint_s \vec{D}\,.\,\vec{ds} = Q^1$$

$$\oint_s D ds \cos\theta \left\{\theta = 0^0,\ because\ \hat{r}\ \&\ \hat{n}\ are\ parallel\right\}$$

$$\int_s D ds \cos 0 = Q^1$$

$$D\, 4\pi r^2 = Q^1$$

$$D\, 4\pi r^2 = Q_v\ X\ \tfrac{4}{3}\pi r^3$$

$$\varepsilon E\, 4\pi r^2 = Q_v\ X\ \tfrac{4}{3}\pi r^3$$

$$E = \frac{Q_V\, r}{3\varepsilon}$$

Since $\vec{E} = \dfrac{\vec{F}}{q}$

Force on the probe charge $= q\left[\dfrac{Q_V\, r}{3\varepsilon}\right]$ newton

Q.38 Prove Ampere's circuital law for time varying field condition in differential form. **(7)**

Ans: $\nabla.\vec{D} = \rho$

$$\frac{\partial}{\partial t}\left(\nabla \vec{D}\right) = \frac{\partial \rho_V}{\partial t}$$

$$\nabla.\left(\frac{\partial \vec{\rho}}{\partial t}\right) = \frac{\partial \rho_V}{\partial t}$$

$$\nabla.\vec{J} = -\frac{\partial \rho_V}{\partial t}\ \{equation\ of\ continuity\}$$

$$\nabla.\vec{J} = -\nabla.\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$\nabla.\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = 0$$

For time varying case $\nabla\vec{J} = 0$, *is changed to* $\nabla.\left(\vec{J} + \dfrac{\partial \vec{D}}{\partial t}\right) = 0$. So $\vec{J}$ must be replaced by $\vec{J} + \dfrac{\partial \vec{D}}{\partial t}$.

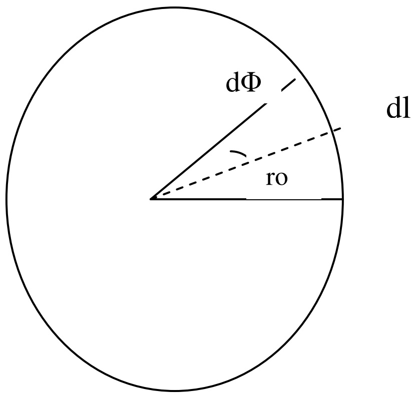
$\therefore$ *Ampere's circuital law in point form becomes*

$$\boxed{\nabla X \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Q.39 A circular conductor of 1cm radius has an internal magnetic field

$$\vec{H} = \frac{1}{r}\left\{\frac{1}{a^2}\sin ar - \frac{r}{a}\cos ar\right\}\vec{i\phi}$$

where a = $\pi/2\, r_o$ and r_o is the radius of the conductor. Calculate the total current in the conductor ($\vec{i\phi}$ is unit vector). **(9)**



Ans: $\oint \vec{H}.\vec{dl} = I$

$r = r_o$

$dl = r_o\, d\phi$

$$H\phi = \frac{1}{r_o}\left\{\frac{1}{a^2}\quad \sin ar_0 - \frac{r}{a}\cos ar_o\quad\right\}$$

$$\oint \vec{H}.\ \vec{dl} = \oint H\ \phi\ r_o\ d\ \phi = I$$

$$\oint \frac{1}{r_o}\left\{\frac{1}{a_2}\sin ar_o - \frac{r_o}{a}\cos ar_o\quad\right\} r_o d\phi = I$$

$$a = \frac{\pi}{2r_o}\ by\ data$$

$$\therefore \oint\left[\frac{1}{a_2}\sin\left(\frac{\pi}{2r_o}r_o\right) - \frac{r_o}{a}\ \cos\left(\frac{\pi}{2r_o}r_o\right)\right) d\phi = I$$

$$\oint \frac{4r_o{}^2}{\pi^2}\, d\phi = I\ |because|\ \left(\pi/_2\right) = 0$$

$$\frac{4r_o^2}{\pi}\oint d\phi = I$$

$$\frac{4r_o{}^2}{\pi^2}\ (2\pi) = I$$

$$I = \frac{8r_o^2}{\pi}$$

$$r = 10^{-2}\ m$$

$$\boxed{I = 2.55 \times 10^{-4}\ A}$$

Q.40 Prove the electric field normal components are discontinuous across the boundary of separation between two dielectrics. **(6)**

Ans: Flux over top surface = $\int \overrightarrow{D1} \, . \, \overrightarrow{ds}$

Flux over bottom surface = $\int \overrightarrow{D_2} \, .\overrightarrow{ds}$

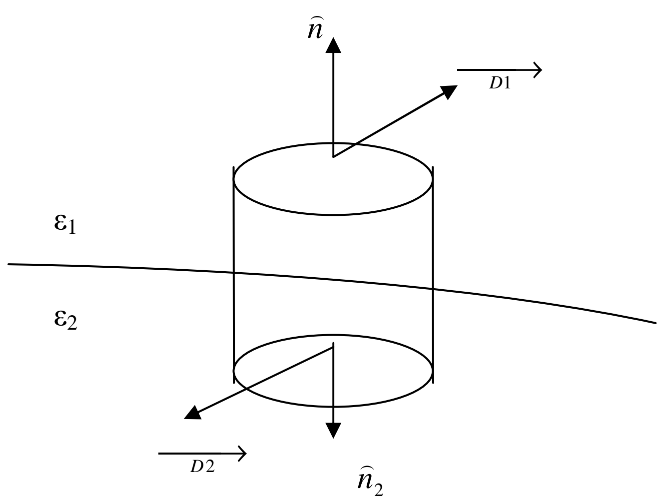
Since height of the box is vanishingly
Small the flux through the sides of box is ignored

Net flux = $\int \overrightarrow{D_1} \, \overrightarrow{ds} - \int \overrightarrow{D_2} \, . \, \overrightarrow{ds}$

Net flux = $\overrightarrow{D_1} \, \overrightarrow{\Delta s} - \overrightarrow{D_2} \, \Delta \vec{s}$

From gauss law net flux = total charge which is equal to $\rho_s \Delta s \left(because \ \rho_s = \dfrac{Q}{\Delta s} \right)$

$\rho_s \ \Delta s = (\overrightarrow{D} \, .\hat{n}_1 - \overrightarrow{D_2} . \hat{n}_2) \Delta s$
$\rho_s \Delta s = (D_{n1} - D_{n2}) \Delta s$
$D_{n1} - D_{n2} = \rho_s$
$\varepsilon_1 E_{n1} - \varepsilon_2 E_{n2} = \rho_s$



Since it is dielectric-dielectric interface $\rho_s = 0$
$\varepsilon_1 \mathrm{E}_{n1} - \varepsilon_2 \ \mathrm{E}_{n2} = 0$

$$\boxed{\text{Since} \ \ E_{n1} \neq E_2}$$

Q.41 Measurement made in the atmosphere show that there is an electric field which varies widely from time to time particularly during thunderstorms. Its average values on the surface of the earth and at a height of 1500m are found to be 100 $\mathrm{V/m}$ and 25 $\mathrm{V/m}$

directed downward respectively. Using Poisson's equation calculate i) the mean space charge in the atmosphere between 0 and 1500 m altitude ii) surface charge density on earth. **(10)**

Ans: $\nabla^2 V = \rho/\varepsilon_o$

$$V = -\rho/\varepsilon_0 \int x dx + \int A dx$$

$$V = -\rho/\varepsilon_0 \frac{x^2}{2} + Ax + B$$

Using V = 0 at X = 0

$$\boxed{\text{We get constant B = 0}}$$

$$\overrightarrow{Ex} = p/\varepsilon_o \ x \ + \ 100$$

At $x = 1500m, \vec{E} = 25 \ v/m$

$$25 = \rho/\varepsilon_o \ x \ 1500 \ + 100$$

$$\rho = \frac{25-100}{1500} x(8.854x10^{-12})$$

$$\rho = -4.427 \ x \ 10^{-13} \ c/m^3$$

To find surface charge density on cases

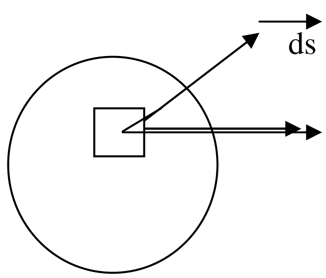
$$\vec{E} \ \frac{\rho_s}{2\varepsilon_o}$$

$$\rho_s \ 2 \ X \ 8.854 \ X \ 10^{-12} \ X \ 100$$

$= 1.771 \text{ x } 10^{-9} \ c/m^2$

Q.42 Derive an expression for equation of continuity and explain its significance. **(6)**

Ans:



Let the charge in elementary volume = ρ_v

Entire charge within the closed surface = $\int_v \rho_v \ dv$

When there is a charge flow out of the surface, then, with time there will be reduction of charge within the surface given by $\frac{-\partial}{\partial t}\int_v \rho_v \; dv$

$$\oint_s \vec{J}.\,d\vec{s} = -\frac{-\partial}{\partial t}\int_v \rho_v dv$$

$$\int_v \nabla.\vec{J}dv = \oint_s \vec{J}.d\vec{s}$$

$$\int_v \overline{V}.\vec{J}\; dv = \frac{-\partial}{\partial t}\int_v \rho_v \; dv$$

$$.\nabla.\vec{J} = -\frac{\partial \rho v}{\partial t}$$

Significance : Divergence of current density $\vec{J}$ represents the net charges which flows out of the enclosed surface and it will be equal to reduction of charge within the surface with respect to time.

Q.43 Do the fields $\vec{E} = E_m \sin x \sin t \; \overrightarrow{a_y}$ and $\vec{H} = \frac{E_m}{m}\cos x \cos t \; \overrightarrow{a_z}$, satisfy Maxwell's equation? **(10)**

Ans: $\nabla \; x \; \vec{E} = \frac{-\partial \vec{B}}{\partial t} = -\mu_0 \frac{\overrightarrow{\partial H}}{\partial t}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{oz} \\ Ex & Ey & Ez \end{vmatrix} = -\; \mu_o \frac{\partial}{\partial t}\left[H_x \hat{x} + H_y \hat{y} + H_z \hat{z}\right]$$

Since wave is traveling in x direction, Ex, Hx components are absent as well $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$ produce null result. Hence

$$\left(\frac{\partial Ez}{\partial x}\right)\hat{y} + \left(\frac{\partial Ey}{\partial x}\right)\hat{z} = -\mu_o\left(\frac{\partial Hy}{\partial t}\right)\hat{y} - \mu_o\left(\frac{\partial Hz}{\partial t}\right)\hat{z}$$

Equating the components along z-direction

$$\frac{\partial Ey}{\partial x} = -\mu_o \frac{\partial Hz}{\partial t} \rightarrow (1)$$

Since Ey= $\vec{E} = Em \sin x \sin t \; \hat{a}y \; (from \; data)$

$$\frac{\partial Ey}{\partial x} = Em \cos x \sin t \rightarrow (A)$$

H_y = $\vec{H} = \frac{Em}{\mu_o} Cosx \cos t \; \hat{a}z \; (from \; data)$

If wave is travelling in x-direction H_x and E_x are absent, as well $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial y}$ produces null result, hence

$$-\left(\frac{\partial Hz}{\partial x}\right)\hat{y}+\left(\frac{\partial Hy}{\partial x}\right)\hat{z}=\varepsilon\left(\frac{\partial Ey}{\partial t}\right)\hat{y}+\varepsilon\left(\frac{\partial Ez}{\partial t}\right)\hat{z}$$

Equating components along y- direction

$$-\frac{\partial Hz}{\partial x}=\varepsilon\frac{\partial Ey}{\partial t}\rightarrow(4)$$

Similarly for z-direction

$$\frac{\partial Hy}{\partial x}=\varepsilon\frac{\partial Ez}{\partial t}$$

Solution for the wave equation for a travelling in x-direction is given by

E_y= f(x-vt)

$$V=\frac{1}{\sqrt{\mu\varepsilon}}$$

Let u = (x-vt)

$E_y=f_1(\mu)$

$$\frac{\partial Hz}{\partial t}=\frac{-E_m}{\mu_o}\cos x\sin t$$

$$-\mu\frac{\partial Hz}{\partial t}=Em\cos x\sin t\rightarrow(B)$$

From equation (A) & (B)

$$\boxed{\frac{\partial Ey}{\partial x}=-\mu o\frac{\partial Hz}{\partial t}}$$

Thus the fields satisfy the maxwell's equations.

Q.44 Write short notes on skin depth and skin effect **(4)**

Ans: $\nabla x\vec{H}=\vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t}$

It wave is travelling in free space where there are no free charges

$$\nabla X\vec{H}=\varepsilon\frac{\partial\vec{E}}{\partial t}$$

$$\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y}\\ H_X & H_Y & H_Z\end{vmatrix}=\varepsilon\frac{\partial}{\partial t}(\vec{E}_X\hat{x}+E_Y\hat{y}+E_Z\hat{z})$$

$$\frac{\partial Ey}{\partial t} = \frac{\partial f_1(u)}{\partial t} = \frac{\partial f_1(u)}{\partial u}\frac{\partial u}{\partial t}$$

$$\frac{\partial Ey}{\partial t} = -Vf_1^1$$

Substituting this in equation (A)

$$-\partial Hz = -\sqrt{\varepsilon/\mu}\, f_1^1\ \partial x$$

$$\boxed{Hz = \sqrt{\frac{\varepsilon}{\mu}}\int f_1^1\ \partial x + C}$$

Differentiating Ey = $f_1(u)$ with respect to x we get

$$\frac{\partial Ey}{\partial x} = \frac{\partial f_1(u)}{\partial x} = \frac{\partial f_1(u)}{\partial u}\frac{\partial u}{\partial x} = f_1^1\frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial x} = 1$$

$$\therefore f_1^1 = \frac{\partial Ey}{\partial x}$$

hence Hz = $\sqrt{\varepsilon/\mu}$ Ey+C

or

Ey = $\sqrt{\mu/\varepsilon}\, Hz \rightarrow (B)$

Similarly we can prove

E_Z = - $\sqrt{\mu/\varepsilon}\, H_Y \rightarrow (C)$

Adding B & C after squaring we get

Ey^2+Ez^2 = $\mu/\varepsilon\,(H_Y^2 + H_Z^2\ )$

$$\boxed{\frac{E}{H} = \sqrt{\mu/\varepsilon}}$$

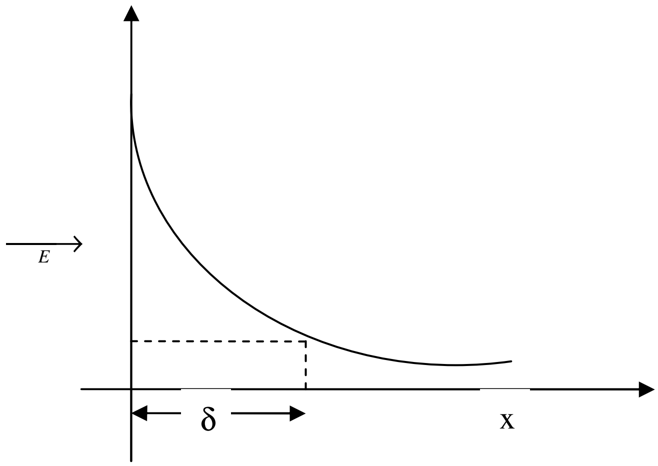
$$\left\{\begin{array}{l}\text{because } \vec{E} = \sqrt{Ey^2 + Ez^2} \\ \qquad\quad \& \ \vec{H} = \sqrt{Hy^2 + Hz^2}\end{array}\right\}$$

Q.45 Prove for a travelling uniform plane wave

$$\frac{\overline{E}}{\overline{H}} = \sqrt{\frac{\mu}{\varepsilon}}$$

where $\overline{E}$ and $\overline{H}$ are amplitudes of $\vec{E}$ and $\vec{H}$, respectively. **(12)**

Ans:



When an electromagnetic wave enters a conducting medium its amplitude decreases exponentially and becomes practically zero after penetrating a small distance. As a result, the current induced by the wave exists only near the surface of the conductor.

This effect is called skin effect

The skin depth is defined as the depth of a conductor at which the amplitude of an incident wave drops (1/e) times its value of amplitude at the time of incidence.

The depth of penetration expression is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

Q.46 Discuss the following terms

(i) Wavelength (ii) Phase velocity

(iii) Group velocity (iv) Propagation constant **(8)**

Ans: **<u>Wave length :</u>** It is defined as the distance in which the phase change of 2π radians is effected by a wave travelling along the line.

$$\lambda = \frac{2\pi}{\beta}$$

<u>Phase velocity :</u> It is defined as the velocity with which a signal of single frequency propagates along the line at a particularly frequency f.

$V_p = \omega/\beta \; km/sec$

<u>Group velocity :</u> If the transmission medium is such that different frequencies travel with different velocities, then the line or the medium is said to be dispersive. In that cases, signals are propagated with a velocity known as group velocity , V_g

$$V_g = \frac{\omega_2 - \omega_1}{\beta_2 - \beta_1}$$

Propagation constant : It reveals the nature in which the waves are propagated along the line that is the manners in which the voltage v and current I vary with distance x.

$$P = \log_e \frac{I_s}{I_R}$$

OR

$$P = \log_e \frac{V_s}{V_R}$$

Q.47 A high frequency transmission line consists of a pair of open wires having a distributed capacitance of 0.01 μF per Km and a distributed inductance of 3mH per Km. What is the characteristic impedance and propagation constant at f=10MHz? **(4)**

Ans: Zo = $\sqrt{\frac{L}{C}} = \sqrt{\frac{3X1\bar{0}^3}{0.01X1\bar{0}^6}}$ = 547.7 ohms

IPI = $\omega\sqrt{LC}$

= $2\pi X 10 X 10^6 \sqrt{(0.01) X (1\bar{0}^6) X (3) X (1\bar{0}^3)}$

= $344.156\ {}^{rad}\!/_{km}$

Q.48 Comment on Impedance matching device. **(4)**

Ans:Impedance matching device : According to maximum power transfer theorem, for maximum power transfer source or generator impedance real part to be equal real part of load impedance and generator impedance imaginary part to be conjugate of imaginary part of load impedance. When load impedance are fixed, one has to employ matching network between source and load to match the impedance of source and load. These networks are called matching impedance devices. For example quarter wave transformer is a impedance matching device.

Q.49 Derive the condition for a distortionless line and comment on the result. **(8)**

Ans:

$$\alpha = \sqrt{\frac{1}{2}\left\{RG - \omega^2 LC + \sqrt{(RC - \omega^2 LC)^2 + \omega^2 (RC + LG)^2}\right\}}$$

In order to eliminate frequency distortion the attenuation constant must be made independent of frequency.

Thus the desired condition

$\sqrt{(RC - \omega^2 LC)^2 + \omega^2 (RC + LG)^2}$ = w^2LC+K

Squaring on both and solving, we get

$R^2G^2+\omega^2(R^2C^2+LG^2) = K^2+2\omega^2LCK$

Which implies K = RG

And $2LCK = R^2C^2+LG^2$

$R^2C^2-2LCRG+L^2G^2= 0$

$(RC - LG)^2 = 0$

$$\boxed{\frac{L}{R} = \frac{C}{G}}$$

Substituting this in the 'α' equation we get α = $\sqrt{RG}$

$$\beta = \sqrt{\frac{1}{2}\left[\left(\omega^2 LC - RG\right) + \sqrt{RG - \omega^2 LC)^2 + \omega^2 (RC + LG)^2}\right]}$$

If phase Vp to be independent of frequency β must become function of frequency

$$\omega^2\left(2K^2 - LC\right) + RG = \sqrt{\left(RG - \omega^2 LC\right)^2 + \omega^2 \ (RC + LG)}$$

Squaring and rearranging we get

$(2K^2 - LC)^2 = L^2C^2$

$K = \sqrt{LC}$

And $2\,RG\,(2K^2\,LC) = R^2C^2 + L^2G^2$

$(RC - LG)^2 = 0$

$$\boxed{RC = LG}$$

Comment : Perfect transmission line is not practically possible. A well constructed transmission line has very small value of G, which requires very large α in order to satisfy RC = LG.

If G is increased attenuation is increased . Resistance R cannot be reduced, because the diameter of conductor increases and it becomes costlier affair. Thus increasing the value of L is the only solution.

Q.50 An antenna array presents an impedance of 300 ohms to the transmission line feeding it. The transmission line consists of two open wire lines whose spacing is 9” apart and the diameter of the wire is 0.1”, calculate the dimension of a quarter wave line required for matching. **(8)**

Ans: The characteristic impedance of the feeder is given by Zo = 276 $\log_{10} {}^{s}\!/_{r}$

S = Spacing between two wires

R = Radius of the wire

$Z_o = 276 \log_{10} {}^{9}\!/_{0.05}$

= 704.6 ohms

$Z_0 = \sqrt{Z_s Z_R}$

$Z_0 = \sqrt{704.6 X 300}$

Z_0= 459.2 ohm

Practically spacing of quarter wave matching line would be same as that of the main line. Hence, if r be the radius of the line, then

$Z_0 = 276 \log_{10} {}^{9}\!/_{r}$

$$\text{Log}\ \frac{9}{r} = \frac{Z_0}{276} = \frac{459.2}{276}$$

r = 0.1950

Diameter = 2xr = 0.3900 = 0.39 “ answer

Q.51 Derive the wave equation for a TE wave and obtain all the field components in a rectangular wave guide. **(11)**

Ans: For a TE wave $E_z = 0$, $H_z \neq 0$

$$\frac{\partial H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$$

$$H_z = XY$$

where x is a pure function of x only and Y is a pure function of y only

$H_z = (C_1 \cos B x + C_2 \sin Bx)(C_3 \cos Ay + Cy \sin Ay)$ Where C_1, C_2, C_3, C_4 are constants and are evaluated using boundary conditions.

Boundary conditions are

1. $E_x = 0$ at $y = 0$ $\forall x \to 0 \; to \, a$
2. $E_x = 0$ at $y = b$ $\forall x \to 0 \; to \, a$
3. $E_y = 0$ at $y = 0$ $\forall x \to 0 \; to \, b$
4. $E_y = 0$ at $x = a \; \forall y \to 0 \; to \; b$

We know

$$E_x = - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_x = \frac{-j\omega\mu}{h^2}\{(C_1 \cos B\,X + C_2\ Sin\ B\,X)(-AC_3 Sin\ Ay + AC_4 \cos Ay)\}$$

Substituting the first boundary condition

$$\boxed{\begin{array}{l} C_4 = 0, \text{ hence} \\ H_z = (C_1 \cos B x + C_2 \sin B x)(C_3 \cos Ay) \end{array}}$$

We also know $E_y = \frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$

$$E_y = \frac{j\omega\mu}{h^2}[-BC, \sin Bx + BC_2 \cos Bx][c_3 \cos Ay]$$

Substituting the second boundary condition

$$\boxed{\begin{array}{l} C_2 = 0 \text{ hence,} \\ H_z = C_1\ C_3 \cos B x \cos A_y \end{array}}$$

Again

$$E_y = \frac{-\gamma}{h^2}\frac{\partial Ez}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial Hz}{\partial x}$$

Differentiating H_z and then substituting we get

$$E_y = -\frac{j\omega\mu}{h^2}\ C_1\ C_3\ B \sin B\ x \cos Ay$$

Substituting 3rd boundary condition

$$\boxed{B = \frac{m\pi}{a}}$$

Again, $E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$

$E_x = \frac{j\omega\mu}{h^2} C_1 \, C_3 \, A \cos B \, x \sin \, Ay$

Substituting 4^{th} boundary condition

$A = \frac{n\pi}{b}$

$\therefore \; H_z = C_1 \, C_3 \cos\left(\frac{m\pi}{a}\right) X \cos\left(\frac{n\pi}{b}\right) y$

$H_z = \left\{ C'' \cos\left(\frac{m\pi}{a}\right) X \cos\left(\frac{n\pi}{b}\right) y \right\} e^{j\omega t - r\, z}$

Knowing H_z, the field components of TE wave are.

$Ex = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$

$Ex = \frac{j\omega\mu}{h^2} C''\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right) X \sin\left(\frac{n\pi}{b}\right) ye^{jwt - rz}$

$Ey = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$

$$\boxed{Ey = - \frac{-j\omega\mu}{h^2} C''\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right) X \cos\left(\frac{n\pi}{b}\right) ye^{j\omega t - \gamma z}}$$

$$\boxed{Hx \frac{\gamma}{h^2} C''\left(\frac{m\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right) X \cos\left(\frac{n\pi}{b}\right) ye^{jwt - rz}}$$

And

$$\boxed{H_y = \frac{\gamma}{h^2} C''\left(\frac{m\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right) X \; Sin\left(\frac{n\pi}{b}\right) y \; e^{jwt - rz}}$$

Q.52 Find the cut-off wavelength in a standard rectangular wave guide for the TE_{11}mode. **(5)**

Ans:

$b = a/2$

$\lambda c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2}} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{a/2}\right)^2}}$

$= \frac{2a}{\sqrt{m^2 + 4n^2}}$

For TE_{11} mode m = 1, n = 1

λc = 0.8944 a meters

Q.53 A long cylinder carries a charge of density $\rho = kr$, find $\vec{E}$ inside the cylinder. **(6)**

Ans: $|E| 2\pi r l = \frac{2}{3}\frac{\pi k l}{\in_0} r^3$

Or $\vec{E} = \frac{1}{3\in_0} k r^2 \hat{r}$

$\oint \vec{E}.\overrightarrow{ds} = \frac{q_{in}}{\in_0}$

Where $q_{in} = \int \rho dv = \int_0^r kr.rdr\, d\phi\, dz$

$= \frac{2}{3}\pi k l r^3$

Q.54 Enunciate and give a simple proof for Ampere's circuital law. **(9)**

Ans: Refer Text Book – II, Chapter – 6.

Q.55 A closed current loop having two circular arcs (of radii 'a' and 'b') and joined by two radial lines as shown in Fig.1. Find the magnetic field 'B' at the centre O.

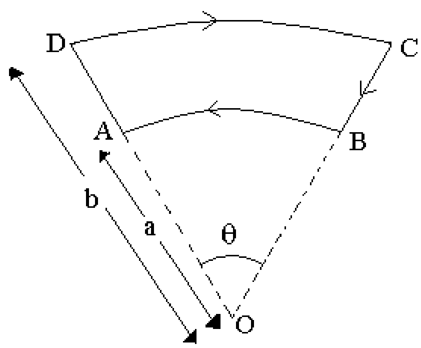


Fig.1

(7)

Ans: The magnetic field at 'O' due to inner arc AB is

$\vec{B}_1 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right)$ (upward)

And the magnetic field at 'O' due to circular arc DC is

$\vec{B}_2 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2b}\right)$ (inward)

Thus the resultant magnetic field at 'O' is

$B = B_1 - B_2 = \frac{\mu_0 i\theta(b-a)}{4\pi ab}$

Q.56 What are the speed, direction of propagation and polarisation of an electro-magnetic wave whose electric field components are given as

$E_x = 4E_o \cos(3x + 4y - 500t)$

$E_y = 3E_o \cos(3x + 4y - 500t + \pi)$

$E_z = 0$ **(6+2+4)**

Ans: $\nabla^2 E_x - \dfrac{1}{{v_x}^2}\dfrac{\partial^2 E_x}{\partial t^2} = 0$ ------------------------ (1)

And $\nabla^2 E_y - \dfrac{1}{{v_y}^2}\dfrac{\partial^2 E_y}{\partial t^2} = 0$ ------------------------ (2)

Now $E_x = 4E_0 \cos(3x + 4y - 500t)$

$\therefore \quad \nabla^2 E_x = -36E_x$

And $\dfrac{\partial^2 E_x}{\partial t^2} = -4(500)^2 E_x$

Substituting there in equation (1)

$-36E_x + \dfrac{4}{{v_x}^2}(500)^2 E_x = 0$

$v_x = \dfrac{500}{3}$

Also $E_y = 3E_0 \cos(3x + 4y - 500t + \pi)$

So $\nabla^2 E_y = -48E_y$

And $\dfrac{\partial^2 E_y}{\partial t^2} = -3 \times (500)^2 E_y$

(2) $\Rightarrow v_y = \dfrac{500}{4}$

$\therefore$ velocity, $v = \sqrt{{v_x}^2 - {v_y}^2}$

$= 2.08 \times 10^{-2}$ m/s.

The divertion of propagation is perpendicular to z axis and in x – y plane.

Polarization,_ $\dfrac{E_y}{E_x} = -\dfrac{3}{4}$

$\therefore \quad \phi = \tan^{-1}\left(\dfrac{3}{4}\right)$

Q.57 A plane electromagnetic wave propagating in the x-direction has a wavelength of 5.0 mm. The electric field is in the y-direction and its maximum magnitude is 30 V/m. Write suitable equations for the electric and magnetic fields as a function of x and t. **(4)**

Ans: $E = E_0 \sin w\left(t - \dfrac{x}{c}\right); \quad B = B_0 \sin w\left(t - \dfrac{x}{c}\right)$

We have $w = 2\pi\nu = \dfrac{2\pi c}{\lambda}$

$E = E_0 \sin \dfrac{2\pi}{\lambda}(ct - x) = (30v/m)\sin\left[\dfrac{2\pi}{5mm}(ct - x)\right]$

$MaxB, B_0 = \dfrac{E_0}{C} = \dfrac{30v/m}{3 \times 10^8 m/s} = 10^{-7}T$

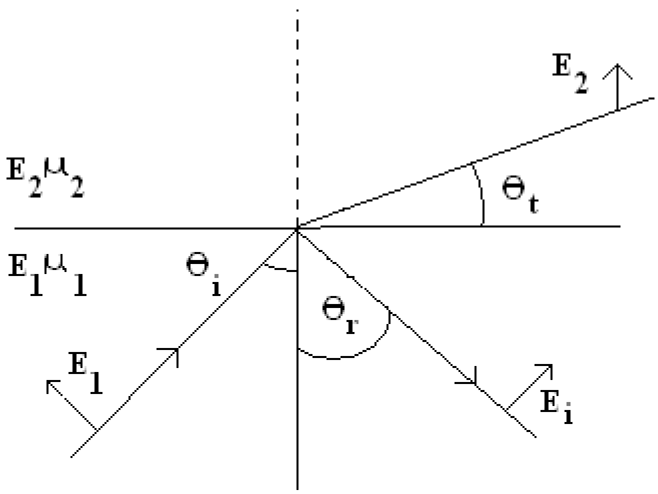
So $B = B_0 \sin\left[\frac{2\pi}{\lambda}(ct - x)\right]$

$$= \left(10^{-7}T\right)\sin\left[\frac{2\pi}{5.0mm}(ct - x)\right].$$

Q.58 Derive a general expression for reflection coefficient and transmission coefficient for $\vec{E}$ and $\vec{H}$ fields when an electromagnetic wave is incident parallel (oblique incidence) on the boundary separating two different perfectly dielectric media and also find the expression for normal incidence. **(8+4)**

Ans: $H_{01} + H'_{01} = H_{02}$

$\left(E_{01} - E'_{01}\right)\cos\theta_i = E_{02}\cos\theta_t$



But $H_{01} = \sqrt{\frac{\in_1}{\mu_1}}$

$$\sqrt{\frac{\in_1}{\mu_1}}E_{01} + \sqrt{\frac{\in_1}{\mu_1}}E'_{01} = \sqrt{\frac{\in_2}{\mu_2}}E_{02}$$

$$\left(\frac{E'_{01}}{E_{01}}\right)_{III} = \frac{\sqrt{\frac{\in_2}{\mu_2}}\cos\theta_i - \sqrt{\frac{\in_1}{\mu_1}}\cos\theta_t}{\sqrt{\frac{\in_2}{\mu_2}}\cos\theta_i + \sqrt{\frac{\in_1}{\mu_1}}\cos\theta_t}$$

$$\left(\frac{E_{02}}{E_{01}}\right)_{III} = \frac{2\sqrt{\frac{\in_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\in_2}{\mu_2}}\cos\theta_i + \sqrt{\frac{\in_1}{\mu_1}}\cos\theta_t}$$

For normal incidence $\theta_i = \theta_t = 0$

$$\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{and} \quad \frac{E_t}{E_i} = \frac{2\eta_1}{\eta_2 + \eta_1}.$$

Q.59 Find the Laplace equation in spherical polar coordinates for

$V = V_a$ at r = a

$= 0$ r = ∞ **(4)**

Ans: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right) = 0$ (for spherical coordinate)

$\Rightarrow v = -\frac{c_1}{r} + c_2$

Given $v = 0$ at $r = \infty \Rightarrow c_2 = 0 \Rightarrow v = -\frac{c_1}{r}$

When $v = v_a$ at $r = a, \Rightarrow v_a = -\frac{c_1}{a}$

And $c_1 = -a v_a$

$\therefore \; v = \frac{a v_a}{r}$.

Q.60 What length of transmission line should be used at 500 MHz and how should it be terminated for use as a

(i) parallel resonant circuit.

(ii) series resonant circuit. **(6)**

Ans: We have seen that for line to act as parallel circuit

$$\lambda = \frac{3\times10^{10}}{500\times10^6} = 60cm$$

Then for short circuit line (odd multiple of $\frac{\lambda}{4}$) $\left.\begin{array}{l}\frac{\lambda}{4} = \frac{60}{4} = 15cm \\ \frac{3\lambda}{4} = 45cm\end{array}\right\}$ short circuit

And for open circuited line the length is (even multiple of $\frac{\lambda}{4}$)

$$\left.\begin{array}{l}\frac{2\lambda}{4} = \frac{2\times60}{4} = 30cm \\ \frac{4\lambda}{4} = 60cm\end{array}\right\} \text{open circuit}$$

Q.61 In a rectangular waveguide for which a = 1.5 cm, b = 0.8 cm, $\sigma = 0$, $\mu = \mu_o$ and $\in = 4\in_o$ and

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\text{Sin}\left(\pi\times10^{11}t - \beta z\right) A/m$$

Find

(i) the mode of operation

(ii) the cut off frequency

(iii) phase constant

(iv) the propagation constant
(v) the intrinsic wave impedance **(8)**

Ans:

(i) TM_{13} or TE_{13}

(ii) $f_{c_{mn}} = \frac{\mu'}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$, $\mu' = \frac{1}{\sqrt{\mu \in}} = \frac{c}{2}$

Or $f_{c_{13}} = 28.57 GH_Z$

(iii) $\beta = W\sqrt{\mu \in}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$= \frac{W\sqrt{\in r}}{C}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$\beta = 1718.81$ rad/m

Where $W = 2\pi f = \pi \times 10^{11}$ or $f = \frac{100}{2} = 50 GH_Z$

(iv) $\gamma = j\beta = j1718.81/m$

(v) $\eta_{TM_{13}} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{377}{\sqrt{\in_r}}\sqrt{1-\left(\frac{28.57}{50}\right)^2} = 154.7\Omega$

Q.62 What is a linear antenna? Find out the expression for effective length of linear antenna when
(i) the current is distributed along its length and
(ii) there is sinusoidal distribution of current.
Also find the total power radiated. **(8)**

Ans: (i) Refer Text Book – II, Chapter – 18, 21.
(ii) Refer Text Book – I, Chapter – 11.

Q.63 A small dipole antenna is carrying a uniform r.m.s current of 10 amperes. Its r.m.s. electric field at a distance 'r' metre in a direction making an angle 'θ' with the conductor is given by

$$E = \frac{200}{r}\text{Sin}\,\theta \ \text{V/m}$$

Find the total power radiated and radiation resistance. **(8)**

Ans: Since $E = \frac{60\pi I_o dl \sin\theta}{\lambda r}\cos\omega\left(t - r/c\right)$

$= \frac{200}{r}\sin\theta$

$P_{av} = EH = \frac{E^2}{120\pi} = \frac{1}{120\pi}\left(\frac{200}{r}\sin\theta\right)^2$

$$= \frac{1000}{3\pi r^2}\sin^2\theta$$

And total power, $P = \int_0^\pi P_{av}.2\pi r^2 \sin\theta d\theta$

$$= \frac{1000}{3\pi r^2}\sin^3\theta.2\pi r^2 d\theta$$

$$= 888.8\omega$$

Q.64 What is skip distance? Find the expression for skip distance and maximum usable frequency considering flat surface and curved surface of earth. **(10)**

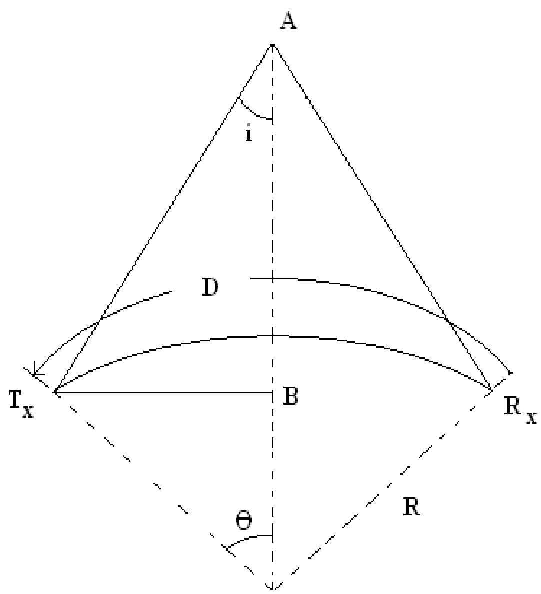
Ans:Refer text Book – II Chapter – 19

Flat Earth => $f_c = f\cos i$

$$D_{skip} = 2h\sqrt{\frac{f^2}{f_c^{\,2}} - 1}$$

$$\tan i = \frac{D}{2h}$$

Curved Earth



$$D = 2R\theta$$

$$\tan i = \frac{BT}{AT} = \frac{R\sin\theta}{h + R - R\cos\theta}$$

Since $\frac{h}{R} << 1, \cos\theta \approx 1$

$$\cos\theta = \frac{R}{R+h} \quad \& \quad \cos\theta = 1 - \frac{\theta^2}{2} = 1 - \frac{h}{R}$$

$$f_{max} = f_c \left[\frac{D^2 + \left(4\left(h^2 + \frac{D^2}{8R}\right)^2\right)}{4\left(h + \frac{D^2}{8R}\right)^2} \right]$$

Q.65 Calculate the value of frequency at which an electro-magnetic wave must be propagated for the D-region having an index of refraction of 0.5 (given N = 400 electrons/cc for D-region) **(6)**

Ans: $\mu = \sqrt{1 - \frac{80.6N}{f^2}}$

For D-region, N = 400 electron/cc

Or $0.5 = \sqrt{1 - \frac{80.6 \times 400}{f^2}}$

Or $f = 207.33\, kc/s$

Q.66 Obtain the expression of electric field of an infinite sheet of charge in xy plane with charge density ρ_s. **(8)**

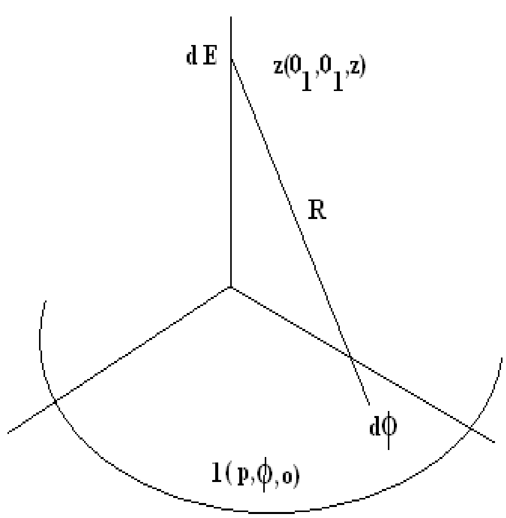
Ans: $\overline{dE} = \frac{\rho_s ds}{4\pi \sum_0 |R_{12}|^2} \left[\frac{R_{12}}{|R_{12}|} \right]$

$\overline{ds} = \rho d\rho d\varphi \overrightarrow{az}$

$ds = \rho d\rho d\varphi$

$R_{12} = \overline{r_2} - \overline{r_1}$

$= -\rho a\overline{\rho} + \tau \overline{a_z}$



$$a_{12} = \frac{\overline{R}_{12}}{|R_{12}|} = \frac{-\rho a\overline{\rho} + \tau\overline{a_z}}{\sqrt{\rho^2 + z^2}}$$

$$\overline{dE} = \frac{\rho_s \rho d\rho d\varphi\left[-\rho a\overline{\rho} + \tau\overline{a_z}\right]}{4\pi\sum_0 \left[\rho^2 + z^2\right]^{3/2}}$$

$$\therefore \quad \overline{E} = \int \overline{dE} = \int_0^\infty \int_0^{2\pi} \frac{\rho_s \rho d\rho d\varphi z\overline{a_z}}{4\pi\sum_0 \left(\sqrt{\rho^2 + z^2}\right)^3}$$

$$= \frac{\rho_s z}{2\sum_0}\left[\frac{-1}{(\rho^2 + z^2)^{1/2}}\right]_0^\infty \overline{a_z}$$

$$= \frac{\rho_s}{z\sum_0}\overrightarrow{a_z}$$

In general

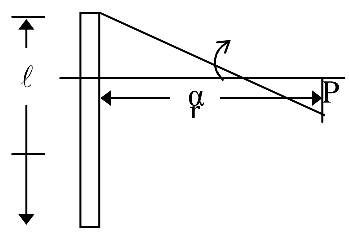
$$E = \frac{\rho_s}{z\sum_0}\vec{a}_N$$

Where $\vec{a}_N$ = unit normal vector.

Q.67 A line of length ℓ carries charge ρ_L C/m. Show that the potential in the median plane can be written as

$$V = \frac{\rho_L}{4\pi\in_0}\ln\frac{1+\sin\alpha}{1-\sin\alpha}$$

Refer to the Figure. **(8)**

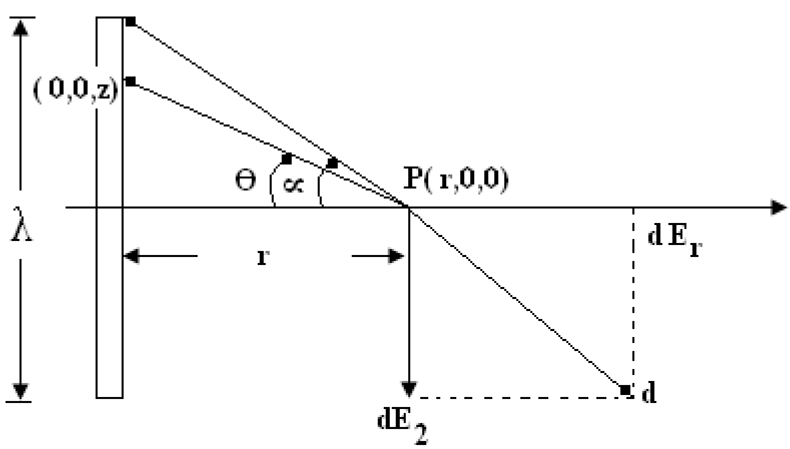


Ans: $dE = \frac{\rho_L dz}{4A\sum l^2}\overrightarrow{a_r}$

$l^2 = r^2 + z^2$

$dEr = dE\cos\theta$

$\quad = dE.r/l$



$$\int dEr = \int_{-\lambda/2}^{\lambda/2} dE.\frac{r}{l}$$

$$= \int_{-\lambda/2}^{\lambda/2} \frac{\rho_L dz}{4\pi \sum l^2}.\frac{r}{l} = \frac{\rho_L r}{4\pi \sum} \int_{-\lambda/2}^{\lambda/2} \frac{dz}{l^3}$$

$$= \frac{\rho_L r}{4\pi \sum} \int_{-\lambda/2}^{\lambda/2} \frac{dz}{\sqrt{r^2+z^2}} = \frac{\rho_L r}{4\pi \sum} \int_{-\lambda/2}^{\lambda/2} \frac{dz}{(r^2+z^2)^{3/2}}$$

$$\therefore\ \overrightarrow{E_r} = \frac{\rho_L \lambda/2}{2\pi \sum r\sqrt{r^2+\left(\lambda/2\right)^2}}\overrightarrow{a_r}$$

$$= \frac{\rho_L \lambda}{4\pi \sum r\sqrt{r^2+\left(\lambda/2\right)^2}}\overrightarrow{a_r}$$

$$V = \int E.dr = \int \frac{\rho_L \lambda}{4\pi \sum r} \frac{dr}{\sqrt{r^2+\left(\lambda/2\right)^2}}$$

$$= \frac{\rho_L \lambda}{4\pi \sum} \int \frac{dr}{r\sqrt{r^2+\left(\lambda/2\right)^2}} = \frac{\rho_L}{4\pi \sum} \int \frac{dt}{(t-a)} - \frac{dt}{(t+a)} \qquad \text{where } t^2 = r^2 + a^2$$

$$= \frac{\rho_L}{4\pi \sum} \ln \frac{1-\sin\alpha}{1+\sin\alpha}.$$

Q.68 Given two mediums having permeabilities μ_1, μ_2, prove that

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$$

where θ_1 and θ_2 represent the angles that the magnetic fields make with the normal to the interface in the two mediums. **(8)**

Ans: Since the normal component of B is continuous and the normal component of H is discontinuous by the ratio μ_1/μ_2

$BN_2 = BN_1$ and

$$HN_2 = \frac{\mu_1}{\mu_2} HN_1$$

$$\therefore\quad H_2 \cos\theta_2 = \frac{\mu_1}{\mu_2} H_1 \cos\theta_1 \qquad \text{---------------------- (1)}$$

Tangential component of H can be written as

$Ht_1 - Ht_2 = K$

For zero current density

$Ht_1 - Ht_2 = 0$

Thus

$H_1 \sin\theta_1 - H_2 \sin\theta_2 = 0$

$$H_1 \sin\theta_1 = H_2 \sin\theta_2 \qquad \text{--------------------- (2)}$$

Thus from (1) and (2)

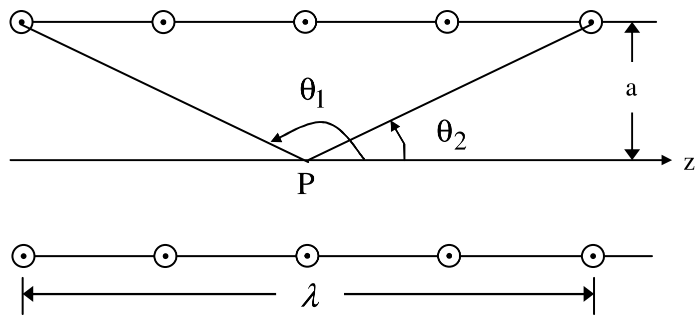
$$\frac{\sin\theta_1}{\sin\theta_2}\times\frac{\cos\theta_1}{\cos\theta_2}=\frac{\mu_1}{\mu_2}$$

$$\therefore\quad \frac{\tan\theta_1}{\tan\theta_2}=\frac{\mu_1}{\mu_2}$$

Q.69 A solenoid of length ℓ and radius 'a' consists of N turns of wire carrying current I, as shown in figure. Show that at point P along its axis (z-axis),

$$\vec{H}=\frac{nI}{2}(\cos\theta_2-\cos\theta_1)\overrightarrow{a_z}$$

where n=N/λ, and θ_1,θ_2 are the angles subtended at P by the end turns. Also show that if λ>>a, at the centre of the solenoid, $\vec{H}=nI\overrightarrow{a_z}$ **(8)**



Ans: Since solenoid consists of circular loops, so we can apply result of

$$dH_2=\frac{Id\lambda a^2}{2\left[a^2+z^2\right]^{3/2}}$$

$$=\frac{za^2ndz}{2\left[a^2+z^2\right]^{3/2}}$$

The contribution to $\vec{H}$ at P by element of solenoid of length $\overline{d_z}$.

Where $d\lambda=ndz=\left(N/\lambda\right)dz$ also $\tan\theta=\frac{a}{z}$

$$dz=-a\cos ec^2 d\theta=\frac{-\left(z^2+a^2\right)^{3/2}}{a^2}\sin\theta d\theta$$

Hence,

$$dH_2=\frac{-nI}{2}\sin\theta d\theta$$

$$H_2=\frac{-nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta d\theta$$

Thus

$$\vec{H}=\frac{nI}{2}(\cos\theta_2-\cos\theta_1)\overrightarrow{a_z}$$

Substituting $n=N/\lambda$

$$\vec{H} = \frac{NI}{2\lambda}(\cos\theta_2 - \cos\theta_1)\overrightarrow{a_z}$$

However at the centre $\cos\theta_2 = -\cos\theta_1$ and

If $\lambda >>$ a or $\theta_2 = 0^0 \quad \theta_1 = 180^0$

$$\therefore \quad \vec{H} = nI\overrightarrow{a_z} = \frac{NI}{\lambda}\overrightarrow{a_z}$$

Q.70 Write down Maxwell's equations in integral as well as differential forms for time-varying fields. **(8)**

Ans: Maxwel's Eqauations for time varying fields:

DIFFERENTIAL FORMS	INTEGRAL FORMS
I. $\nabla.e = \frac{1}{\Sigma_0}\rho$	$\oint E.da = \frac{1}{\varepsilon_0}\int \rho dV$
II. $\nabla.B = 0$	$\oint B.dA = 0$
III. $\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint E.dl = -\int \frac{\partial B}{\partial t}.dA$
IV. $\nabla \times B = \mu_0 J + \mu_0 \Sigma_0 \frac{\partial E}{\partial t}$	$\oint B.dl = \int\left(\mu_0 J + \varepsilon_0 \frac{\mu_0 \partial E}{\partial t}\right).dA$.

Q.71 In a medium characterized by the parameters σ=0, μ_0 and $\in_0$, and an electric field given by

$\vec{E} = 20\sin(10^8 t - \beta z)\overrightarrow{a_z}$ V/m, calculate β and $\vec{H}$. **(8)**

Ans: $\nabla.\vec{E} = \frac{dt_y}{d_y} = 0$ (as $\sigma = 0 \; \nabla.D_{\rho v} = 0$)

From Faraday's law

$$\nabla.\vec{E} = -\mu\frac{dH}{dt}$$

$$H = \frac{-1}{\mu}\int(\nabla \times \overline{E})dt$$

But

$$\nabla \times \vec{E} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 0 & 0 & E_z \end{vmatrix}$$

$$= \frac{dE_z}{dy}\overrightarrow{a_x} - \frac{d}{d_z}E_z\overrightarrow{a_y}$$

$$= 0 - 20\beta\cos(10^8 t - \beta_z)\overrightarrow{a_y}$$

$$\vec{H} = \frac{20\beta}{\mu}\int \cos(10^8 t - \beta z)dt\overrightarrow{a_y}$$

$$= \frac{\beta \times 20}{\mu 10^8}\sin(10^8 t - \beta z)\overrightarrow{a_y}$$

Also

$$\vec{E} = \frac{1}{\in}\int (\nabla \times \vec{H})dt$$

$$\nabla \times \vec{H} = -\frac{dH_y}{dz}\overrightarrow{a_x} = \frac{\beta^2 \times 20}{\mu 10^8}\cos(10^8 t - \beta z)\overrightarrow{a_x}$$

$$\vec{E} = \frac{\beta^2}{\in} \times \frac{20}{\mu 10^8}\int \cos(10^8 t - \beta z)dt\overrightarrow{a_x}$$

$$= \frac{\beta^2 \times 20}{\in \mu 10^8 \times 10^8}\sin(10^8 t - \beta z)\overrightarrow{a_x}$$

Comparing for E we get

$$\frac{20\beta^2}{\mu \in \times 10^{16}} = 20$$

$$\beta^2 = \mu \in \times 10^{16}$$

$$\beta = \pm 10^8\sqrt{\mu \in}$$

$$= \pm 10^8\sqrt{\mu_0 \in_0}$$

$$= \pm\frac{10^8}{C}$$

$$= \pm\frac{10^8}{3 \times 10^8}$$

$$= \pm 1/3$$

$$\vec{H} = \frac{1}{3} \times \frac{20}{4\pi \times 10^{-7} \times 10^8}\sin\left(10^8 t \pm \frac{z}{3}\right)\overrightarrow{a_y}$$

$$= \frac{1}{6\pi}\sin\left(10^8 t \pm \frac{z}{3}\right)\overrightarrow{a_y}.$$

Q.72 Obtain an expression for the propagation constant in good conductors. Explain skin effect. **(8)**

Ans: Helmholtz equation

$$\nabla^2 E - y^2 E = 0$$

Let $y' = \alpha + j\beta$ (propagation constant)

Since $y^2 = j\omega\mu(\sigma + j\omega\in)$

Where α = attenuation constant

B = phase shift constant.

Uniform wave travelling in *x* direction

$$\frac{\partial^2 E}{\partial x^2} = y^2 E$$

$E(x) = E_0 e^{-yz}$

Time varying form

$\vec{E}(x,t) = \text{Re}\{E_0 e^{+j\omega t - yx}\}$

$= e^{-\alpha x}\ \text{Re}\{E_0 e^{j(\omega t - \beta x)}\}$

$\because \quad \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\in)}$

$\alpha^2 + \beta^2 + j2\alpha\beta = j\omega\mu(\sigma + j\omega\in)$

$= j\omega\mu \times j\omega\in \left[1 + \dfrac{\sigma}{j\omega\in}\right]$

$= -\omega^2\mu\in \left[1 + \dfrac{\sigma}{j\omega\in}\right]$

Equating real and imaginary part.

$\alpha^2 - \beta^2 = -\omega^2\mu\in$

$2\alpha\beta = j\omega\mu\sigma$.

Solving for α and β, we get

$$\alpha = \omega\sqrt{\frac{\mu\in}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\in}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2} + 1\right]}$$

SKIN EFFECT:

Skin depth ∂ is defined as the distance it takes to reduce the amplitude to 1/e about 3.7%

$$\partial = \frac{1}{\alpha} = \frac{1}{k} \simeq \sqrt{\frac{2}{\omega\mu\sigma}}$$

So, in good conductor, the depth of penetration decreases with frequency low. In term of wave length $\partial = \lambda/2\pi$ i.e. about $1/\sigma^{th}$ of a wave length. So a wave gets attenuated even before one cycle. This phenomena of conductor is known as skin effect. So what is a good conductor at low or audio frequencies may become poor at high frequencies.

Q.73 A lossy dielectric has an intrinsic impedance of 200 $\angle 30°$ Ω at a particular frequency. If at that frequency, the plane wave propagating through the dielectric has the magnetic field component

$\vec{H} = 10e^{-\alpha x}\cos\left(\omega t - \frac{1}{2}x\right)\overrightarrow{a_y}$ A/m, Find $\vec{E}$ and α. Determine the skin depth and wave polarization. **(8)**

Ans: Wave travels along $\overrightarrow{a_x}$ so that $\overline{K} = \overrightarrow{a_x}$; $\vec{H} = \overrightarrow{a_y}$

$\therefore \quad -\vec{E} = \overline{K} \times \vec{H} = \overrightarrow{a_x} \times \overrightarrow{a_y} = \overrightarrow{a_z}$

$\vec{E} = -\overrightarrow{a_z}$

Also $Ho = 10$ so $E/_{Ho} = \eta = 200\angle 30 = 200e^{j\pi/6}$

$Eo = 2000\,e^{j\pi/6}$

$\therefore \quad \vec{E} = \mathrm{Re}\left[2\times 10^3 e^{j\pi/6}.e^{-\gamma x}.e^{j\omega t}.\vec{E}\right]$

So except for amplitude and phase difference $\vec{E}$ and $\vec{H}$ have same form

$\vec{E} = -2\times 10^3\ e^{-ax}\cos\left(\omega t - \frac{x}{2} + \frac{\pi}{6}\right)\overrightarrow{a_z}$

Knowing that $\beta = 1/2$, so α will be

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega t}\right)^2} - 1}{\sqrt{1+\left(\frac{\sigma}{\omega t}\right)^2} + 1}\right]^{1/2}$$

Where $\frac{\sigma}{\omega \in} = \tan 60 = \sqrt{3}$

So $\frac{\alpha}{\beta} = \left[\frac{2-1}{2+1}\right]^{1/2} = \frac{1}{\sqrt{3}}$

$\alpha = \frac{1}{2}\times\frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}}$

$\delta = \frac{1}{\alpha} = 2\sqrt{3}\,m$.

Q.74 What are standing waves? How do they arise? Discuss their characteristics. **(8)**

Ans: When a line is terminated in an iimpedance other than Z_c, there is a mismatch and consequent reflection of voltage or current from the load end. If there is a mismatch at sending end also, the wave is re-reflected. Thus, there will be multiple reflections and standing waves on the line.

On a reflected wave, the distance between successive maxima or minima is $\lambda/_2$. Similarly, the distance between a minimum and maximum is $\lambda/_4$.

1. The maximum amplitude is

   $V_{\max} = Ae^{-ax} + Be^{ax}$

   Where $\beta x = n\pi$ where $n = 0, \pm 1, \pm 2$ etc.

2. The maximum amplitude is

   $V_{\min} = Ae^{-ax} - Be^{+ax}$

   Where $\beta = (2n-1)\pi/_2$ where $n = 0, \pm 1, \pm 2$ etc.

3. The distance between any two successive maxima or minima is

   $\beta x = n\pi; x = \frac{n\pi}{\beta} = n\lambda/_2$

Q.75 A 30-m long lossless transmission line with Z_0=50Ω operating at 2 MHz is terminated with a load Z_L=60+j40 Ω. If the velocity of propagation on the line is 0.6 times that of light, find the following without using Smith-chart:

(i) The reflection coefficient Γ
(ii) The standing wave ratio S
(iii) The input impedance **(8)**

Ans:

(i) Reflection coefficient Γ

$$\Gamma = \frac{60 + j40 - 50}{60 + j40 + 50}$$
$$= \frac{10 + j40}{110 + j40}$$
$$= 0.3523 \angle 56^0$$

(ii) VSWR = $\frac{1+\Gamma}{1-\Gamma} = 2.088$

(iii) $Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}\right]$

$$\beta d = \frac{\omega l}{\mu} = \frac{2\pi(2\times10^6)(30)}{0.6\times(3\times10^8)} = 120^0$$

$$Z_{in} = 50\left[\frac{60 + j40 + j50\tan(120)}{60 + j[60 + j40]\tan(120)}\right]$$
$$= 23.97 + j1.35.$$

Q.76 What is a cavity resonator? Derive an expression for the frequency of oscillation of a rectangular cavity resonator. **(8)**

Ans: Cavity Resonator:A cavity resonator is a hollow inductor blocked at both ends and along which are electromagnetic wave can be supported. It can be viewed as a wave guide short circuited at both ends. The cavity has interior surfaces which reflects a wave of a specific frequency. Where a wave that is resonant with the cavity enters, it bounces back and forth within the cavity, with low loss. As more wave energy enters the cavity, it combines with and reinforces the standing wave, increasing its intensity.

Q.77 In a rectangular waveguide for which a=1.5cm, b=0.8cm, $\sigma = 0, \mu = \mu_0$,

$$\text{and} \in = 4\in_0, H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin\left(\pi x10^{11}t - \beta z\right) \text{ A/m}$$

Determine

(i) The mode of operation
(ii) The cutoff frequency
(iii) The phase constant β
(iv) The intrinsic wave impedance η **(8)**

Ans:

(i) The wave guide is operating at TM_{13} or TE_{13} mode.

(ii) $$fc_{mm} = \frac{\mu'}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

$$= \frac{C}{4}\sqrt{\frac{1}{\left[1.5\times10^{-2}\right]^2}+\frac{9}{\left[0.8\times10^{-2}\right]^2}}$$

$$= 28.57\, GH_z$$

(iii) $$\beta = \omega\sqrt{\mu\in}\sqrt{1-\left(\frac{fc}{f}\right)^2}$$

$$= \frac{\omega\sqrt{\in r}}{C}\sqrt{1-\left(\frac{fc}{f}\right)^2}$$

$$\beta = \frac{\pi\times10^{11}(2)}{3\times10^8}\sqrt{1-\left(\frac{28.57}{50}\right)^2}$$

= 1718.81 rad/m

(iv) $$\eta_{TM_{13}} = \eta'\sqrt{1-\left(\frac{fc}{f}\right)^2}$$

$$= \frac{377}{\sqrt{\in r}}\sqrt{1-\left(\frac{28.57}{50}\right)^2}$$

$= 154.7\Omega$.

Q.78 Explain the principle of a phased array, and write a note on the log-periodic dipole array. **(8)**

Ans:PHASED ARRAY: Phased Array is designed with more energy radiated in some particular directions and less in other directions, i.e. the radiation pattern be concentrated in the direction of interests.

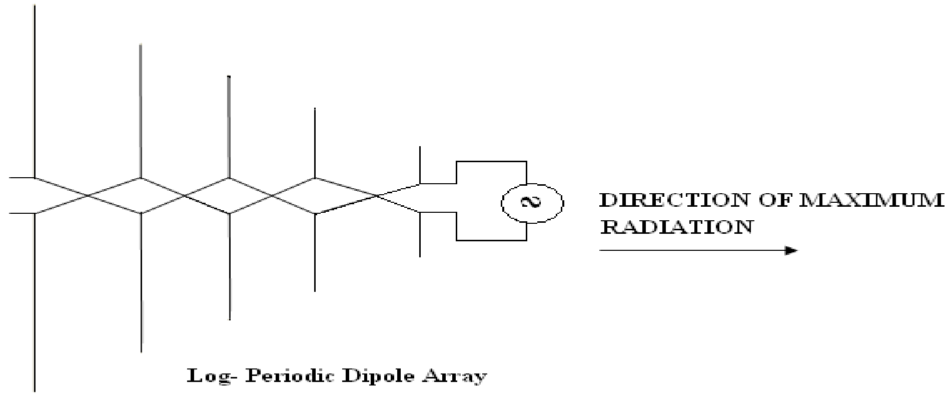
A phased array is used to obtain greater directivity that cab be obtained with a single antenna element.

A phased array is a group of radiating elements arranged so as to produce some particular radiation characteristics. It is practical and convenient that the array consists of identical elements, but this is not fundamentally required.

LOG PERIODIC DIPOLE ARRAY:

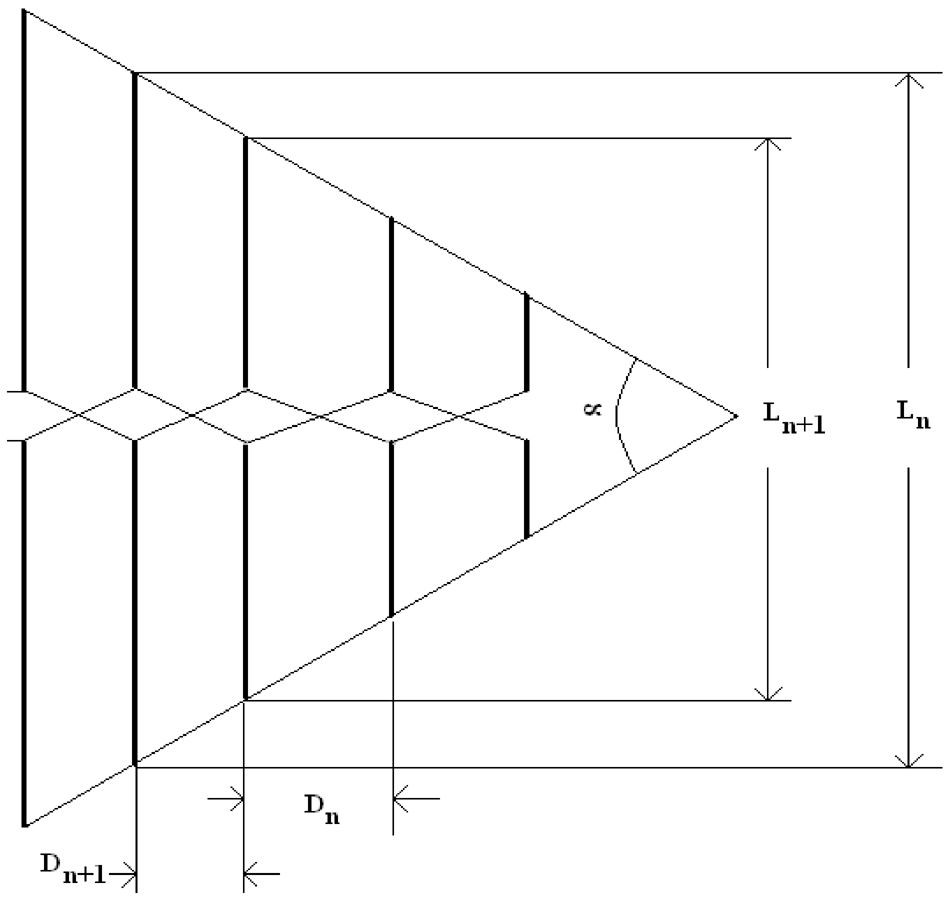
The log periodic dipole array (LPDA) is one antenna that almost everyone over 40 years old has seen. They were used for years as TV antennas. The chief advantage of an LPDA is that it is frequency-independent. Its input impedance and gain remain more or less constant over its operating bandwidth, which can be very large. Practical designs can have a bandwidth of an octave or more.

Although an LPDA contains a large number of dipole elements, only 2 or 3 are active at any given frequency in the operating range. The electromagnetic fields produced by these active elements end of the array. The radiation in the opposite direction is typically 15 – 20 dB below the maximum. The ratio of maximum forward to minimum rearward radiation is called the Front-to-Back (FB) ration and is normally measured in dB.



Log- Periodic Dipole Array

The log periodic antenna is characterized by three interrelated parameters, α, σ, and τ as well as the minimum and maximum operating frequencies, f_{MIN} and f_{MAX}. The diagram below shows the relationship between these parameters.



Unlike many antenna arrays, the design equations for the LPDA are relatively simple to work with.

Q.79 A magnetic field strength of 5μA/m is required at a point $\theta = \frac{\pi}{2}$, 2 km from an antenna in air. Neglecting ohmic loss, how much power must the antenna transmit if it is

(i) A Hertizian dipole of $\frac{\lambda}{25}$

(ii) A half-wave dipole

(iii) A quarter-wave monopole. **(8)**

Ans:

(i) For a Hertizian dipole

$$|H_{\varphi s}| = \frac{I_0 \beta dl \sin\theta}{4\pi r} \quad \text{where } \partial l = \lambda/25 \qquad \text{------------------ (1)}$$

$$\beta dl = \frac{2\pi}{\lambda}.\frac{\lambda}{25} = \frac{2\pi}{25}$$

Thus from (1)

We get $I_0 = 0.5A$

$$P_{rad} = 40\pi^2\left[\frac{dl}{\lambda}\right]^2 I_0^{\,2} = \frac{40\pi^2 (0.5)^2}{(25)^2}$$

$= 158\,m\omega$

(ii) For a $\lambda/2$ dipole

$$|H_{\varphi s}| = \frac{I_0 \cos\left(\frac{\pi}{2}.\cos\theta\right)}{2\pi r \sin\theta}$$

$\Rightarrow I_0 = 20\pi\, mA$

$$P_{av} = \frac{1}{2} I_0^{\,2} R_{rad} = \frac{1\times(20\pi)^2}{2}.10^{-6}$$

$= 144\,m\omega$.

(iii) For a $\lambda/4$ monopole

$I_0 = 20\pi\, mA$

As in part (b)

$$P_{rad} = \frac{1}{2} I_0^{\,2} R_{rad} = \frac{1}{2}(20\pi)^2 \times 10^{-6} \times 36.56$$

$= 72\,m\omega$

Q.80 Explain Poisson's and Laplace's equations with suitable applications. **(8)**

Ans:POISSON'S AND LAPLACE'S EQUATIONS:

Point form of Gauss Law

$\overline{\nabla}.\overline{D} = \rho_v$

Relation between $\overline{D}$ & $\overline{E}$

$\overline{D} = \in \overline{E}$

Relation between $\overline{E}$ & V

$\overline{E} = -\overline{\nabla} V$

Hence $\overline{\nabla}.\overline{D} = \overline{\nabla}.\left(\in \overline{E}\right) = -\overline{\nabla}.\left(\in \overline{\nabla} V\right) = \rho_v$

$$\overline{\nabla}.\overline{\nabla}.V = -\rho_v/\in$$

For a homogenous region in which $\in$ is a constant. The last equation is known as poisson's equation. In cartessan co-ordinate system

$$\overline{\nabla}.\overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\overline{\nabla}.V = \frac{\partial v}{\partial x}\overline{a}_x + \frac{\partial v}{\partial y}\overline{a}_y + \frac{\partial v}{\partial z}\overline{a}_z$$

And therefore

$$\overline{\nabla}.\overline{\nabla}V = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$$

Usually the operation $\overline{\nabla}.\overline{\nabla}$ is $\nabla^2 V$, hence

$$\nabla^2 V = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$$

$$\overline{\nabla}^2 V = \frac{-\rho_v}{\in}$$

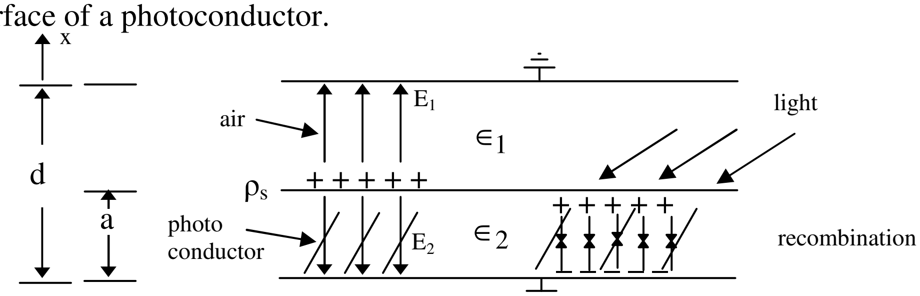
If $\rho_v = 0$

$$\overline{\nabla}^2 V = 0$$

Which is the Laplace's equation.

The $\nabla^2 V$ is called the laplacian of V.

Q.81 The surface of the photoconductor in a xerographic copying machine is charged uniformly with surface charge density ρ_s as shown in figure. When the light from the document to be copied is focused on the photoconductor, the charges on the lower surface combine with those on the upper surface to neutralize each other. The image is developed by pouring a charged black powder over the surface of the photopowder, which is later transferred to paper and melted to form a permanent image. Determine the electric field below and above the surface of a photoconductor. **(8)**



Ans: Since $\rho_v = 0$ in this case, we apply Laplace's equation. Also the potential depends only on *x*. Thus

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} = 0$$

Integrating twice gives

$V = Ax + B$

Let the potential above and below be V_1 and V_2, respectively.

$V_1 = A_1 x + B_1$, x > a

$V_2 = A_2x + B_2$, x < a 1

The boundary condition at the grounded electrodes are

$V_1(x = d) = 0$

$V_2(x = 0) = 0$ 2

At the surface of the photoconductor,

$V_1(x = a) = V_2(x = a)$

$D_{1n} - D_{2n} = \rho_s\big|_{x=a}$ 3

We use the four conditions in eqs. (2) and (3) to determine the four unknown constants A_1, A_2, B_1 and B_2. From eqs. (1) and (2),

$0 = A_1d + B_1 \rightarrow B_1 = -A_1d$

$0 = 0 + B_2 \rightarrow B_2 = 0$

From eqs. (1) and (2),

$A_1a + B_1 = A_2a$

To apply eq. (2), recall that $D = \varepsilon E = -\varepsilon\nabla V$ so that

$$\rho_s = D_{1n} - D_{2n} = \varepsilon_1E_{1n} - \varepsilon_2E_{2n} = -\varepsilon_1\frac{dV_1}{dx} + \varepsilon_2\frac{dV_2}{dx}$$

Or

$\rho_s = -\varepsilon_1A_1 + \varepsilon_2A_2$

Solving for A_1 and A_2 in eqs. (6.2.4) to (6.2.6), we obtain

$$E_1 = -A_1a_x = \frac{\rho_s a_x}{\varepsilon_1\left[1 + \frac{\varepsilon_2}{\varepsilon_1}\frac{d}{a} - \frac{\varepsilon_2}{\varepsilon_1}\right]}$$

$$E_2 = -A_2a_x = \frac{-\rho_s\left(\frac{d}{a} - 1\right)a_x}{\varepsilon_1\left[1 + \frac{\varepsilon_2}{\varepsilon_1}\frac{d}{a} - \frac{\varepsilon_2}{\varepsilon_1}\right]}$$

Q.82 What is Gauss law? How gauss law is applicable to point charge and infinite line charge. **(4)**

Ans: The generalization of Faraday's experiment lead to the following statement known as Gauss's Law "The electric flux passing through any closed surface is equal to the total charge enclosed by that surface." For a given fig if the total charge is φ, then φ of electric flux pass through the enclosing surface. Let at point P, $\overline{a_n}$ is a unit vector normal to differential area Δs located at point P. Then $\overline{\Delta s} = \Delta s\overline{a_n}$

$\Delta\varphi$ = flux crossing $\overline{\Delta s}$

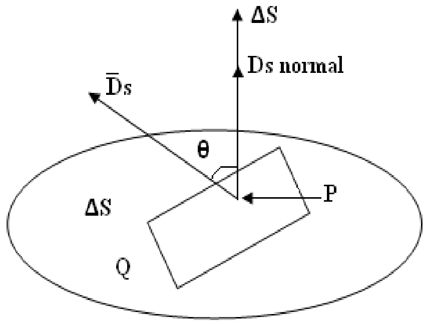
= Ds noun Δs = Ds cos θ Δs

And the total flux is given by

$$\varphi = \int d\varphi = \oint_s \overline{Ds}.\overline{ds}$$

Thus mathematical formulation of Gauss's law is

$\varphi = Q = \phi_s\ \overline{Ds}.\overline{ds}$



For point charge:

$$\varphi = \oint_s Ds.ds = \oint_{spn} Dsds = Ds\int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta d\theta d\varphi$$

$$= 4\pi r^2 \text{ Ds.}$$

Hence

$$Ds = \frac{Q}{4\pi r^2}. \quad \& \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}\overrightarrow{ar}$$

For infinite line charge:

$$Q = \oint_{cyc} Ds.ds = Ds\int_{sides} ds + 0\int_{top} ds + 0\int_{bottom} ds$$

$$= Ds\int_0^L\int_0^{2\pi} \rho d\varphi dz = Ds2\pi\rho L$$

$$Ds = \frac{Q}{2\pi\rho L}$$

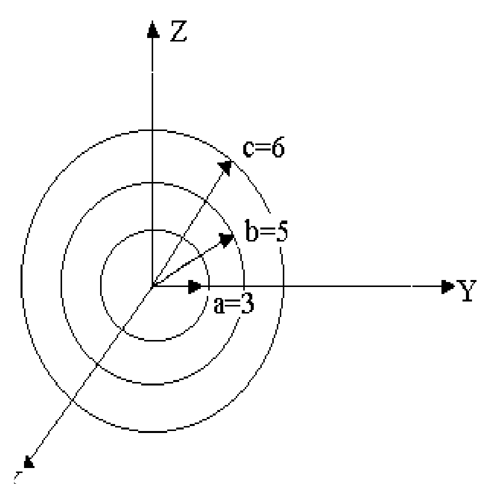
Hence

$$\overline{E} = \frac{Q}{2\pi\rho\varepsilon_0 L}\vec{\upsilon} = \frac{\rho_L \times L}{2\pi\rho\varepsilon_0 L}\vec{\upsilon}$$

$$= \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{\upsilon}.$$

Q.83 The spherical region $0 \le r \le 3$ contains uniform volume charge density of $\rho_v = 2\ C/m^3$ and $\rho_v = 1\ C/m^3$ for $5 \le r \le 6$. Use law to find D_r for

(i) $r \le 3$
(ii) $3 \le r \le 5$
(iii) $5 \le r \le 6$
(iv) r>>6



Ans:

(i) For $r \le 3$

$$\psi = \int_0^r\int_0^{\pi}\int_0^{2\pi} \rho_v dv = \int_0^r\int_0^{\pi}\int_0^{2\pi} 2r^2 \sin\theta d\theta d\varphi$$

$$= \frac{2r^3}{3} \times 4\pi$$

$$\overrightarrow{Dr} = \frac{\psi}{area} = \frac{\frac{2r^3}{3} \times 4\pi}{4\pi r^2} = \frac{2r}{3}\overrightarrow{ar}$$

(ii) For $3 \le r \le 5$

Will be same as (i)

$$\overrightarrow{Dr} = \frac{2r}{3}\overrightarrow{ar}$$

(iii) For $5 \le r \le 6$

$$\psi = \frac{2r^3}{3} \times 4\pi + \int_0^r \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\theta d\varphi$$

$$= \frac{2r^3}{3} \times 4\pi + \frac{r^3}{3} \times 4\pi$$

$$D = \frac{\psi}{area} = \frac{4\pi r^3}{4\pi r^2}\overrightarrow{ar} = r\overrightarrow{ar}$$

(iv) For r >> 6 it will same as for r = $5 \le r \le 6$

Thus $\overrightarrow{Dr} = \vec{r}.\overrightarrow{ar}$.

Q.84 Write Laplaces equation in Cartesian, Cylindrical, Spherical coordinates. **(4)**

Ans:CARTESIAN

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial^2 V}{\partial \varphi^2}.$$

Q.85 Define Biot Savert law. Calculate the magnetic field of line current along a thin straight wire of infinite length. **(6)**

Ans: Biot-Savert Law may be written as

$$\overline{dH} = \frac{I\overrightarrow{dL} \times \overrightarrow{a_{12}}}{4\pi.d^2}$$

Where

d = distance of point where it is to be find out from element dl.

$\overrightarrow{a_{12}}$ = unit vector from point 1 to point 2.

I = current in the filamentary conductor.

FOR WIRE OF INFINITE LENGTH:

Suppose a wire of infinite length is kept on z axis carries a current I. Then any point 1(0,0,z) on the z axis at which a differential length.

$\overline{dL} = \overline{d_z}\,\overline{a_z}$ is located. The point at which $\vec{H}$ has to be calculated is $2(\rho, \varphi, \tau)$

$$\overline{R_{12}} = \overline{r_2} - \overline{r_1}$$

$$= \rho\vec{\upsilon} + z\overline{a_z} - z\overline{a_z}$$

$$= \rho\vec{\upsilon} + (z - z)\overrightarrow{a_z}$$

$$d = \left|\overline{R_{12}}\right| = \sqrt{\rho^2 + (Z - z)^2}$$

$$\therefore \quad \overline{dH} = \frac{Idz\overline{a_z} \times \left[\rho\overline{\upsilon} + (Z - z)\overline{a_z}\right]}{4\pi\left[\rho^2 + (Z - z)^2\right]^{3/2}}$$

$$\therefore \quad \overline{H} = \int_{-\infty}^{\infty} \frac{Idz(\rho\overline{a\varphi})}{4\pi\left[\rho^2 + (Z - z)^2\right]^{3/2}}$$

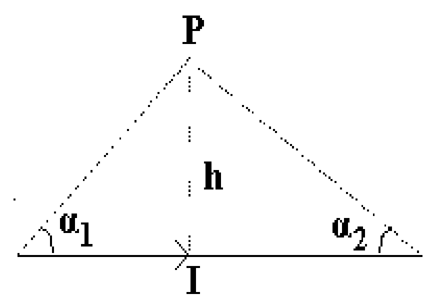
$$= \frac{I\rho\overline{a\varphi}}{4\pi}\left[-\frac{1}{\rho^2}\frac{Z - z}{\sqrt{\rho + (Z - z)^2}}\right]_{-\infty}^{\infty}$$

$$= \frac{I}{2\prod\rho}\overline{a\varphi}$$

In general the magnetic field intensity due to infinitely long filamentary conductor is given by $\overline{H} = \frac{I}{2\pi d}\left(\overline{a_1} \times \overline{a_{12}}\right)$

Where $\overline{a_1}$, is a unit vector in the direction of current, point 2 is that point at which magnetic field intensity is derived, point 1 is the foot of perpendicular from point 2 on the filamentary conductor and d is the distance between the joints 1 & 2.

Q.86 Find the magnetic flux density and field intensity at a point P due to a straight conductor carrying a current I as **(10)**



Ans: Magnetic field due to length element $\overline{dl}$ i

$$\overrightarrow{dH} = \frac{I\overrightarrow{dL} \times \overrightarrow{ar}}{4\pi R^2}$$

$$= \frac{I\overline{dL} \times \sin(\alpha)\overrightarrow{a\varphi}}{4\pi R^2}$$

Where $Rd\alpha = dl\sin\alpha$

And $R = h/\sin\alpha$

$$\overrightarrow{dH} = \frac{IR\,d\alpha/\sin\alpha \times \sin\alpha\overrightarrow{a\varphi}}{4\pi R^2} = \frac{Id\alpha}{4\pi R}\overrightarrow{a\varphi}$$

$$= \frac{Id\alpha}{4\pi\left(h/\sin\alpha\right)}\overrightarrow{a\varphi}$$

$$H = \frac{I}{4\pi h}\int_{\alpha_1}^{\alpha_2} \sin\alpha d\alpha \overrightarrow{a\varphi}$$

$$= \frac{I}{4\pi h}\left[-\cos\alpha\right]_{\alpha_1}^{\alpha_2}\overrightarrow{a\varphi}$$

$$= \frac{I}{4\pi h}\left[\cos\alpha_1 - \cos\alpha_2\right]\overrightarrow{a\varphi}$$

Q.87 Given that $E = 50\pi e^{j(\omega t - \beta z)}(u_x)$

$H = H_m e^{j(\omega t-\beta z)}(u_y)$ in free space where $\omega = 10^9$.

Evaluate H_m and β $(\beta > 0)$ **(8)**

Ans: Given $\omega = 10^9$ $\quad\therefore\quad \beta = \dfrac{\omega}{c} = \dfrac{10^9}{3\times 10^8} = 10/3$

Now

$$\vec{E} = \frac{1}{\varepsilon}\int \vec{\nabla}\times\vec{H}dt \quad \because \angle = 0$$

$$\vec{\nabla}\times\vec{H} = \frac{-\partial}{\partial^2}H_y\overrightarrow{a_x} + \overrightarrow{a_z}\frac{\partial}{\partial x}H_y$$

$$= \beta H_m e^{j(\omega t-\beta z)}\overrightarrow{a_x}$$

$$\vec{E} = \frac{1}{\varepsilon}\int \beta H_m e^{j(\omega t-\beta z)}dt\overrightarrow{a_x}$$

$$= \frac{\beta H_m}{\omega\varepsilon}e^{j(\omega t-\beta z)}\overrightarrow{a_x}$$

Comparing with $\vec{E}$ we get

$$\frac{\beta H_m}{\omega\varepsilon} = 50\pi$$

$$H_m = \frac{50\pi\omega\varepsilon}{\beta} = \frac{50\pi}{10}\times 3\times\omega\varepsilon$$

$$= 15\pi\times 10^9\times 8.854\times 10^{-12}$$

$$= 416.3\times 10^{-3}$$

$$= 0.4163 \text{ A/m.}$$

Q.88 Show that E and H fields constitute a wave travelling in Z-direction. Verify that the wave speed and E/H depend only on the properties of free space. **(8)**

Ans: Relation between H & E for a wave propagation in z direction can be written as

$E_x(z,t) = E_{X_0}\cos(\omega t - k_0 z)$ --------------------- (1)

Applying maxwells equation H can be easily found out as

$H_y(z,t) = E_{X_0}\sqrt{\dfrac{\varepsilon_0}{\mu_0}}\cos(\omega t - k_0 z)$ --------------------- (2)

Therefore from above relations we find that for an X directed E field that propagates the wave in z direction, the direction of H field comes out to be y directed. Thus a uniform plane wave is transverse in nature.

Taking ratio of (1) & (2) we get

$$\frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad \text{----------------------- (3)}$$

Also $k_0 = \omega/c$ and $\lambda = 2\lambda/k_0$

$v = \omega/\beta$ $\qquad \beta = \omega\sqrt{\mu_0\varepsilon_0}$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

From above relation we find that wave speed and E/H depends only on the properties of free space,

Q.89 Define polarization of waves, linear polarization, elliptical polarization, circular polarization. **(4)**

Ans: Polarization refers to time varying behaviour of electric field strength vector at some fixed point in space.

LINEAR POLARIZATION:

For a wave to have linear polarization, the time phase difference between two component must be

$\delta = \delta_y - \delta_y = n\pi$

Where n = 0,1,2,3,4----------

A linearly polarized wave can be resolved into a light hand circularly polarized wave and left hand circularly polarized wave of equal amplitude.

ELLIPTICAL POLARIZATION:

$E_0 = \overline{a_x}A + j\overrightarrow{a_y}B$ & $\overline{E}(o,t) = A\cos\omega t\,\overline{a_x} - B\sin\omega t\,\overline{a_y}$

$\overline{E_x} = A\cos\omega t \quad E_y = -B\sin\omega t$

$\therefore \ \frac{E_x^{\ 2}}{A^2} + \frac{E_y^{\ 2}}{B^2} = 1 \quad \therefore$ END points $\overline{E}(o,t)$ traces out an Ellipse.

CIRCULAR POLARIZATION:

If y component leads x by 90^0 $\ E_a$ = amplitude

$E_0 = \left(\overline{a_x} + j\overline{a_y}\right).\ E_a \Rightarrow \overline{E}(o,t) = \left(\cos\omega t\,\overrightarrow{a_x} - \sin\omega t\,\overrightarrow{a_y}\right)E_a$

$$\left.\begin{aligned} E_x &= E_a\cos\omega t \\ E_y &= -E_a\sin\omega t \end{aligned}\right\} E_x^{\ 2} + E_y^{\ 2} = E_a^{\ 2}$$

End points of $\overline{E}(o,t)$ traces out a circle of radius E_a.

Q.90 A magnetic field strength of $5\mu\,A/m$ is required at a point on $\theta = \pi/2$, 2 Km from an antenna in air. Neglecting ohmic loss how much power must antenna transmit if it is

(i) a hertzian dipole of length $\lambda/25$.

(ii) A half wave dipole. **(4+4)**

Ans:

(i) For a Hertzian dipole

$$|H_{\varphi s}| = \frac{I_0 \beta dl \sin\theta}{4\pi r}$$

Where

$dl = \lambda/25$

$$\beta dl = \frac{2\pi}{\lambda} \times \frac{\lambda}{25} = \frac{2\pi}{25}$$

$$5\times10^{-6} = \frac{I_0 . 2\pi/25(1)}{4\pi\left(2\times10^3\right)} = \frac{I_0}{10^5}$$

$I_0 = 0.5A$

$$P_{rad} = 40\pi^2\left(\frac{dl}{\lambda}\right)^2 I_0{}^2 = \frac{40\pi^2(0.5)^2}{(25)^2}$$

= 158 mw.

(ii) For a λ/2 dipole

$$|H_{\varphi s}| = \frac{I_0 \cos\left(\frac{\pi}{2}.\cos\theta\right)}{2\pi r \sin\theta}$$

$$5\times10^{-6} = \frac{I_0 . 1}{2\pi(2\times10^3)(1)}$$

$I_0 = 20\pi \; mA$

$$P_{av} = \frac{1}{2} I_0{}^2 R_{rad} = \frac{1}{2}(20\pi)^2 \times 10^{-6}$$

= 144 mw.

Q.91 Find the characteristic impedance of lossless transmission line having R=5Ω, L=40H and C=10F having frequency of 10Hz. **(4)**

Ans: A transmission line is said to be lossless if both its conductor & dielectric loss are zero, R = 0 and G = 0

∴ Characteristic impedance = $\sqrt{\frac{L}{C}}$

$= \sqrt{\frac{40}{10}} = \sqrt{4}$

= 2 Ω.

Q.92 What is standing wave ratio? Calculate reflection coefficient having SWR of 1.5. **(4)**

Ans: Standing Wave ratio: The ratio of maximum to minimum voltages along a finite terminated line is called standing wave ratio.

$$SWR = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+\Gamma_L}{1-\Gamma_L}$$

$\therefore \quad \Gamma_L =$ reflection coefficient is

$$= \frac{SWR-1}{SWR+1}$$

$$= \frac{1.5-1}{1.5+1} = \frac{.5}{1.5} = 1/3 .$$

Q.93 Define cut-off wavelength for a rectangular wave guide. A rectangular wave guide measures 3 x 4.5 cm internally and has a 10GHz signal propagated in it. Calculate the cut-off wavelength, the guide wavelength and characteristic wave impedance for TE_{10} mode. **(8)**

Ans: For TE_{10} mode

$f' = 10\,GH_z$

$\lambda_g = 0.8757\lambda_c$ for TE_{10} mode

$$\lambda_0 = \frac{c}{f} = \frac{3\times10^8}{10\times10^9}\times100$$

$= 3$ cm.

$$\left(\frac{\lambda_g}{\lambda_0}\right)^2 = 1 + \left(\frac{\lambda_g}{\lambda_c}\right)^2$$

$= 1+(0.8757)^2$

$= 1.767$

$$\frac{\lambda_g}{\lambda_0} = 1.329 \quad \& \quad \lambda_g = 1.329\times\lambda_0 \quad = 4.00 \text{ cm}$$

$$\lambda_c = \frac{\lambda_g}{0.8757} = \frac{4}{0.8757} = 4.571 \text{ cm}$$

$$\eta_{TE10} = 120\times\pi\times\frac{4}{3} = 160\pi .$$

Q.94 Explain Hertizan dipole. Show time variation of current and charge in Hertizan dipole. **(8)**

Ans: Hertzian dipole – By Hertzian dipole, we mean an infinite current element idl. Although such a current element does not exist in real life, it serves as building block from which the field of a practical antenna can be calculated by integration. Consider a hertzian dipole as in figure.

Magnetic vector potential at point P, due to dipole is

$$A = \frac{\mu I dl}{4\pi r} a \overleftarrow{z}$$

Where I is given by

$$[I] = I_0 \cos\left(wt - \frac{wr}{a}\right)$$

$$= I_0 \cos(wt - \beta r)$$

We find the E field using $\nabla \times H$ as

$$Exs = \frac{\eta I_0 dl}{2\pi}\cos\theta\left[\frac{1}{r^2} - \frac{1}{\beta r^3}\right]e^{-j\beta r}$$

$$Eys = \frac{\eta I_0 dl}{4\pi}\sin\theta\left[\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3}\right]e^{-j\beta r}$$

$$Ezs = 0$$

Power dissipated in fictitions Resistance R_{rad} .

$$P_{rad} = \frac{1}{2} I_0^{\,2} R_{rad}$$

Where $R_{rad} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2$.

Q.95 Define radiation resistance and directivity. Calculate the radiation resistance of an antenna having wavelength $\lambda = 5$ and length 25cm. **(8)**

Ans: $R_{rad} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2$

$= 80\pi^2\left[\frac{25}{5}\right]^2$

$= 2000\pi^2$

Directivity –
The directivity D of an antenna is the ratio of maximum radiation intensity to the average radiation intensity.

Q.96 Write short notes
(i) Space wave propagation
(ii) Skip distance
(iii) Ground wave propagation
(iv) Antenna Array **(16)**

Ans:
(i) SPACE (DIRECT) WAVE PROPAGATION
Space Waves, also known as direct waves, are radio waves that travel directly from the transmitting antenna to the receiving antenna. In order for this to occur, the two antennas must be able to "see" each other; that is there must be a line of sight path between them. The diagram on the next page shows a typical line of sight. The maximum line of sight distance between two antennas depends on the height of each antenna. If the heights are measured in feet, the maximum line of sight, in miles, is given by;

$$d = \sqrt{2h_t} + \sqrt{2h_r}$$

Because a typical transmission path is filled with buildings, hills and other obstacles, it is possible for radio waves to be reflected by these obstacles, resulting in radio waves that arrive at the receive antenna from several different directions. Because the length of each path is different, the waves will not arrive in phase. They may reinforce each other or cancel each other, depending on the phase differences. This situation is known as multipath propagation. It can cause major distortion to certain types of signals. Ghost images seen on broadcast TV signals are the result of multipath-one picture arrives slightly later than the other and is shifted in position on the screen. Multipath is very troublesome for mobile communications. When the transmitter and/or receiver are in motion, the path lengths are continuously changing and the signal fluctuates wildly in amplitude. For this reason, NBFM is used almost exclusively for mobile communications. Amplitude variations caused by multipath that make AM unreadable are eliminated by the limiter stage in an NBFM receiver. An interesting example of direct communications is satellite communications. If a satellite is placed in an orbit 22,000 miles above the equator, it appears to stand still in the sky, as viewed from the ground. A high gain antenna can be pointed at the satellite to transmit signals to it. The satellite is used as a relay station, from which approximately ¼ of the earth's surface is visible. The satellite receives signals from the ground at one frequency, known as the uplink frequency, translates this frequency to a different frequency, known as the downlink frequency, and retransmits the signal. Because two frequencies are used, the reception and transmission can happen simultaneously. A satellite operating in this way is known as a transponder. The satellite has a tremendous line of sight from its vantage point in space and many ground stations can communicate through a single satellite.

(ii) SKIP DISTANCE/SKIP ZONE –

The SKIP DISTANCE is the distance from the transmitter to the point where the sky wave is first returned to Earth. The size of the skip distance depends on the frequency of the wave, the angle of incidence, and the degree of ionization present.

The SKIP ZONE is a zone of silence between the point where the ground wave becomes too weak for reception and the point where the sky wave is first returned to Earth. The size of the skip zone depends on the extent of the ground wave coverage and the skip distance. When the ground wave coverage is great enough or the skip distance is short enough that no zone of silence occurs, there is no skip zone.

Occasionally, the first sky wave will return to Earth within the range of the ground wave. If the sky wave and ground wave are nearly of equal intensity, the sky wave alternately reinforces and cancels the ground wave, causing severe fading. This is caused by the phase difference between the two waves, a result of the longer path travelled by the sky wave.

PROPAGATION PATH: The path that a refracted wave follows to the receiver depends on the angle at which the wave strikes the ionosphere. You should remember, however, that the rf energy radiated by a transmitting antenna spreads out with distance. The energy therefore strikes the ionosphere at many different angles rather than a single angle. After the rf energy of a given frequency enters an ionospheric region, the paths that this energy might follow are many. It may reach the receiving antenna via two or more paths through a single layer.

(iii) GROUND WAVE PROPAGATION

Ground Waves are radio waves that follow the curvature of the earth. Ground waves are always vertically polarized, because a horizontally polarized ground wave would be shorted out by the conductivity of the ground. Because ground waves are actually in contact with the ground, they are greatly affected by the ground's properties. Because ground is not a perfect electrical conductor, ground waves are attenuated as they follow the earth's surface. This effect is more pronounced at higher frequencies, limiting the usefulness of ground wave propagation to frequencies below 2 MHz. Ground waves will propagate long distances over sea water, due to its high conductivity.

Ground waves are used primarily for local AM broadcasting and communications with submarines. Submarine communications take place at frequencies well below 10KHz, which can penetrate sea water (remember the skin effect?) and which are propagated globally by ground waves.

Lower frequencies (between 30 and 3,000 KHz) have the property of following the curvature of the earth via groundwave propagation in the majority of occurrences.

In this mode the radio wave propagates by interacting with the semi-conductive surface of the earth. The wave "clings" to the surface and thus follows the curvature of the earth. Vertical polarization is used to alleviate short circuiting the electric field through the conductivity of the ground. Since the ground is not a perfect electrical conductor, ground waves are attenuated rapidly as they follow the earth's surface. Attenuation is proportional to the frequency making this mode mainly useful for LF and VLF frequencies.

Today LF and VLF are mostly used for time signals, and for military communications, especially with ships and submarines. Early commercial and professional radio services relied exclusively on long wave, low frequencies and ground-wave propagation. To prevent interference with these service, amateur and experimental transmitters were restricted to the higher (HF) frequencies, felt to be useless since their ground-wave range was limited. Upon discovery of the other propagation modes possible at medium wave and short wave frequencies, the advantages of HF for commercial and military purposes became apparent. Amateur experimentation was then confined only to authorized frequency segments in the range.

(iv) ANTENNA ARRAY

An antenna array is an antenna that is composed of more than one conductor. There are two types of antenna arrays:

Driven arrays – all elements in the antenna are fed RF from the transmitter.

Parasitic arrays – only one element is connected to the transmitter. The other elements are coupled to the driven element through the electric fields and magnetic fields that exist in the near field region of the driven element.

There are many types of driven arrays. The four most common types are:

COLLINEAR ARRAY

The collinear array consists of $\lambda/2$ dipoles oriented end-to-end. The center dipole is fed by the transmitter and sections of shorted transmission line known as phasing lines connect the ends of the dipoles.

BROADSIDE ARRAY

A broadside array consists of an array of dipoles mounted one above another as shown below. Each dipole has its own feed line and the lengths of all feed lines are equal so that the currents in all the dipoles are in phase.

LOG PERIODIC DIPOLE ARRAY

The lop periodic dipole array (LPDA) is one antenna that almost everyone over 40 years old has seen. They were used for years as TV antennas. The chief advantage of an LPDA is that it is frequency-independent. Its input impedance and gain remain more or less constant over its operating bandwidth, which can be very large. Practical designs can have a bandwidth of an octave or more.

YAGI-UDA ARRAY (YAGI)

The Agi-Uda array, named after the two Japanese physicists who invented it, is the most common antenna array in use today. In contrast to the other antenna arrays that we have already looked at, the Yagi has only a single element that is connected to the transmitter, called the driver or driven element. The remaining elements are coupled to the driven element through its electromagnetic field. The other elements absorb some of the electromagnetic energy radiated by the driver and re-radiate it. The fields of the driver and the remaining elements sum up to produce a unidirectional pattern. The diagram below shows the layout of elements in a typical Yagi.

Q.97 Prove that energy density stored in an electric field of magnitude **E** is proportional to $\mathbf{E}^2$. **(8)**

Ans: Refer text Book 1, Section 4.8 (Page No. 111)

Q.98 A circular ring of radius 'a' carries a uniform charge ρ_L C/m and is placed on xy-plane with the axis the same as the z-axis

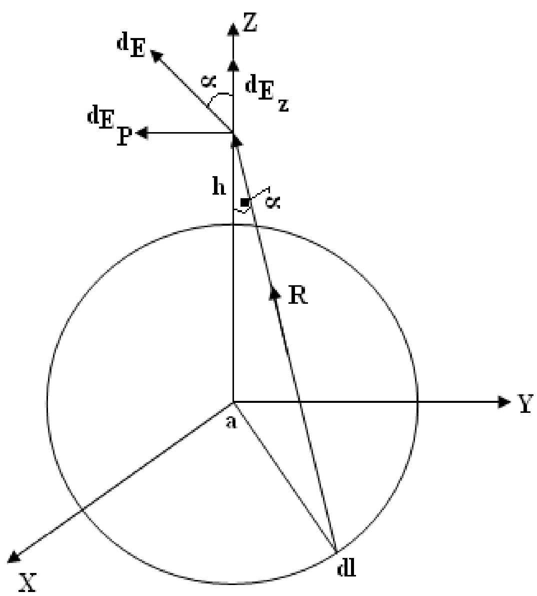
(i) Find $\overline{E}(0,0,h)$

(ii) What value of h gives maximum value of $\overline{E}$?

(iii) If the total charge on the ring is Q, find $\overline{E}$ as 'a' tends to 0. **(8)**

Ans:

(i) Consider the figure:



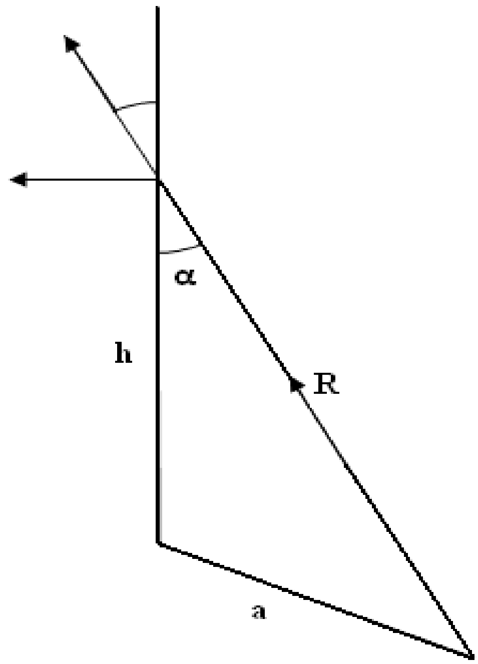
$$dl = ad\phi \qquad \overline{R} = a(-\overline{a}_P) + h\overline{a}_z$$

$$R = |\overline{R}| = \left[a^2 + h^2\right]^{1/2}, \quad \overline{a}_R = \frac{\overline{R}}{R}$$

$$\frac{a_R}{R^2} = \frac{\overline{R}}{|\overline{R}|^3} = \frac{-a\overline{a}_P + h\overline{a}_z}{\left[a^2 + h^2\right]^{3/2}}$$

$$\overline{E} = \frac{P_L}{4\pi\in_0}\int_{\phi=0}^{2\pi}\frac{-a\overline{a}_P + h\overline{a}_z}{\left[a^2+h^2\right]^{3/2}}ad\phi$$

By symmetry, contributions along $\overline{a}_P$ add up to zero.
Thus we are left with the z-component.



$$\overline{E}(0,0,h) = \frac{P_L a h \overline{a}_z}{4\pi\in_0\left[h^2+a^2\right]^{3/2}}\int_0^{2\pi}d\varphi = \frac{P_L a h\overline{a}_z}{2\in_0\left[h^2+a^2\right]^{3/2}} \text{ v/m.}$$

(ii) $$\frac{d\left|\overline{E}\right|}{dh} = \frac{P_L a}{2E_0}\left\{\frac{\left[h^2+a^2\right]^{3/2}(1) - \frac{3}{2}(h)(2h)\left[h^2+a^2\right]^{1/2}}{\left[h^2+a^2\right]^3}\right\}$$

For maximum $\overline{E}$, $\frac{d\left|\overline{E}\right|}{dh} = 0$

$\therefore \quad \left[h^2+a^2\right]^{1/2}\left[h^2+a^2-3h^2\right] = 0$

$\therefore \quad a^2 - 2h^2 = 0 \quad \text{or} \quad h = \pm\frac{a}{\sqrt{2}}m$

(iii) Since the charge is uniformly distributed, the line charge density is $P_L = \frac{Q}{2\pi a}$

$\therefore \quad \overline{E} = \frac{Qh}{4\pi\in_0\left[h^2+a^2\right]^{3/2}}\overline{a_z}$

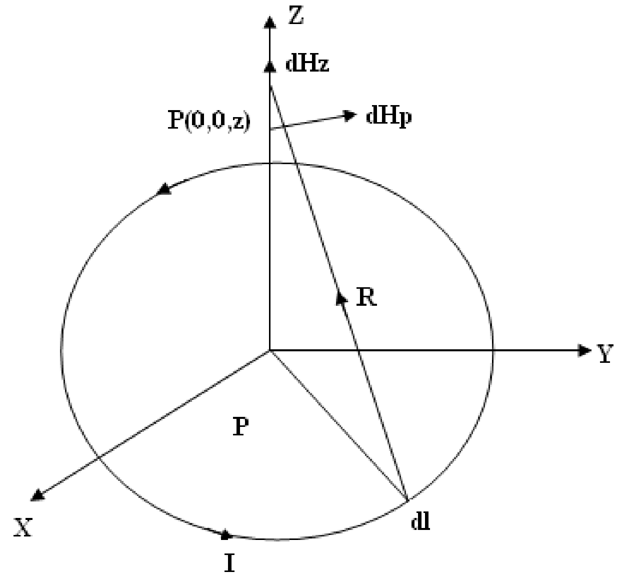
As $a \to 0$, $\overline{E} = \frac{Q}{4\pi\in_0 h^2}\overline{a_z}$ v/m

Q.99 Derive an expression for the magnetic field due to an infinite plane sheet of uniform urface current density. **(8)**

Ans: Refer text Book 1, Section 8.2 (Page No. 237)

Q.100 A circular loop located on $x^2 + y^2 = 9,\ z = 0$ carries a direct current of 10A along $\hat{a}_\phi$. Determine $\overline{H}$ at (0, 0, 4) and (0, 0, −4). **(8)**

Ans: Consider the circular loop shown in Fig.



$$d\overline{H} = \frac{Id\bar{l} \times \overline{R}}{4\pi R^3}$$

$$d\bar{l} = \rho d\phi \bar{a}_\varphi,\quad \overline{R} = (0,0,h) - (x,y,0) = -\rho \bar{a}_p + h\bar{a}_z$$

$$d\bar{l} \times \overline{R} = \begin{vmatrix} \bar{a}_\rho & \bar{a}_\phi & \bar{a}_z \\ 0 & \rho d\phi & 0 \\ -\rho & 0 & h \end{vmatrix} = \rho h d\phi \bar{a}_\rho + P^2 d\phi \bar{a}_z$$

$$\therefore\ d\overline{H} = \frac{I}{4\pi[\rho^2 + h^2]^{3/2}}(\rho h d\phi \bar{a}_\rho + P^2 d\phi \bar{a}_z) = d\overline{H}_\rho \bar{a}_\rho + d\overline{H}_z \bar{a}_z$$

By symmetry, the contributions along $\bar{a}_\rho$ add up to zero

$$\therefore\quad \overline{H}_\rho = 0$$

$$\overline{H} = \int dH_z \bar{a}_z = \int_0^{2\pi} \frac{I\rho^2 d\phi \bar{a}_z}{4\pi\left[\rho^2 + h^2\right]^{3/2}} = \frac{I\rho^2 2\pi \bar{a}_z}{4\pi\left[\rho^2 + h^2\right]^{3/2}}$$

$$\therefore\quad \overline{H} = \frac{I\rho^2 \bar{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$$

I = 10A, $\rho = 3$, h = 4

$$\therefore\quad H(0,0,4) = \frac{10(3)^2 \bar{a}_z}{2[9+16]^{3/2}} = 0.36\bar{a}_z \text{ A/m}$$

For H(0,0,4), h is released by –h

$\therefore \quad \overline{H}(0,0,-4) = \overline{H}(0,0,4) = 0.36\overline{a}_z$ A/m.

Q.101 State and explain Maxwell's equation in their Integral and differential forms. Derive the corresponding equations for fields varying harmonically with time. **(8)**

Ans: Refer text Book 1, Section 10.4, 10.5 (Page No. 334 - 337)

Q.102 A conducting bar P can slide freely over two conducting rails as shown in Fig.1. Calculate the induced voltage in the bar

(i) If the bar is stationed at y = 8 cm and $\overline{B} = 4\cos 10^6 t\, \hat{a}_z$ mWb/m^2.

(ii) If the bar slides at a velocity $\overline{v} = 20\ \hat{a}_y$ m/s and $\overline{B} = 4\,\hat{a}_z$ mWb/m^2.

(iii) If the bar slides at a velocity $\overline{v} = 20\,\hat{a}_y$ m/s and $\overline{B} = 4\cos\left(10^6 t - y\right)\hat{a}_z$ mWb/m^2. **(8)**

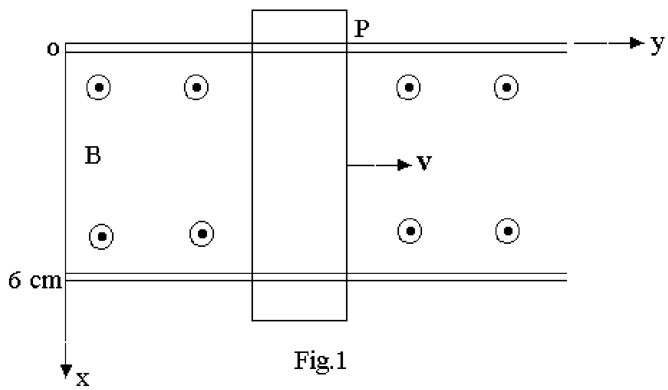


Fig.1

Ans: Consider the fig.

(i) $\overline{B} = 4\cos\left(10^6 t\right)\overline{a}_z \dfrac{mWb}{m^2}$

$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t}.ds = \int_{y=0}^{0.08}\int_{x=0}^{0.06} 4(10^{-3})(10^6)\sin(10^6 t)dxdy$$

$$= 4(10^3)(0.08)(0.06)\sin(10^6 t)$$

$$V_{emf} = 19.2\sin(10^6 t)V$$

(ii) $\overline{u} = 20\overline{a}_y m/s$, $\overline{B} = 4\overline{a}_z mWb/m^2$

$$V_{emf} = \int\left(\overline{u} + \overline{B}\right).d\overline{l} = \int_{x=l}^{0}\left(u\overline{a}_y \times B\overline{a}_z\right).dx\overline{a}_x$$

$$= -uBl = -20(4)(10^{-3})(0.06)$$

$$\therefore \quad V_{emf} = -4.8mV$$

(iii) $\overline{u} = 20\overline{a}_y m/s$, $\overline{B} = 4\cos\left(10^6 t - y\right)\overline{a}_z \dfrac{mWb}{m^2}$

$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t}.ds + \int (\overline{u} \times \overline{B}).d\overline{l}$$

$$= \int_{x=0}^{0.06} \int_{0}^{y} (4)(10^{-3})(10^{6}) \sin(10^{6}t - y')dy'dx$$

$$+ \int_{0.06}^{0} \left[20\overline{a}_y \times (4)(10^{-3}) \cos(10^6 t - y)\overline{a}_z \right] dx \overline{a}_x$$

$$= 240 \cos(10^6 t - y')\Big|_0^y - 80(10^{-3})(0.06)\cos(10^6 t - y)$$

$$= 240\cos(10^6 t - y) - 240\cos 10^6 t - 4.8(10^{-3})\cos(10^6 t - y)$$

$$V_{emf} \simeq 240\cos(10^6 t - y) - 240\cos 10^6 t$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\therefore \quad V_{emf} = 480\sin\left(10^6 t - \frac{y}{2}\right)\sin\frac{y}{2} V .$$

Q.103 State and prove Poynting theorem. Explain the physical interpretation of each terms in it. **(8)**

Ans: Refer text Book 1, Section 11.3 (Page No. 365 - 369)

Q.104 Given a uniform plane wave in air as

$$\overline{E_i} = 40\cos(\omega t - \beta z)\hat{a}_x + 30\sin(\omega t - \beta z)\hat{a}_y \text{ V/m}$$

a. Find $\overline{H_i}$.
b. If the wave encounters a perfectly conducting plate normal to the z-axis at z = 0, find the reflected wave $\overline{E_r}$ and $\overline{H_r}$.
c. What are the total $\overline{E}$ and $\overline{H}$ fields for $z \le 0$?
d. Calculate the time-average Poynting vectors for $z \le 0$ and $z \ge 0$. **(8)**

Ans: $\overline{E}_i = 40\cos(\omega t - \beta z)\overline{a}_x + 30\sin(\omega t - \beta z)\overline{a}_y \, V/m$

(i) we may treat the wave as consisting of two waves,

$\overline{E}_{i1} = 40\cos(\omega t - \beta z)\overline{a}_x$, $\quad \overline{E}_{i2} = 30\sin(\omega t - \beta z)\overline{a}_y$

At atmospheric pressure, air has $\varepsilon_r = 1.0006 \simeq 1$.

Thus air may be regarded as free space.

Let $\overline{H}_i = \overline{H}_{i1} + \overline{H}_{i2}$

$$\overline{H}_{i1} = \overline{H}_{i10}\cos(\omega t - \beta z)\overline{a}_{H_1}$$

$$\overline{H}_{i10} = \frac{\overline{E}_{i10}}{\eta_0} = \frac{40}{120\pi} = \frac{1}{3\pi}$$

$$\overline{a}_{H_1} = \overline{a}_k \times \overline{a}_E = \overline{a}_z \times \overline{a}_x = \overline{a}_y$$

$$\therefore \quad \overline{H}_{i1} = \frac{1}{3\pi}\cos(\omega t - \beta z)\overline{a}_y$$

Similarly $\overline{H}_{i2} = H_{i20}\sin(\omega t - \beta z)\overline{a}_{H_2}$

$$H_{iz0} = \frac{E_{i20}}{\eta_0} = \frac{30}{120\pi} = \frac{1}{4\pi}$$

$$\overline{a}_{H_z} = \overline{a}_k \times \overline{a}_E = \overline{a}_z \times \overline{a}_y = -\overline{a}_x$$

$$\therefore \quad \overline{H}_{i2} = -\frac{1}{4\pi}\sin(\omega t - \beta z)\overline{a}_x$$

$$\overline{H} = \overline{H}_{i1} + \overline{H}_{i2} = -\frac{1}{4\pi}\sin(\omega t - \beta z)\overline{a}_x + \frac{1}{3\pi}\cos(\omega t - \beta z)\overline{a}_y$$

(ii) Since the medium 2 is perfectly conducting

$$\frac{\sigma_2}{\omega \in_2} >> 1 \Rightarrow \eta_2 << \eta_1$$

$$\Gamma \overset{\omega}{=} -1, \quad \tau = 0$$

Showing that incident $\overline{E}$ and $\overline{H}$ fields are totally reflected.

$$E_{ro} = \Gamma E_{io} = -E_{io}$$

$$\therefore \quad \overline{E}_r = -40\cos(\omega t + \beta z)\overline{a}_x - 30\sin(\omega t + \beta z)\overline{a}_y \; V/m$$

$\overline{H}_r$ can be found from $\overline{E}_r$ just as in part (i)

$$\overline{H}_r = \frac{1}{3\pi}\cos(\omega t + \beta z)\overline{a}_y - \frac{1}{4\pi}\sin(\omega t + \beta z)\overline{a}_x \; A/m$$

(iii) The total fields in air

$\overline{E}_1 = \overline{E}_i + \overline{E}_r$ and $\overline{H}_1 = \overline{H}_i + \overline{H}_r$

The total fields in the conductor are

$\overline{E}_2 = E_t = 0$, $\overline{H}_2 = \overline{H}_t = 0$

(iv) $z \le 0$

$$P_{1arc} = \frac{|E_{1s}|^2}{2\eta_1}\overline{a}_k = \frac{1}{2\eta_0}\left[E_{i0}^2\overline{a}_z - E_{r0}^2\overline{a}_z\right]$$

$$= \frac{1}{240\pi}\left[\left(40^2 + 30^2\right)\overline{a}_z - \left(40^2 + 30^2\right)\overline{a}_z\right]$$

$$P_{1arc} = 0$$

For $z \ge 0$

$$P_{2arc} = \frac{|E_{2s}|^2}{2\eta_2}\overline{a}_k = \frac{E_{t0}^2}{2\eta_2}\overline{a}_2 = 0$$

Because the whole incident power is reflected.

Q.105 Discuss the derivation of the transmission-line equations from field equations by considering a parallel-plate line. Also model the line as a distributed circuit. **(8)**

Ans: Refer text Book 1, Section 13.1 (Page No. 436)

Q.106 A distortionless line has $z_o = 60\Omega, \alpha = 20\,\text{mNp/m}$, v = 0.6 c, where c is the speed of light in vacuum. Find R, L, G, C and λ at 100 MHz frequency. **(8)**

Ans: For a distortion less line, $RC = GL$ or $G = \frac{RC}{L}$

$Z_0 = \sqrt{\frac{L}{C}}$, $\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$

$R = \alpha Z_0$

But $u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

$R = \alpha Z_0 = (20\times10^{-3})(60) = 1.2\ \Omega/m$

$L = \frac{Z_0}{u} = \frac{60}{0.6(3\times10^8)} = 333\ \eta H/m$

$G = \frac{\alpha^2}{R} = \frac{400\times10^{-6}}{1.2} = 333\ \mu\,\text{mho}/m$

$uZ_0 = \frac{1}{C}$

$\therefore\quad C = \frac{1}{4Z_0} = \frac{1}{0.6(3\times10^8)(60)} = 92.59\ \phi F/m$

$\lambda = \frac{u}{f} = \frac{0.6(3\times10^8)}{10^8} = 1.8\ m$.

Q.107 Explain the Terms-dominant mode, cut-off frequency, guide wavelength and characteristic Impedance. Discuss them for both TE and TM modes. **(8)**

Ans: Refer text Book 1, Section 9.1.4 (Page No. 721)

Q.108 Consider a parallel-plate waveguide as shown in Fig.2. Find the power reflection coefficients for $TE_{1,0}$ and $TM_{1,0}$ waves at frequency f = 5000 MHz incident on the junction from free space side. **(8)**

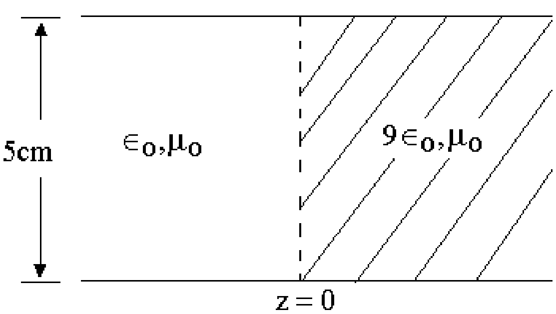


Fig.2

Ans: Consider the fig.

For $TE_{1,0}$ mode or for $TM_{1,0}$ mode

For f = 5000 MH_z

$\lambda_1 = \frac{3\times10^8}{5\times10^9} = 6\ cm$

$$\lambda_2 = \frac{3\times10^8}{\sqrt{9}\times5\times10^9} = \frac{6}{3} = 2\ cm$$

Since λ & λ_c in both sections, $TE_{1,0}$ and $TM_{1,0}$ modes propagate in both sections.

For $TE_{1,0}$ mode

$$\eta_{9_1} = \frac{\eta_1}{\sqrt{1-\left(\lambda_1/\lambda_c\right)^2}} = \frac{120\pi}{\sqrt{1-\left(6/10\right)^2}} = 471.24\Omega$$

$$\eta_{9_2} = \frac{\eta_2}{\sqrt{1-\left(\lambda_2/\lambda_c\right)^2}} = \frac{120\pi/\sqrt{9}}{\sqrt{1-\left(2/10\right)^2}} = \frac{40\pi}{\sqrt{1-0.04}} = 12825\Omega$$

$$\Gamma^2_{TE_{1,0}} = \left(\frac{\eta_{9_2}-\eta_{9_1}}{\eta_{9_2}+\eta_{9_1}}\right)^2$$

$$\Gamma^2_{TE_{1,0}} = (-0.572)^2 = 0.327$$

For $TM_{1,0}$ mode

$$\eta_{9_1} = \eta_1\sqrt{1-\left(\lambda_1/\lambda_c\right)^2} = 301.59\Omega$$

$$\eta_{9_2} = \eta_2\sqrt{1-\left(\lambda_2/\lambda_c\right)^2} = 123.12\Omega$$

$$\Gamma^2_{TM_{1,0}} = \left(\frac{\eta_{9_2}-\eta_{9_1}}{\eta_{9_2}+\eta_{9_1}}\right)^2$$

$$\Gamma^2_{TM_{1,0}} = (-0.42)^2 = 0.176.$$

Q.109 Discuss the concept of unit and group patterns and their multiplications to obtain the resultant pattern of an array. **(8)**

Ans: Refer text Book 1, Section 10.30, 10.30.1 (Page No. 826 - 827)

Q.110 The radiation intensity of an antenna is given by,

$$U(\theta,\phi) = \begin{cases} 2\sin\theta\sin^3\phi, & 0\le\theta\le\pi, \quad 0\le\phi\le\pi, \\ 0, & \text{otherwise.} \end{cases}$$

Determine the directivity of the antenna. **(8)**

Ans: For a certain antenna radiation intensity

$$U(\theta,\phi)=\begin{cases}2\sin\theta\sin^3\theta & 0\le\theta\le\pi, 0\le\phi\le\pi\\ 0 & elsewhere\end{cases}$$

Directivity $D=\dfrac{U_{\max}}{U_{arc}}$

$U_{\max}=2$

$$V_{arc}=\frac{1}{4\pi}\int U\,d\Omega\left(=P_{rad}/4\pi\right)$$

$$=\frac{1}{4\pi}\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi}2\sin\theta\sin^3\phi\,\sin\theta\,d\theta\,d\phi$$

$$=\frac{1}{2\pi}\int_0^{\pi}\sin^2\theta\,d\theta\int_0^{\pi}\sin^3\phi\,d\phi$$

$$=\frac{1}{2\pi}\int_0^{\pi}\frac{1}{2}(1-\cos 2\theta)\,d\theta\int_0^{\pi}(1-\cos^2\phi)\,d(1-\cos\phi)$$

$$=\frac{1}{2\pi}\frac{1}{2}\left(\theta-\frac{\sin 2\theta}{2}\right)\Bigg|_0^{\pi}\left(\frac{\cos^3\phi}{3}-\cos\phi\right)\Bigg|_0^{\pi}$$

$$\therefore\quad V_{arc}=\frac{1}{2\pi}\left(\frac{\pi}{2}\right)\left(\frac{4}{3}\right)=\frac{1}{3}$$

$$\therefore\quad D=\frac{U_{\max}}{V_{arc}}=\frac{2}{\left(1/3\right)}=6\,.$$

Q.111 State the different layers of Ionosphere. Which layer disappears at night, also explain why the ground wave propagation called medium wave propagation?

(4+2)

Ans: Refer text Book 1, Section 11.5.2 and 11.4.1 (Page No. 946 and 938)

Q.112 Determine the electric potential energy of a uniformly charged sphere of radius R. **(8)**

Ans: Electric potential energy is given by

$$We=\frac{\varepsilon_0}{2}\int\left|\overline{E}\right|^2dv$$

$$=\frac{\varepsilon_0}{2}\int\left|\frac{\overline{D}}{\varepsilon_0}\right|^2dv$$

$$=\frac{1}{2\varepsilon_0}\int\left|\overline{D}\right|^2dv$$

Where $\overline{D}=\dfrac{Q}{4\pi R^2}\overrightarrow{ar}$, R = radius of sphere.

And $dv=R^2\sin\theta\,d\theta\,d\varphi\,dR$

$$We=\frac{1}{2\varepsilon_0}\int_R^{\infty}\int_0^{2\pi}\int_0^{\pi}\left|\frac{Q}{4\pi R^2}\right|^2\times R^2\sin\theta\,dR\,d\varphi\,d\theta$$

$$= \frac{1}{2\varepsilon_0}\int_R^{\infty}\int_0^{2\pi}\int_0^{\pi}\frac{Q^2}{16\pi^2R^2}\times R^2\sin\theta\,dR\,d\varphi\,d\theta$$

$$= \frac{1}{2\varepsilon_0}\times\frac{Q}{16\pi^2}\left[-\frac{1}{R}\right]_R^{\infty}[-\cos\theta]_0^{\pi}[\varphi]_0^{2\pi}$$

$$= \frac{1}{2\varepsilon_0}\times\frac{Q}{16\pi^2}\left[\frac{1}{R}\right][2][2\pi]$$

$$= \frac{Q}{8\pi\varepsilon_0}\times\frac{1}{R}.$$

Q.113 Find the electric field at a point P on the perpendicular bisector of a uniformly charged rod. The length of the rod is L, the charge on it is Q and the distance of P from the centre of the rod is a. **(8)**

Ans: $dE = \frac{\rho_L dz}{4\pi\varepsilon l^2}\overrightarrow{ar}$

$l^2 = z^2 + a^2$

$\rho_L = Q/L$

$dE_r = dE\cos\theta$

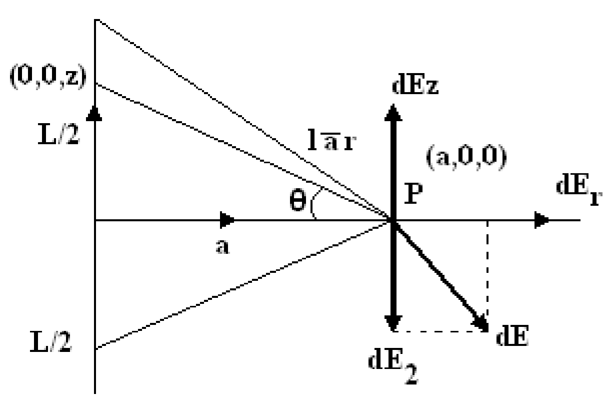
$= dE.a/l$

$$\int dE_r = \int_{-L/2}^{L/2} dE.\frac{a}{l} = \int_{-L/2}^{L/2}\frac{\rho_L dz}{4\pi\varepsilon l^2}.\frac{a}{l}$$

$$= \frac{\rho_L a}{4\pi\varepsilon}\int_{-L/2}^{L/2}\frac{dz}{l^3} = \frac{\rho_L a}{4\pi\varepsilon}\int_{-L/2}^{L/2}\frac{dz}{\left(\sqrt{a^2+z^2}\right)^3}$$

$$\therefore\quad E_r = \frac{\rho_L\times L/2}{2\pi\varepsilon a\sqrt{\left(L/2\right)^2+a^2}} \qquad \because\quad \rho_L = Q/L$$

$$= \frac{Q}{4\pi\varepsilon_0 a\sqrt{\left(L/2\right)^2+a^2}}$$



As seen above, the resultant Z component E_z of field at point P is zero. Therefore the total field E at point along the r-axis is

$$E_r = \frac{Q}{4\pi\varepsilon_0} \times \frac{1}{a} \times \frac{1}{\sqrt{\left(L/2\right)^2 + a^2}}.$$

Q.114 Prove that a uniform plane wave is transverse in nature and $\frac{|\bar{E}|}{|\bar{H}|} = \sqrt{\frac{\mu}{\in}}$. **(12)**

Ans: Helmholtz equation can be written as

$$\nabla^2 E_s = -K_0{}^2 E_s \qquad \text{----------------------- (1)}$$

When $K_0 = \omega\sqrt{\mu_0\varepsilon_0}$, the free space wave number.

Thus x component of (1) becomes

$$\nabla^2 E_{xs} = -K_0{}^2 E_{xs}$$

Or $$\frac{\partial^2 E_{xs}}{\partial x^2} + \frac{\partial^2 E_{xs}}{\partial y^2} + \frac{\partial^2 E_{xs}}{\partial z^2} = -K_0{}^2 E_{xs} \qquad \text{--------------------- (2)}$$

The solution of (2) which does not vary with x and y, so that corresponding two derivatives are zero, leads to

$$\frac{d^2 E_{xs}}{dz^2} = -K_0{}^2 E_{xs} \qquad \text{--------------------- (3)}$$

Thus solution of (3) can be written as

$$E_{xs} = E_{xo} e^{-jK_o z} \qquad \text{--------------------- (4)}$$

Next, we insert the $e^{j\omega t}$ factor and take the real part

$$E_x(z,t) = E_{xo}\cos(\omega t - K_o z) \qquad \text{--------------------- (5)}$$

Now from Maxwells equations E_s & H_s is most easily obtained from

$$\nabla \times E_s = -j\omega\mu_o H_s$$

$$\therefore \quad \frac{dE_{xs}}{dz} = -j\omega\mu_o H_{ys}$$

From (4) for E_{xs}, we have

$$H_{ys} = -\frac{1}{j\omega\mu_o}(-jK_o)E_{xo}e^{-jK_o z}$$

$$= E_{xo}\sqrt{\frac{\varepsilon_0}{\mu_0}} e^{-jK_o z}$$

In real instantaneous form, this becomes

$$H_y(z,t) = E_{xo}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos(\omega t - K_o z) \qquad \text{---------------------- (6)}$$

We therefore find the x-directed E field that propagates in the +ve z direction is accompanied by a y directed H field. Moreover, the ratio of the electric and magnetic field intensities, given by the rates of (5) to (6)

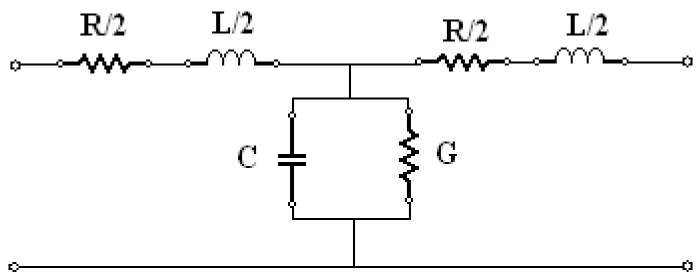
$$\frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

This shows that the uniform plane wave is a transverse electromagnetic wave.

Q.115 Derive expression for attenuation constant (α) and phase constant (β) for lossless transmission lines. **(4)**

Ans: Propagation constant for transmission line can be written as

$Y = \sqrt{zy}$ ---------------- (1)



Where

$Z = R + jwL$

$Y = (G + jwC)$

Hence substituting in (1)

$Y = \sqrt{(R + jwL)(G + jwC)}$

Since $Y = \alpha + j\beta$

Where α = attenuation constant

β = Phase Constant

$\alpha + j\beta = \sqrt{(R + jwL)(G + jwC)}$

Since for a lossless transmission line R = 0 & G = 0

$\alpha + j\beta = \sqrt{(jwL)(jwC)}$

$\alpha + j\beta = jw\sqrt{LC}$

$\therefore \quad \alpha = 0$

$\beta = w\sqrt{LC}$.

Q.116 Explain critical frequency and maximum usable frequency. Determine the value of frequency at which an Electromagnetic wave must be propagated for D-region having refractive index of 0.5. (given N=400 electrons/cc for D-region). **(8)**

Ans: Critical Frequency: The critical frequency (fc) for a given layer is the highest frequency that will be returned down to each by that layer after having been beamed straight up at it.

Maximum Usable frequency (MUF): It is also a limiting frequency, but this time for some specific angle of incidence other than normal. If the angle of incidence (between incidence say and the normal) is θ, it follows that,

$$MUF = \frac{critical\ frequency}{\cos\theta}$$

$= fc\sec\theta$.

Given $\mu = 0.5$ and $N = 400\ electrons/cc$

$$\mu = \sqrt{1 - \frac{80.6N}{f^2}}$$

$$0.5 = \sqrt{1 - \frac{80.6 \times 400}{f^2}}$$

$$f = 207.33\, kc/s\,.$$

Q.117 Derive the wave equations from Maxwell's equation for free space, lossless charge free region. **(8)**

Ans: Maxwells equation in phasor form can be written as

$\overline{\nabla} \times \overline{E}_s = -j\omega\mu \overline{H}_s$ ----------------- (1)

$\overline{\nabla} \times \overline{H}_s = (\sigma + j\omega\varepsilon)\overline{E}_s$ ----------------- (2)

$\overline{\nabla}.\overline{D}_s = \rho_v$ ----------------- (3)

$\overline{\nabla}.\overline{B}_s = 0$ ----------------- (4)

Using vector identity

$$\overline{\nabla} \times \overline{\nabla} \times \overline{E}_s = \overline{\nabla}(\overline{\nabla}.\overline{E}_s) - \overline{\nabla}^2 \overline{E}_s$$

LHS

$$\overline{\nabla} \times \overline{\nabla} \times \overline{E}_s = \overline{\nabla} \times (-j\omega\mu \overline{H}_s)$$
$$= -j\omega\mu \overline{\nabla} \times \overline{H}_s$$
$$= -j\omega\mu(\sigma + j\omega\varepsilon)\overline{E}_s$$

RHS

$$\overline{\nabla}(\overline{\nabla}.\overline{E}_s) - \overline{\nabla}^2.\overline{E}_s = \overline{\nabla}\left(\overline{\nabla}.\frac{\overline{D}_s}{\varepsilon}\right) - \overline{\nabla}^2 \overline{E}_s$$
$$= \frac{1}{\varepsilon}\overline{\nabla}(\rho_v) - \overline{\nabla}^2 \overline{E}_s$$

Assuming $\rho_v = 0$ for change free region

RHS = $-\overline{\nabla}^2 \overline{E}_s$

$$\therefore \quad -\overline{\nabla}^2 \overline{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\overline{E}_s$$
$$\overline{\nabla}^2 \overline{E}_s = j\omega\mu(\sigma + j\omega\varepsilon)\overline{E}_s$$

Let $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

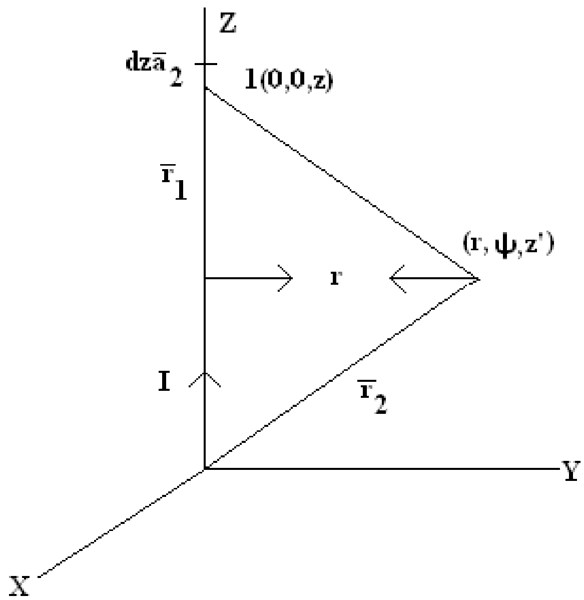
$\therefore \quad \overline{\nabla}^2 \overline{E}_s = \gamma^2 \overline{E}_s$ -------------------- (5)

This is the wave equation for electric field intensity. Similarly for magnetic field intensity, the wave equation can be written as

$$\overline{\nabla}^2 \overline{H}_s = \gamma^2 \overline{H}_s\,.$$

Q.118 Find the magnetic field at a distance r from a long straight wire carrying a steady current I. **(5)**

Ans: Figure shows a long straight wise on z axis carrying a current I.



Consider any point 1(o,o,z) on the z axis at which a differential length $\overline{d}_L = d_z \overline{a}_z$ is located. The point at which $\overline{H}$ has to be calculated is z(r,φ,z`)

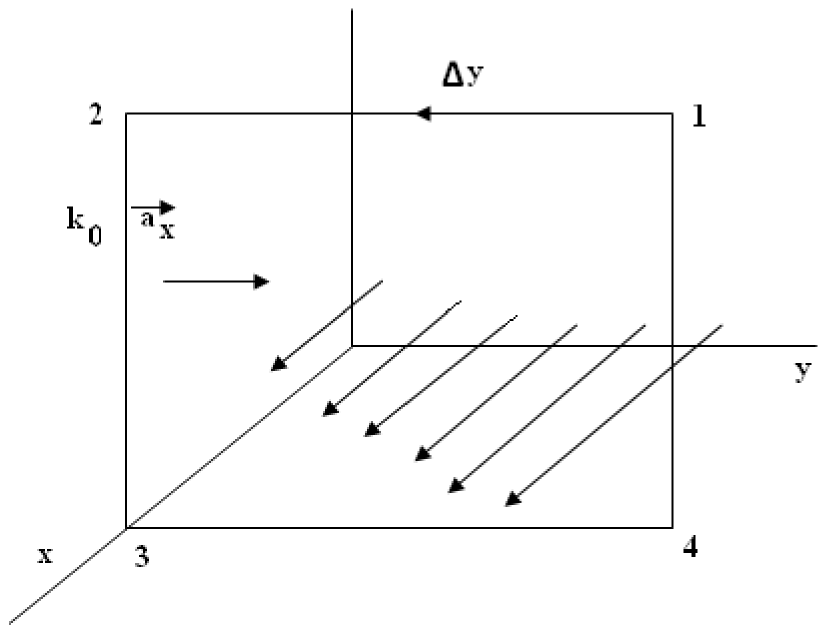
$$\overline{R}_{12} = \overline{r}_2 - \overline{r}_1$$
$$= \left(\gamma \overline{a}_p + Z'\overline{a}_z\right) - Z\overline{a}_z$$
$$= \gamma \overline{a}_p + \left(Z' - Z\right)a_z)$$
$$d = \left|\overline{R}_{12}\right|$$
$$= \sqrt{\gamma^2 + \left(Z' - Z\right)^2}$$
$$\overline{d}H = \frac{Id_z\overline{a}_z \times \left[\gamma\overline{\upsilon} + \left(Z' - Z\right)\overline{a}_z\right]}{4\pi\left[\gamma^2 + \left(Z' - Z\right)^2\right]^{3/2}}$$
$$\therefore \quad H = \int_{-\infty}^{\infty} \frac{Id_z[\gamma a\varphi]}{4\pi\left[\gamma^2 + \left(Z' - Z\right)^2\right]^{3/2}}$$
$$= \frac{I\rho\overline{a}_\varphi}{4\pi}\left[-\frac{1}{\gamma^2}\frac{Z' - Z}{\sqrt{\gamma^2 + \left(Z' - Z\right)^2}}\right]_{-\infty}^{\infty}$$
$$H = \frac{I}{2\pi r}\overline{a}_\varphi .$$

Q.119 Find the magnetic field due to an infinite uniform surface current $k\hat{i}$ covering the x – y plane. **(5)**

Ans: Let us assume $\overline{K} = K_0\overline{a}_x \; A/m$ in x-y plane.

Note: as current is in $\overline{a}_x$ direction *x* component of *H* is absent.

Let the point at which H is to be calculated lies above the Z=0 plane. Thus to symmetrical distribution of current, Z component of H will get cancel out and only Y-component (H_y) will be present.



$\therefore$ Amperes circuit law

$\oint H.dl = I$ to the closed part 1-2-3-4-1.

Hence

$$-H_{y,1-2}\Delta y + H_{y,3-4}\Delta y = K_0\Delta y$$

Since

$$-H_{y,3-4} = H_{y,1-2} \text{ ; we have}$$

$$-H_{y,1-2}\Delta y - H_{y,1-2}\Delta y = K_0\Delta y$$

$$-2H_{y,1-2} = K_0$$

Hence

$$H_{y,1-2} = -K_0/2$$

Thus

$$H = -\frac{K_0}{2}\bar{a}_y$$

In general $\overline{H} = \dfrac{\overline{K}\times\bar{a}_N}{2}$

Where $\bar{a}_N$ is a unit vector perpendicular to the plane in which surface current density lies and directed towards the point at which $\overline{H}$ is desired.

Q.120 Evaluate $\oint_S \overline{B}\cdot d\bar{s}$ for a cubical box bounded by x = 0 & x = 1, y = 0 & y = 1, z = 0 & z = 1 and the magnetic field is given by $\vec{B} = (x+2)\hat{a}_x + (1-3y)\hat{a}_y + 2z\hat{a}_z$. **(6)**

Ans: The six surfaces are located by x = 0, x = 1, y = 0, y = 1 and z = 0, z =1

$$\int_{x=0}\overline{B}.ds+\int_{x=1}B.ds+\int_{y=0}B.ds+\int_{y=1}B.ds+\int_{z=0}B.ds+\int_{z=1}B.ds$$

$$= \int_0^1\int_0^1 \underset{(x=0)}{-(x+2)dydz} + \int_0^1\int_0^1 \underset{(x=1)}{(x+2)dydz} +$$

$$\int_0^1\int_0^1 \underset{(y=0)}{-(1-3y)dxdz} + \int_0^1\int_0^1 \underset{(y=1)}{(1-3y)dxdz} +$$

$$\int_0^1\int_0^1 \underset{(z=0)}{-2zdxdy} + \int_0^1\int_0^1 \underset{(z=1)}{2zdxdy}$$

$$= -2[y]_0^1[z]_0^1 + 3[y]_0^1[z]_0^1 +$$

$$-1[x]_0^1[z]_0^1 + (-2)[x]_0^1[z]_0^1 +$$

$$0 \quad + 2[x]_0^1[y]_0^1$$

= -2 +3 + (-1) + (-2) +2

= 0.

Q.121 An electromagnetic wave travel in free space with the electric component $E_S = 100e^{j(0.866y+0.5z)}\,\hat{a}_x$ V/m. Determine

(i) ω and λ

(ii) the magnetic field component.

(iii) time average power in wave. **(3+3+4)**

Ans:

(i) $E_s = 100e^{j(0.866y+0.5z)}\overline{a}_x\,V/m = E_0 e^{j\vec{k}.\vec{r}}$

$$\therefore \quad k = \sqrt{\left(\sqrt{3}/2\right)^2 + \left(1/2\right)^2} = 1$$

But in free space, $k = \beta = \omega\sqrt{\mu_0\varepsilon_0} = \dfrac{\omega}{c}$

$$\therefore \quad \omega = kc = 1\times 3\times 10^8 \text{ rad/sec}$$

$$= 3\times 10^8 \text{ rad/sec.}$$

$$\lambda = \frac{2\pi}{k} \qquad \because \quad k = 1$$

$$= 2\pi = 6.28 \text{ m.}$$

(ii) $H_s = \dfrac{1}{\mu\omega}\vec{k}\times\vec{E}_s$

$$= \frac{(0.866\overline{a}_y + 0.5\overline{a}_z)\times 100\overline{a}_x e^{j\vec{k}.\vec{r}}}{4\pi\times 10^{-7}\times 3\times 10^8}$$

$$= \frac{\left(0.5\overline{a}_y - 0.866\overline{a}_z\right)100e^{j(.866\overline{y}+0.5\overline{z})}}{120\pi}$$

$$= \left(0.1327\overline{a}_y - 0.229\overline{a}_z\right)e^{j(0.866\overline{y}+0.5\overline{z})}$$

$$= 132.7\overline{a}_y - 229\overline{a}_z e^{j(0.866\overline{y}+0.5\overline{z})}\ mA/m$$

(iii) $P_{av} = \frac{E_0^{\ 2}}{2\eta}\hat{k}$

$= \frac{(100)^2}{2\times 120\pi}\left(0.866\overline{y} + 0.5\overline{z}\right)$

$= 13.26\left(0.866\overline{y} + 0.5\overline{z}\right)$

$= 11.5\overline{y} + 6.63\overline{z}\ w/m^2$.

Q.122 In a loss less medium for which $\eta = 60\pi$, $\mu_0 = 1$ and H = 0.1 Cos $(\omega t - z)$ $\hat{a}_x$ +0.5 Sin $(\omega t - z)\hat{a}_y$. Calculate electric field intensity. **(6)**

Ans: Let $H = H_x\overline{a}_x + H_y\overline{a}_y + H_z\overline{a}_z$

$= 0.1\cos(\omega t - z)\overline{a}_x + 0.5\sin(\omega t - z)\overline{a}_y$

Where $H_x = 0.1\cos(\omega t - z)$

$H_y = 0.5\sin(\omega t - z)$

$H_z = 0$

Now $\nabla\times\overline{H} = \sigma\overline{E} + \frac{\partial\overline{D}}{\partial t}$

$\because\ \sigma = 0\ \ \vec{E} = \frac{1}{\varepsilon}\int\nabla\times\overline{H}dt$

$\nabla\times\overline{H} = -\frac{\partial H_y}{\partial z}\overrightarrow{a_x} + \frac{\partial H_x}{\partial z}\overrightarrow{a_y}$

$= 0.5\cos(\omega t - z)\overline{a}_x + 0.1\sin(\omega t - z)\overline{a}_y$

$\vec{E} = \frac{1}{\varepsilon}\int(\nabla\times H)dt$

$= \frac{1}{\varepsilon}\int\left[0.5\cos(\omega t - z)\overrightarrow{a_x}dt + 0.1\sin(\omega t - z)\overline{a_y}dt\right]$

$= \frac{1}{\varepsilon}\left[\frac{0.5\sin(\omega t - z)\overrightarrow{a_x}}{\omega} + \frac{(-0.1)\cos(\omega t - z)a_y}{\omega}\right]$

$= \frac{0.5}{\varepsilon\omega}\sin(\omega t - z)a_x - \frac{0.1}{\varepsilon\omega}\cos(\omega t - z)\overline{a_y}$

Where

$\eta = \sqrt{\mu/\varepsilon}$

$= \frac{120\pi\sqrt{1}}{\sqrt{\varepsilon_r}}$

$60\pi = \frac{120\pi}{\sqrt{\varepsilon_r}} \rightarrow \sqrt{\varepsilon_r} = 2$

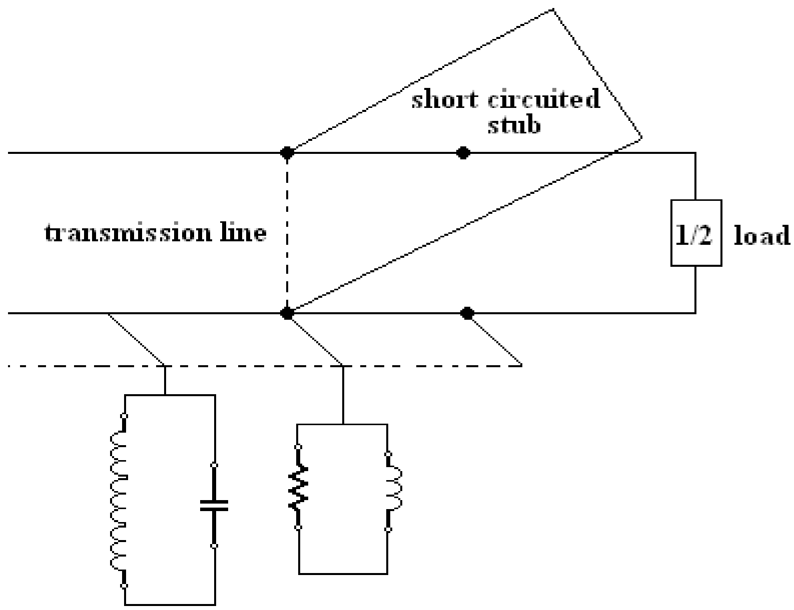
$\varepsilon_r = 4$

$$\omega = \frac{\beta c}{\sqrt{\mu_r \varepsilon_r}} = \frac{3\times10^8\times1}{\sqrt{1\times4}}$$

$$= \frac{3}{2}\times10^8\ rad/s = 1.5\times10^8\ rad/s\ .$$

Q.123 What is stub matching? Outline the solution for the single stub matching problem. **(8)**

Ans: Stub Matching : when sections of open circuited or short circuited line known as stub is connected in shunt with the main line at a certain point or points to effect the matching, the matching is known as stub matching.



SINGLE STUB MATCHING: The principal element of the transformer shown in fig. is a short circuit section of line where open and connected to the main line at a particular distance from the load and, where the input conductance at that point is equal to the characteristic conductance of the line, and the stub length is adjusted to provide a susceptance which is equal in value but opposite in sign, to the input susceptance of the main line at that point of attachment is zero. The combination of stub which is equal to the characteristic conductance of the line i.e. main length of H.F. transmission line will be matched.

Q.124 A 30m long loss less transmission line with $Z_o = 50\Omega$ operating at 2 MHz is terminated with a load $Z_L = 60 + j40\Omega$. If $v = 0.6\ c$ on the line, find

a. the reflection coefficient, (K).

b. the voltage standing wave ratio, (S). **(4+4)**

Ans:

a. The reflection coefficient (K)

$$K = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{60 + j40 - 50}{60 + j40 + 50}$$

$$= \frac{10 + j40}{110 + j40}$$

$$= 0.3523\angle 56^0$$

b. VSWR(S)

$$S = \frac{1+K}{1-K}$$

$$= 2.088$$

Q.125 Discuss the propagation characteristics of a rectangular wave guide propagating under TM mode and hence explain propagation constant, guide wavelength and cut off frequency. **(10)**

Ans: The propagation constant for TM_{mn} mode can be written as

$$\rho = \sqrt{\left(\frac{m\Pi}{a}\right) + \left(\frac{n\pi}{b}\right) - \omega^2 \mu \in}$$

At low frequencies, $\omega^2 \mu \in$ is small as ω is comparatively low. As such ρ will be a real no.

$\rho = \alpha + j\beta$

Thus α = attenuation constant is real

So β = phase constant = 0.

This (β = 0) represents that there is no phase shift along the rectangular wave guide. It means there can be no wave motion along the rectangular wave guides at low frequencies, a stage is reached when ρ becomes zero. This particular value of ω is critical frequency ω_c and is known as cut off frequency. This is given by

$$f_c = \frac{v_\rho}{2ab}\sqrt{m^2 b^2 + h^2 a^2}$$

$$v_\rho = \frac{1}{\sqrt{\mu\varepsilon}} = \text{phase velocity}$$

The corresponding cut off wave length below which wave propagation does occur

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

The rectangular wave guide behaves as a high pass filter because it allows to work only after a higher critical frequency.

Q.126 A rectangular wave guide has the following characteristics: b = 1.5 cm, a = 3 cm, $\mu_r = 1$, $\in_r = 2.25$

a. Calculate the cutoff frequency for TE_{10}, TE_{20} and TM_{11} modes.

b. Calculate the guide wavelength and characteristic impedance, Z_o at 4.0 GHz for TE_{10} modes. **(6)**

Ans:

a. Cut off frequency for TE & TM mode

TE_{10} **Mode:**

m = 1 n = 0

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{\upsilon\rho}{2a} = \frac{c}{2a}$$

$$= \frac{3\times10^8}{2\times3\times10^{-2}} = 0.5\times10^{10}\ Hz$$

T_{20} **Mode:**

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{2\pi}{a}\right)^2} = \frac{c}{a}$$

$$= \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\frac{(2\pi)^2}{(3\times10^{-2})^2}} = 10^{10}\ Hz$$

TM_{11} **Mode:**

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi}{3\times10^{-2}}\right)^2 + \left(\frac{\pi}{1.5\times10^{-2}}\right)^2}$$

$$= 2.02\times10^{10}\ Hz$$

b. TE_{10}:

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{1}{3\times10^{-2}}\right)^2}} = 2\times\left(3\times10^{-2}\right) = 6\times10^{-2}\,m = 6cm$$

$$\lambda = \frac{\upsilon\rho}{\xi} = \frac{3\times10^8\times10^2}{4\times10^9} = \frac{300}{40}cm$$

$$\eta_{TE_{10}} = 120\pi.\frac{X_g}{\lambda}$$

For TE_{10} mode $\lambda_g = 0.8757\ \lambda_c$

$$\lambda_g = 0.8757\times6$$

$$= 5.2542 \text{ cm.}$$

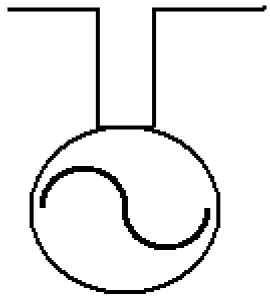
$$\eta_{TE_{10}} = 120\pi\times\frac{5.25}{300}\times40$$

$$= 94.10\pi .$$

Q.127 Establish the expression for normalised E-field and H-field pattern for a half wave dipole. **(4+4)**

Ans:THE HALF WAVE DIPOLE (HERTZ ANTENNA)

The dipole antenna dates back to the early RF experiments of Heinrich Hertz in the late 19th century. It consists of a conductor that is broken in the centre so that RF power can be applied to it. one can think of the half wave dipole as an open circuited transmission line that has been spread out, so that the transmission line can radiate a signal into space.



A dipole can be any length, but it most commonly is just under ½ wavelength log. A dipole with this length, known as a resonant or half wave dipole, has an input impedance that is purely resistive and lies between 30 and 80 ohms, which provides a good match to commercially available 50 ohms coaxial cables as well as commercial transmitters and receivers, most of which have 50 ohms output and input impedances. The length of a dipole can be approximately determined from the following formula:

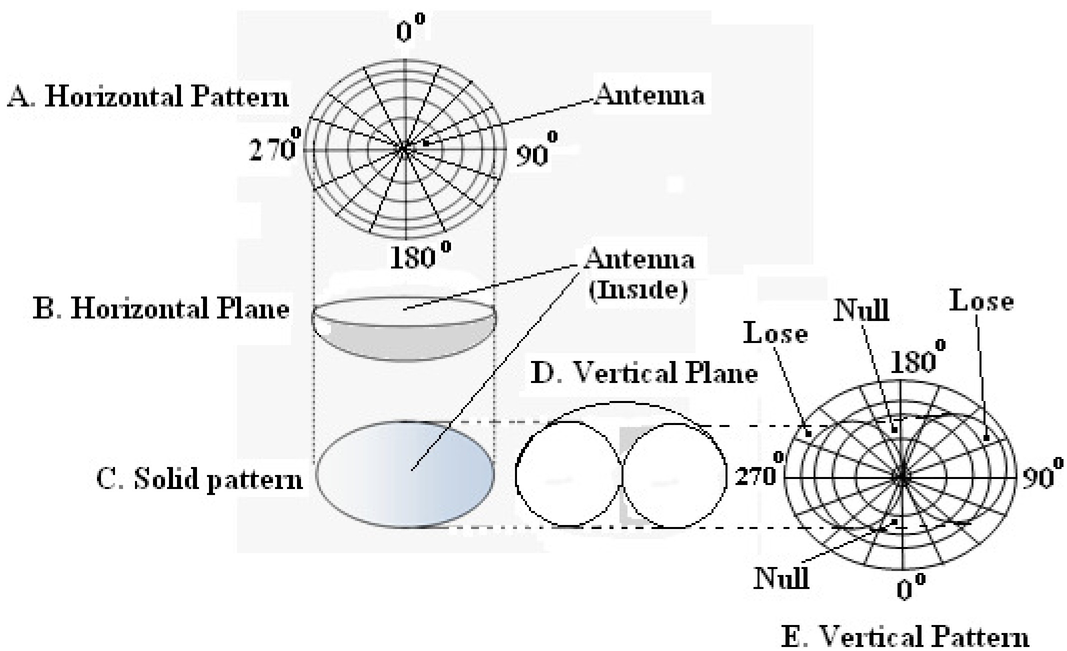
$l = 468/f$

Where:

l is the length in feet and

f is the frequency in MHz.

The radiation pattern of a $\lambda/2$ dipole in free space is shown below



Q.128 What is the criterion for a material to be a good conductor? **(2)**

Ans: Refer section 5.**4**, Page No 282, 283 of "Elements of engineering Electromagnetics" .