TYPICAL QUESTIONS & ANSWERS

PART I

OBJECTIVE TYPE QUESTIONS

Each Question carries 2 marks.

Choose correct or the best alternative in the following:

Q.1 The poles of the impedance Z(s) for the network shown in Fig.1 below will be real and coincident if

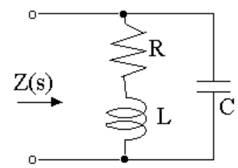


Fig.1

(A) $R = 2\sqrt{\frac{L}{C}}$. **(B)** $R = 4\sqrt{\frac{L}{C}}$.

(C) $R = \frac{1}{2}\sqrt{\frac{L}{C}}$. **(D)** $R = 2\sqrt{\frac{C}{L}}$.

Ans: A

The impedance Z(s) for the network is

$$Z(s) = \frac{(R+SL)\times\frac{1}{SC}}{(R+SL)+\frac{1}{SC}}$$

$$= \frac{R+SL}{SRC+S^2LC+1}$$

OR

$$Z(s) = \frac{S+\frac{R}{L}}{S^2+\frac{R}{L}S+\frac{1}{LC}}$$

The network function has zeros at

$S = -\frac{R}{L}$ & $S = \infty$ and

Poles at $S = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$

$$= -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$

The poles will coincidence, if $R = 2\sqrt{\frac{L}{C}}$

i.e. $S = -\frac{R}{2L} \pm \sqrt{\frac{1}{LC}-\frac{1}{4L^2}\left(2\sqrt{\frac{L}{C}}\right)^2}$

$$= -\frac{R}{2L} \pm \sqrt{\frac{1}{LC} - \frac{1\left(4.\frac{L}{C}\right)}{4L^2}}$$

$$= -\frac{R}{2L} \pm \sqrt{\frac{1}{LC} - \frac{1}{LC}}$$

$$S = -\frac{R}{2L} \pm 0$$

So the Poles will coincidence if $R = 2\sqrt{\frac{L}{C}}$

Q.2 The network shown in part a has zeros at

(A) s = 0 and s = ∞. **(B)** s = 0 and s = $-R/L$.

(C) s = ∞ and s = $-R/L$. **(D)** s = ∞ and s = $-\frac{1}{CR}$.

Ans: C

Q.3 Of the two methods of loop and node variable analysis

(A) loop analysis is always preferable.
(B) node analysis is always preferable.
(C) there is nothing to choose between them.
(D) loop analysis may be preferable in some situations while node analysis may be preferable in other situations.

Ans: B

Q.4 In a double tuned circuit, consisting of two magnetically coupled, identical high-Q tuned circuits, at the resonance frequency of either circuit, the amplitude response has

(A) a peak, always. **(B)** a dip, always.
(C) either a peak or a dip. **(D)** neither a peak nor a dip.

Ans: A

This is because Quality Factor $Q = \frac{fr}{B.W}$ where fr is the resonant frequency. When Q is high. Fr is mole.

Q.5 In a series RLC circuit with output taken across C, the poles of the transfer function are located at $-\alpha \pm j\beta$. The frequency of maximum response is given by

(A) $\sqrt{\beta^2 - \alpha^2}$. **(B)** $\sqrt{\alpha^2 - \beta^2}$.

(C) $\sqrt{\beta^2 + \alpha^2}$. **(D)** $\sqrt{\alpha\beta}$.

Ans: A

Q.6 A low-pass filter (LPF) with cutoff at 1 r/s is to be transformed to a band-stop filter having null response at ω_0 and cutoff frequencies at ω_1 and ω_2 $(\omega_2 > \omega_1)$. The complex frequency variable of the LPF is to be replaced by

(A) $\dfrac{s^2+\omega_0^2}{(\omega_2-\omega_1)s}$. **(B)** $\dfrac{(\omega_2-\omega_1)s}{s^2+\omega_0^2}$.

(C) $\left(\dfrac{s}{\omega_1}+\dfrac{\omega_2}{s}\right)\omega_0$. **(D)** $\left(\dfrac{s}{\omega_2}+\dfrac{\omega_1}{s}\right)\omega_0$.

Ans: C

Q.7 For an ideal transformer,

(A) both z and y parameters exist.
(B) neither z nor y parameters exist.
(C) z-parameters exist, but not the y-parameters.
(D) y-parameters exist, but not the z-parameters.

Ans: A

Q.8 The following is a positive real function

(A) $\dfrac{(s+1)(s+2)}{(s^2+1)^2}$. **(B)** $\dfrac{(s-1)(s+2)}{s^2+1}$.

(C) $\dfrac{s^4+s^2+1}{(s+1)(s+2)(s+3)}$. **(D)** $\dfrac{(s-1)}{(s^2-1)}$.

Ans: C

Q.9 The free response of RL and RC series networks having a time constant τ is of the form:

(A) $A+Be^{-\frac{t}{\tau}}$ **(B)** $Ae^{-\frac{t}{\tau}}$

(C) $Ae^{-t}+Be^{-\tau}$ **(D)** $(A+Bt)e^{-\frac{t}{\tau}}$

Ans: B

Q.10 A network function can be completely specified by:

(A) Real parts of zeros **(B)** Poles and zeros
(C) Real parts of poles **(D)** Poles, zeros and a scale factor

Ans: D

Q.11 In the complex frequency $s=\sigma+j\omega$, ω has the units of rad/s and σ has the units of:

(A) Hz **(B)** neper/s
(C) rad/s **(D)** rad

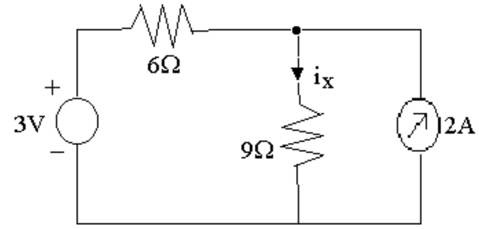
Ans: B

Q.12 The following property relates to LC impedance or admittance functions:

(A) The poles and zeros are simple and lie on the $j\omega$-axis.
(B) There must be either a zero or a pole at origin and infinity.
(C) The highest (or lowest) powers of numerator or denominator differ by unity.
(D) All of the above.

Ans: D

Q.13 The current i_x in the network is:



(A) 1A **(B)** $\frac{1}{2}$A
(C) $\frac{1}{3}$A **(D)** $\frac{4}{5}$A

Ans: A

With the voltage source of 3v only, the current $i_x^{'}$ is

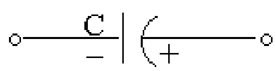
$$i_x^{'} = \frac{3}{6+9} = \frac{3}{15}A$$

With the current source of 2A only, the current $i_x^{''}$ is

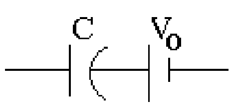
$$i_x^{''} = 2\times\frac{6}{6+9} = 2\times\frac{6}{15} = \frac{12}{15}A.$$

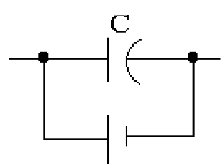
Thus, by Superposition principle, the current i_x through the resistance 9Ω is

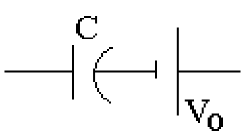
$$i_x = i_x^{'} + i_x^{''} = \frac{3}{15} + \frac{12}{15} = \frac{3+12}{15} = \frac{15}{15}A = 1A$$

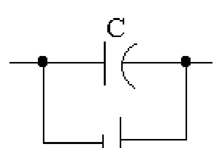
Q.14 The equivalent circuit of the capacitor  shown is

$$V_0 = \frac{q_0}{C}$$

(A)


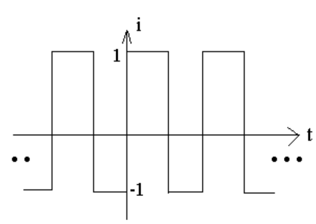
(B)


(C)


(D)


Ans: C

Q.15 The value of $\left(\frac{I_{rms}}{I_{max}}\right)$ for the wave form shown is



(A) $\sqrt{2}$ **(B)** 1.11
(C) 1 **(D)** $1/\sqrt{2}$

Ans: D

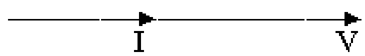
$I_{rms} = \dfrac{I_{max}}{\sqrt{2}}$

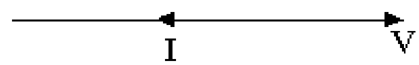
For the given wave form $I_{max} = 1A$

The value of $\left(\dfrac{I_{rms}}{I_{max}}\right)$ *is* $\left(\dfrac{I_{max}}{\sqrt{2}} \cdot \dfrac{1}{I_{max}}\right)$

$= \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{2} = \dfrac{1}{\sqrt{2}}$.

Q.16 The phasor diagram for an ideal inductance having current I through it and voltage V across it is :

(A) 

(B) 

(C) 

(D) 

Ans: D

Q.17 If the impulse response is realisable by delaying it appropriately and is bounded for bounded excitation, then the system is said to be :

(A) causal and stable **(B)** causal but not stable
(C) noncausal but stable **(D)** noncausal, not stable

Ans: C

Q.18 In any lumped network with elements in b branches, $\sum_{k=1}^{b} \upsilon_k(t) . i_k(t) = 0,$ for all t, holds good according to:

(A) Norton's theorem. **(B)** Thevenin's theorem.
(C) Millman's theorem. **(D)** Tellegen's theorem.

Ans: D

Q.19 Superposition theorem is applicable only to networks that are:

(A) linear. **(B)** nonlinear.
(C) time-invariant. **(D)** passive.

Ans: A

Q.20 In the solution of network differential equations, the constants in the complementary function have to be evaluated from the initial conditions, and then the particular integral is to be added. This procedure is

(A) correct.

(B) incorrect.
(C) the one to be followed for finding the natural response.
(D) the one to be followed for finding the natural and forced responses.

Ans: A

Q.21 Two voltage sources connected in parallel, as shown in the Fig.1, must satisfy the conditions:

(A) $v_1 \neq v_2$ but $r_1 = r_2$.   **(B)** $v_1 = v_2$, $r_1 \neq r_2$.
(C) $v_1 = v_2, r_1 = r_2$.   **(D)** $r_1 \neq 0$ or $r_2 \neq 0$ if $v_1 \neq v_2$

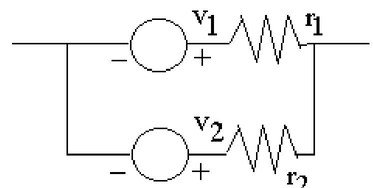


Fig.1

Ans: D

Q.22 The rms value of the a-c voltage $v(t) = 200 \sin 314\,t$ is:
(A) 200 V.   **(B)** 314 V.
(C) 157.23 V.   **(D)** 141.42 V.

Ans: D

$$V_{RMS} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = \frac{200}{1.44} = 141.42V$$

Q.23 In a 2-terminal network containing at least one inductor and one capacitor, resonance condition exists only when the input impedance of the network is:
(A) purely resistive.   **(B)** purely reactive.
(C) finite.   **(D)** infinite.

Ans: A

Q.24 If a network function has zeros only in the left-half of the s-plane, then it is said to be
(A) a stable function.   **(B)** a non-minimum phase function.
(C) a minimum phase function.   **(D)** an all-pass function.

Ans: C

Q.25 Zeros in the right half of the s-plane are possible only for
(A) d.p. impedance functions.   **(B)** d.p. admittance functions.
(C) d.p. impedance as well as admittance functions.   **(D)** transfer functions.

Ans: D

Q.26 The natural response of a network is of the form $\left(A_1 + A_2 t + A_3 t^2\right)e^{-t}$. The network must have repeated poles at s = 1 with multiplicity
(A) 5   **(B)** 4
(C) 3   **(D)** 2

Ans: D

Q.27 The mutual inductance M associated with the two coupled inductances L_1 and L_2 is related to the coefficient of coupling K as follows:

(A) $M = K\sqrt{L_1 L_2}$ **(B)** $M = \dfrac{K}{\sqrt{L_1 L_2}}$

(C) $M = \dfrac{K}{L_1 L_2}$ **(D)** $M = KL_1 L_2$

Ans: A

Q.28 An L-C impedance or admittance function:

(A) has simple poles and zeros in the left half of the s-plane.
(B) has no zero or pole at the origin or infinity.
(C) is an odd rational function.
(D) has all poles on the negative real axis of the s-plane.

Ans: A

Q.29 The Thevenin equivalent resistance R_{th} for the given network is equal to

(A) 2Ω.
(B) 3Ω.
(C) 4Ω.
(D) 5Ω.

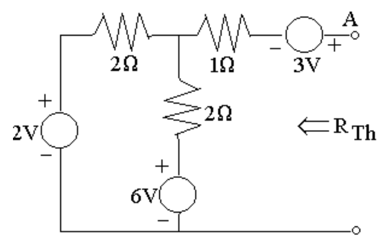


Fig.1

Ans: A

$$R_{TH} = (2\|2) + 1 = \frac{2\times 2}{2+2} + 1 = 2\Omega$$

Q.30 The Laplace-transformed equivalent of a given network will have $\frac{5}{8}$ F capacitor replaced by

(A) $\dfrac{5}{8s}$. **(B)** $\dfrac{5s}{8}$.

(C) $\dfrac{8s}{5}$. **(D)** $\dfrac{8}{5s}$.

Ans: D

Laplace Transform of the capacitor C is $\dfrac{1}{CS} = \dfrac{1}{\frac{5S}{8}} = \dfrac{8}{5S}$

Q.31 A network function contains only poles whose real-parts are zero or negative. The network is

(A) always stable.
(B) stable, if the $j\omega$-axis poles are simple.
(C) stable, if the $j\omega$-axis poles are at most of multiplicity 2
(D) always unstable.

Ans: B

The network is stable; if the jω-axis poles are simple.

Q.32 Maximum power is delivered from a source of complex impedance Z_S to a connected load of complex impedance Z_L when

(A) $Z_L = Z_S$ **(B)** $|Z_L| = |Z_S|$
(C) $\angle Z_L = \angle Z_S$ **(D)** $Z_L = Z_S$ *

Ans: D

Q.33 The admittance and impedance of the following kind of network have the same properties:
(A) LC **(B)** RL
(C) RC **(D)** RLC

Ans: A

Q.34 The Q-factor (or figure of merit) for an inductor in parallel with a resistance R is given by
(A) $\frac{\omega L}{R}$. **(B)** $\frac{R}{\omega L}$.
(C) ωLR **(D)** $\frac{1}{\omega LR}$.

Ans: A

Q.35 A 2-port network using z-parameter representation is said to be reciprocal if
(A) $z_{11} = z_{22}$. **(B)** $z_{12} = z_{21}$.
(C) $z_{12} = -z_{21}$. **(D)** $z_{11}z_{22} - z_{12}z_{21} = 1$.

Ans: B

Q.36 Two inductors of values L_1 and L_2 are coupled by a mutual inductance M. By inter connection of the two elements, one can obtain a maximum inductance of
(A) $L_1 + L_2 - M$ **(B)** $L_1 + L_2$
(C) $L_1 + L_2 + M$ **(D)** $L_1 + L_2 + 2M$

Ans: D

Q.37 The expression $(s^2 + 2 + 1)(s + 1)$ is
(A) a Butterworth polynomial.
(B) a Chebyshev polynomial.
(C) neither Butterworth nor Chebyshev polynomial.
(D) not a polynomial at all.

Ans: A

Q.38 Both odd and even parts of a Hurwitz polynomial P(s) have roots
(A) in the right-half of s-plane. **(B)** in the left-half of s-plane.
(C) on the σ-axis only. **(D)** on the jω-axis only.

Ans: D

Q.39 The minimum amount of hardware required to make a lowpass filter is

(A) a resistance, a capacitance and an opamp.
(B) a resistance, an inductance and an opamp.
(C) a resistance and a capacitance.
(D) a resistance, a capacitance and an inductance.

Ans: C

A low pass filter consists of resistance and capacitance is shown in Fig

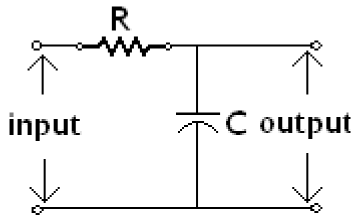


Fig.

Q.40 A system is described by the transfer function $H(s) = \frac{1}{s-1}$. The value of its step response at very large time will be close to

(A) -1 **(B)** 0
(C) 1 **(D)** ∞

Ans: A

Q.41 A network N is to be connected to load of 500 ohms. If the Thevenin's equivalent voltage and Norton's equivalent current of N are 5Volts and 10mA respectively, the current through the load will be

(A) 10mA **(B)** 5mA
(C) 2.5mA **(D)** 1mA

Ans: B

Given that $V_{TH} = 5V$, $I_N = 10mA$ and $R_L = 500\Omega$

Now $R_{TH} = \frac{V_{TH}}{I_N} = \frac{5}{10mA} = 0.5k\Omega$ and

Therefore, the current through the load I_L is

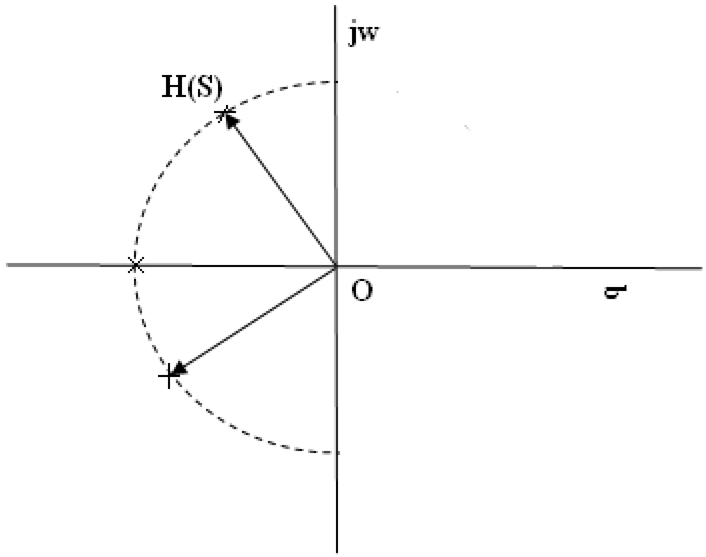
$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{5}{0.5\times10^3 + 500} = \frac{5}{1000} = 5\times10^{-3}$$

Or $I_L = 5mA$

Q.42 A unit impulse voltage is applied to one port network having two linear components. If the current through the network is 0 for t<0 and decays exponentially for *t>0* then the network consists of

(A) R and L in series **(B)** R and L in parallel
(C) R and C in parallel **(D)** R and C in series

Ans: D



Q.43 The two-port matrix of an *n:1* ideal transformer is $\begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$. It describes the transformer in terms of its

(A) *z*-parameters. **(B)** *y*-parameters.
(C) *Chain*-parameters. **(D)** *h*-parameters.

Ans: C
The chain parameters or ABCD parameters for the ideal transformers for the Fig.2 is

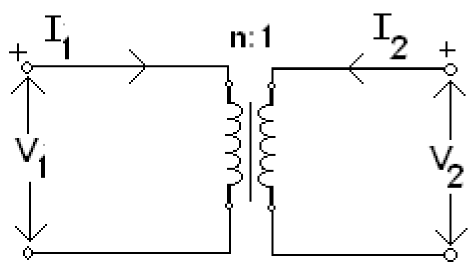


Fig.2

$V_1 = nV_2$ &

$I_1 = \frac{1}{n}(-I_2)$

If we express the above two equations in Matrix form, we have

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} n & o \\ o & \frac{1}{n} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

So that the transmission matrix of the ideal transformer is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} n & o \\ o & \frac{1}{n} \end{bmatrix}$$

Q.44 If $F(s)$ is a positive-real function, then $\mathrm{Ev}\{F(s)\}\big|_{s=j\omega}$

(A) must have a single zero for some value of ω.
(B) must have a double zero for some value of ω.
(C) must not have a zero for any value of ω.
(D) may have any number of zeros at any values of ω but $\mathrm{Ev}\{F(s)\}\big|_{s=j\omega} \geq 0$ for all ω.

Ans: D

Q.45 The poles of a Butterworth polynomial lie on

(A) a parabola. **(B)** a left semicircle.
(C) a right semicircle. **(D)** an ellipse.

Ans: B
The poles of Butterworth polynomial lie on a left semicircle

Q.46 A reciprocal network is described by $z_{21} = \dfrac{s^3}{3s^2+2}$ and $z_{22} = \dfrac{s^3+4s}{3s^2+2}$. Its transmission zeros are located at

(A) $s = 0$ **(B)** $s = \pm j2$
(C) $s = 0$ and at $s = \pm j2$ **(D)** $s = 0$ and at $s = \infty$

Ans: A

Q.47 In order to apply superposition theorem, it is necessary that the network be only

(A) Linear and reciprocal.
(B) Time-invariant and reciprocal.
(C) Linear and time-invariant.
(D) Linear.

Ans: D

Q.48 The Q-factor of a parallel resonance circuit consisting of an inductance of value 1mH, capacitance of value 10^{-5}F and a resistance of 100 ohms is

(A) 1 **(B)** 10
(C) 20π **(D)** 100

Ans: B
The Q-factor of a parallel resonant circuit is

$$Q = R\sqrt{\frac{C}{L}} = 100\sqrt{\frac{10^{-5}}{1\times10^{-3}}} = 100\sqrt{10^{-2}} = \sqrt{10000\times10^{-2}} = \sqrt{100} = 10$$

Q.49 Power in 5Ω resistor is 20W. The resistance R is

(A) 10Ω.
(B) 20Ω.
(C) 16Ω.
(D) 8Ω.

Ans: C

Given that the power in 5Ω resistance is 20w
But the power is given by the relation.
$P = I_1^2 R$

Or $20 = I_1^2 5 \Rightarrow I_1^2 = \frac{20}{5} = 4$

Or $I_1 = \sqrt{4} = 2A$

Therefore, the current through the resistance 5Ω is 2A. Now the current flowing through 20Ω resistance is I_2 and the voltage drop is 4 times more than 5Ω resistance (i.e. $5\Omega \times 4 = 20\Omega$)
Hence the current through 20Ω resistance is I_2 and the voltage drop is 4 times more than 5Ω resistance (i.e. 5Ω x 4 = 20Ω)
Hence the current through 20 Ω reistance is

$I_2 = \frac{2}{4} = 0.5A$

Now the total current $I_T = I_1 + I_2 = 2 + 0.5 = 2.5A$
But the total current in the circuit is

$I_T = \frac{50V}{R_{eff}}$ or $R_{eff} = \frac{50}{I_T} = \frac{50}{2.5} = 20\Omega$

But the parallel combination of 5Ω and 20Ω resistance is

$\frac{5 \times 20}{5 + 20} = \frac{100}{25} = 4\Omega$

Now the resistance R is
4 + R = 20
Or R = 20 – 4 = 16Ω

Q.50 The Thevenin's equivalent circuit to the left of AB in Fig.2 has R_{eq} given by

(A) $\frac{1}{3}\Omega$ **(B)** $\frac{1}{2}\Omega$

(C) $1\,\Omega$ **(D)** $\frac{3}{2}\Omega$

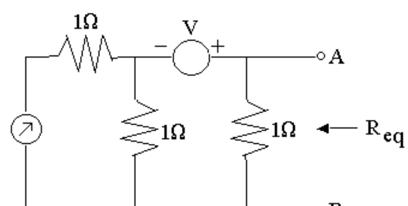


Fig.2

Ans: B

Q.51 The energy stored in a capacitor is

(A) $\frac{1}{2}ci^2$ **(B)** $\frac{1}{2}\frac{1}{c}i^2$

(C) $\frac{1}{2}\frac{v^2}{c}$ **(D)** $\frac{1}{2}cv^2$

Ans: D
The energy stored by the capacitor is

$W = \int_0^t Pdt = \int_0^t VC\frac{dV}{dt}dt$ [∵ Power absorbed by the capacitor P is $P = Vi = VC\frac{dV}{dt}$]

Or $W = \frac{1}{2}CV^2$

Q.52 The Fig.3 shown are equivalent of each other then

(A) $i_g = -\dfrac{v_g}{R_g}$ **(B)** $i_g = \dfrac{v_g}{R_g}$

(C) $i_g = v_g R_g$ **(D)** $i_g = \dfrac{R_g}{v_g}$

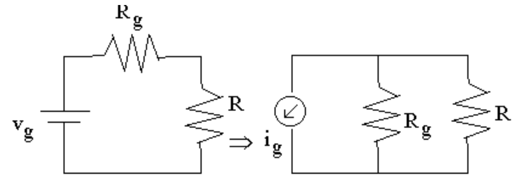


Fig.3

Ans: B

The voltage source V_g in series with resistance R_g is equivalent to the current source $i_g = \dfrac{V_g}{R_g}$ in parallel with the resistance R_g.

Q.53 For the circuit shown in Fig.4, the voltage across the last resistor is V. All resistors are of 1Ω. The V_S is given by

(A) 13V. **(B)** 8V.
(C) 4V. **(D)** 1V.

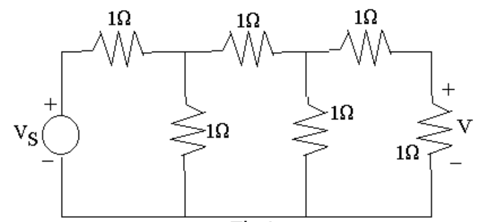


Fig.4

Ans: A

Assume $V_1 = 1V$, so that $I_1 = \dfrac{V_1}{R_1} = \dfrac{1}{1} = 1A$

From Fig.4 $I_2 = I_1 = 1Amp$

And $V_2 = R_2 I_2 = 1\times 1 = 1V$ & $V_3 = V_1 + V_2 = 1+1 = 2V$ & $I_3 = \dfrac{V_3}{R_3} = \dfrac{2}{1} = 2A$

Also $I_4 = I_2 + I_3 = 1+2 = 3$; $V_4 = R_4 I_4 = 3V$; $V_5 = V_3 + V_4 = 2+3 = 5V$

$I_5 = \dfrac{V_5}{R_5} = \dfrac{5}{1} = 5V$; $I_6 = I_4 + I_5 = 3+5 = 8A$

$V_6 = R_6 I_6 = 1\times 8 = 8V$

Now $V_5 = V_5 + V_6 = 8+5 = 13V$

Q.54 In the circuit shown in Fig.5, the switch s is closed at t = 0 then the steady state value of the current is

(A) 1 Amp. **(B)** 2 Amp.

(C) 3 Amp. **(D)** $\dfrac{4}{3}$ Amp.

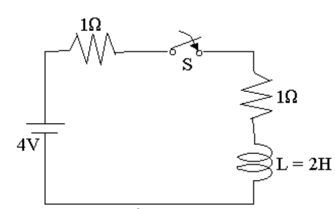


Fig.5

Ans: B

The equivalent circuit at steady state after closing the switch is shown in fig.5.1

$$i(\infty) = \frac{4}{1+1} = \frac{4}{2} = 2Amp$$

Q.55 The z parameters of the network shown in Fig.6 is

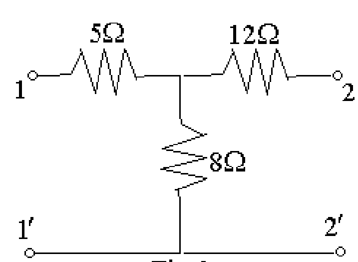


Fig.6

(A) $\begin{bmatrix} 5 & 8 \\ 8 & 20 \end{bmatrix}$ **(B)** $\begin{bmatrix} 13 & 8 \\ 8 & 20 \end{bmatrix}$

(C) $\begin{bmatrix} 8 & 20 \\ 13 & 12 \end{bmatrix}$ **(D)** $\begin{bmatrix} 5 & 8 \\ 8 & 12 \end{bmatrix}$

Ans: B

$Z_{11} = Z_A + Z_C = 5 + 8 = 13\Omega$

$Z_{21} = Z_{12} = Z_C = 8\Omega$

$Z_{22} = Z_B + Z_C = 12 + 8 = 20\Omega$

Q.56 For the pure reactive network the following condition to be satisfied

(A) $M_1(J\omega)M_2(J\omega) + N_2(J\omega)N_1(J\omega) = 0$
(B) $M_1(J\omega)N_1(J\omega) - N_2(J\omega)M_2(J\omega) = 0$
(C) $M_1(J\omega)M_2(J\omega) - N_1(J\omega)N_2(J\omega) = 0$
(D) $M_1(J\omega)N_2(J\omega) - N_1(J\omega)M_2(J\omega) = 0$

Where $M_1(J\omega)$ & $M_2(J\omega)$ even part of the numerator and denominator and N_1 N_2 are odd parts of the numerator & denominator of the network function.

Ans: C

Q.57 The network has a network function $Z(s) = \frac{s(s+2)}{(s+3)(s+4)}$. It is

(A) not a positive real function. **(B)** RL network.
(C) RC network. **(D)** LC network.

Ans: A

The given network function Z(s) is

$Z(s) = \frac{s(s+2)}{(s+3)(s+4)} = \frac{s^2+2s}{s^2+7s+2}$

This equation is in the form $\frac{s^2+a_1s+a_0}{s^2+b_1s+b_0}$

Where $a_1 = 2$; $a_0 = 0$; $b_1 = 7$ & $b_0 = 2$. The first condition that the equation to be positive real function is $a_0a_1, b_1 \,\&\, b_0 > 0$.

Here a_0 is not greater than zero.

Q.58 The Q factor for an inductor L in series with a resistance R is given by

(A) $\frac{\omega L}{R}$ **(B)** $\frac{R}{\omega L}$

(C) ωLR **(D)** $\frac{1}{\omega LR}$

Ans: A

Quality factor of the coil $Q = 2\pi \times \frac{\textit{Maximum energy stored}}{\textit{energy dissipated / cycle}}$

$$Q = 2\pi \times \frac{\frac{1}{2}LI^2}{\frac{I^2R}{2} \times \frac{1}{f}} = \frac{2\pi fL}{R} = \frac{\omega L}{R}$$

Q.59 The value of $z_{22}\ (\Omega)$ for the circuit of Fig.1 is:

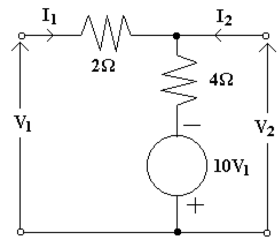


Fig.1

(A) $\frac{4}{11}$ **(B)** $\frac{11}{4}$

(C) $\frac{4}{9}$ **(D)** $\frac{9}{4}$

Ans: A

To find $Z_{22}(\Omega)$ in Fig.1. Open-circuit V_1 i.e., when V_1 = open and $I_1 = 0$, so that

$V_2 = 4I_2 - 10V_2$

Or $4I_2 = 10V_2 + V_2$

Or $4I_2 = 11V_2$

Or $I_2 = \frac{11}{4}V_2$

Now $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{4}{11}\Omega$

Q.60 A possible tree of the topological equivalent of the network of Fig.2 is

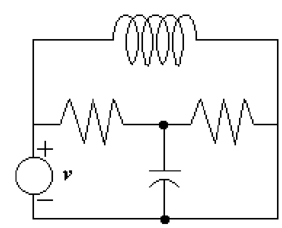
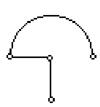
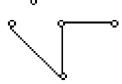


Fig.2

(A) 

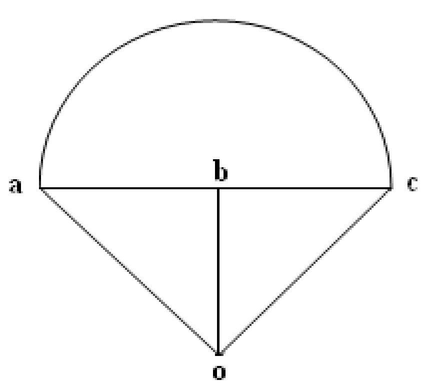
(B) 

(C) Neither **(A)** nor **(B)**

(D) Both **(A)** and **(B)**

Ans: C

Topology equivalent for the given network is:



Q.61 Given $F(s)=\dfrac{5s+3}{s(s+1)}$ then $f(\infty)$ is

(A) 1 **(B)** 2
(C) 0 **(D)** 3

Ans: D

$$f(\infty)=\lim_{s\to 0} sF(s)=\lim_{s\to 0} s\left[\frac{5s+3}{s(s+1)}\right]$$

$$\lim_{s\to 0}\left[\frac{5s+3}{s+1}\right]=\frac{5\times 0+3}{0+1}=\frac{3}{1}=3$$

Q.62 The two-port matrix of an *n:1* ideal transformer is $\begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$. It describes the transformer in terms of its

(A) *z*-parameters. **(B)** *y*-parameters.
(C) *Chain*-parameters. **(D)** *h*-parameters.

Ans: C

Q.63 The value of i_x(A) (in the circuit of Fig.3) is

(A) 1 **(B)** 2
(C) 3 **(D)** 4

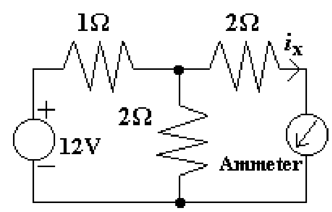


Fig. 3

Ans: C

The current i_x' for the current of Fig.3 is

$$i_x'=\frac{12}{1+(2\parallel 2)}=\frac{12}{1+\frac{4}{4}}=\frac{12}{2}=6$$

Hence the current i_x for the given circuit is

$$i_x=i_x'.\frac{2}{2+2}=6.\frac{2}{4}=6.\frac{1}{2}=3$$

Q.64 To effect maximum power transfer to the load, $Z_L(\Omega)$ in Fig.4 should be

(A) 6
(B) 4
(C) $7.211\angle 33.69^\circ$
(D) $7.211\angle -33.69^\circ$

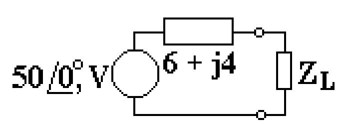


Fig.4

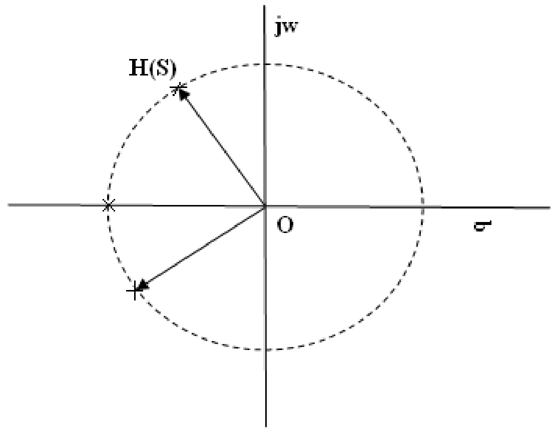
Ans: A

Maximum Power will be transferred from source to the load for the circuit of Fig..4, if $R_L=R_S=6\Omega$ (i.e., the load resistance should be equivalent to the source resistance)

Q.65 The poles of a stable Butter worth polynomial lie on

(A) parabola **(B)** left semicircle
(C) right semicircle **(D)** an ellipse

Ans: B



Q.66 If $F_1(s)$ and $F_2(s)$ are p.r., then which of the following are p.r. (Positive Real)?

(A) $\frac{1}{F_1(s)}$ and $\frac{1}{F_2(s)}$ **(B)** $F_1(s)+F_2(s)$

(C) $\frac{F_1(s)\cdot F_2(s)}{F_1(s)+F_2(s)}$ **(D)** All of these

Ans: D

Q.67 For the pole-zero of Fig.5, the network function is

(A) $\frac{s^2(s+1)}{(s+3)(s+2+j)(s+2-j)}$

(B) $\frac{s^2(s+2+j)(s+2-j)}{(s+1)(s+3)}$

(C) $\frac{(s+1)(s^2+4s+5)}{s^2(s+3)}$

(D) $\frac{s^2(s+3)}{(s+1)(s^2+4s+5)}$

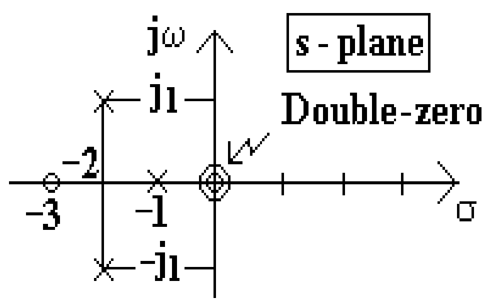


Fig.5

Ans: D

Q.68 For a series R-C circuit excited by a d-c voltage of 10V, and with time-constant τ, s, the voltage across C at time $\mathbf{t} = \tau$ is given by

(A) $10(1-e^{-1})$, V **(B)** $10(1-e)$, V

(C) $10-e^{-1}$, V **(D)** $1-e^{-1}$, V

Ans: A
The voltage across C for a series R-C circuit when excited by a d-c voltage of 10v is
$V_C = 10\left(1-e^{-t/RC}\right)$
Given that the time constant $RC = T$ seconds i.e.

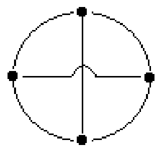
$V_C = 10\left(1 - e^{-t/T}\right)$

Now the voltage across C at time t = T is given by

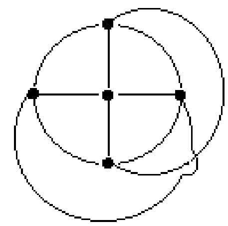
$V_C = 10\left(1 - e^{-T/T}\right) = 10\left(1 - e^{-1}\right)V$

Q.69 Example of a planar graph is

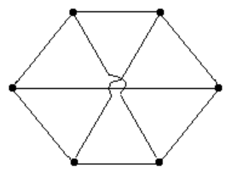
(A)



(B)



(C)



(D) None of these

Ans: A

Q.70 The value of $R_{eq}(\Omega)$ for the circuit of Fig.1 is

(A) 200 **(B)** 800
(C) 600 **(D)** 400

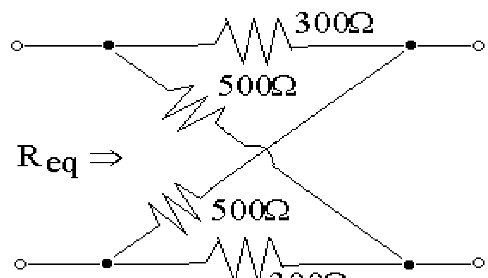


Fig.1

Ans: D

Q.71 A 2 port network using Z parameter representation is said to be reciprocal if

(A) $Z_{11} = Z_{22}$ **(B)** $Z_{12} = Z_{21}$
(C) $Z_{12} = -Z_{21}$ **(D)** $Z_{11}Z_{22} - Z_{12}Z_{21} = 1$

Ans: B

Q.72 The phasor diagram shown in Fig.3 is for a two-element series circuit having

(A) R and C, with tan $\theta = 1.3367$
(B) R and C, with tan $\theta = 4.2635$
(C) R and L, with tan $\theta = 1.1918$
(D) R and L, with tan $\theta = 0.2345$

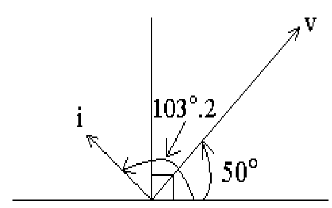


Fig.3

Ans: A

Q.73 The condition for maximum power transfer to the load for Fig.4 is

(A) $R_\ell = R_s$
(B) $X_\ell = -X_s$
(C) $Z_\ell = Z_s^*$
(D) $Z_\ell = Z_s$

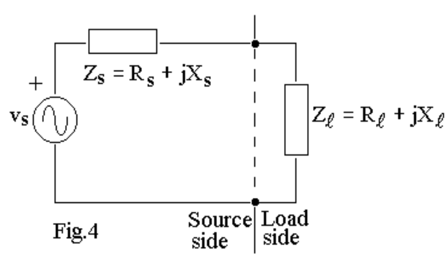


Fig.4

Ans: C

Q.74 The instantaneous power delivered to the 5Ω resistor at t=0 is (Fig.5)

(A) 35W **(B)** 105W
(C) 15W **(D)** 20W

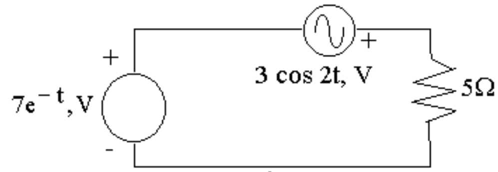


Fig.5

Ans: D

Q.75 Of the following, which one is not a Hurwitz polynomial?

(A) $(s+1)(s^2+2s+3)$ (B) $(s+3)(s^2+s-2)$

(C) $(s^3+3s)\left(1+\frac{2}{s}\right)$ **(D)** $(s+1)(s+2)(s+3)$

Ans: B

Q.76 Which of these is not a positive real function?

(A) $F(s)=Ls(L \rightarrow Inductance)$ **(B)** $F(s)=R(R \rightarrow Resistance)$

(C) $F(s)=\frac{K}{s}(K \rightarrow constant)$ **(D)** $F(s)=\frac{s+1}{s^2+2}$

Ans: D

Q.77 The voltage across the 3Ω resistor e_3 in Fig.6 is :

(A) $6\sin t, v$
(B) $4\sin t, v$
(C) $3\sin t, v$
(D) $12\sin t, v$

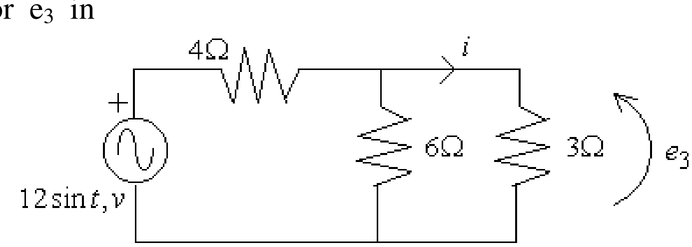


Fig.6

Ans: B

Q.78 A stable system must have

(A) zero or negative real part for poles and zeros.
(B) atleast one pole or zero lying in the right-half s-plane.
(C) positive real part for any pole or zero.
(D) negative real part for all poles and zeros.

Ans: A

PART II

NUMERICALS

Q.1 In the circuit shown in Fig.2 below, it is claimed that $\sum_{k=1}^{6} v_k i_k = 0$. Prove OR disprove. **(7)**

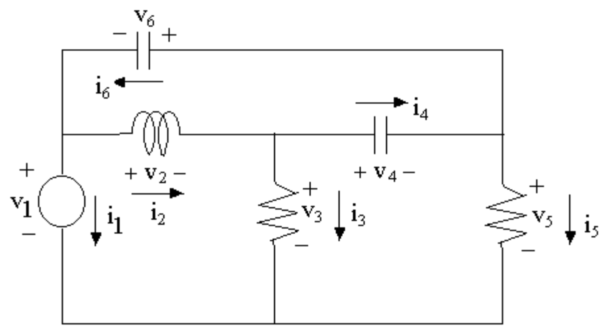


Fig.2

Ans:
The given circuit is shown in Fig.2.1 with their corresponding voltages and currents. In this figure, 'O' as the reference node (or) datum node, and V_A, V_B, V_C be the voltages at nodes A, B, and C respectively with respect to datum O.

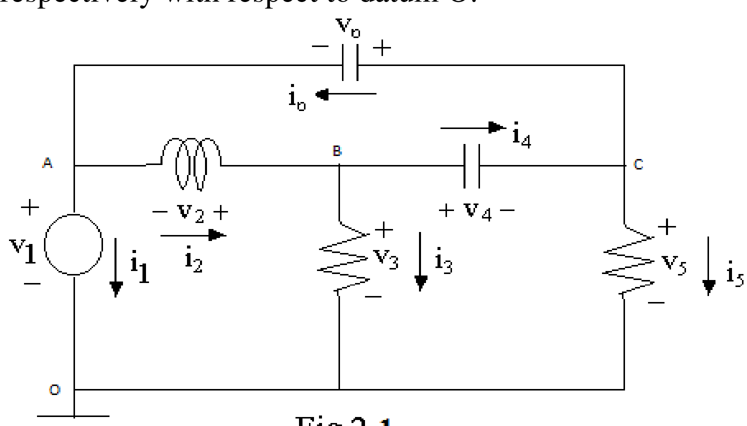


Fig.2.1

By calculating the instantaneous Power $V_K . i_K$ for each of the branch of the network shown in Fig.2.1.
We have

$$V_2.i_2 = (V_A - V_B)i_2 \quad [\because \ (V_A - V_B) \text{ is the voltage across induction}]$$

Similarly,

$$V_4.i_4 = (V_B - V_C)i_4$$
$$V_0.i_0 = (V_C - V_A)i_0$$
$$V_{V_1}.i_{V_1} = V_A.i_1$$
$$V_3.i_3 = V_B.i_3$$
$$V_5.i_5 = V_C.i_5$$

Therefore,

$$\sum_{K=1}^{6} V_K.i_K = V_2.i_2 + V_4.i_4 + V_0.i_0 + V_{V_1}.i_{V_1} + V_3.i_3 + V_5.i_5$$

$$= (V_A - V_B)i_2 + (V_B - V_C)i_4 + (V_C - V_A)i_0 + V_A.i_1 + V_B.i_3 + V_C.i_5$$

$$= V_A(i_2 - i_0 + i_1) + V_B(-i_2 + i_4 + i_3) + V_C(-i_4 + i_0 + i_5) \quad \text{--------------- (1)}$$

By applying Kirchhoff's Current Law at the node A, B and C, we have

At node A, $i_0 = i_1 + i_2$ --------------- (2)

At node B, $-i_2 + i_4 + i_3 = 0$ --------------- (3) and

At node C, $-i_4 + i_5 + i_0 = 0$ --------------- (4)

By substituting the equations (2), (3) and (4) in equation (1), we have

$$\sum_{K=1}^{6} V_K.i_K = V_A \times 0 + V_B \times 0 + V_C \times 0 = 0$$

Hence it is proved.

Q.2 A voltage source V_1 whose internal resistance is R_1 delivers power to a load $R_2 + jX_2$ in which X_2 is fixed but R_2 is variable. Find the value of R_2 at which the power delivered to the load is a maximum. **(7)**

Ans:

A voltage source V_1 whose internal resistance is R_1 delivers Power to a load $R_2 + jX_2$ in which X_2 is fixed but R_2 is variable is shown in Fig.2.2.

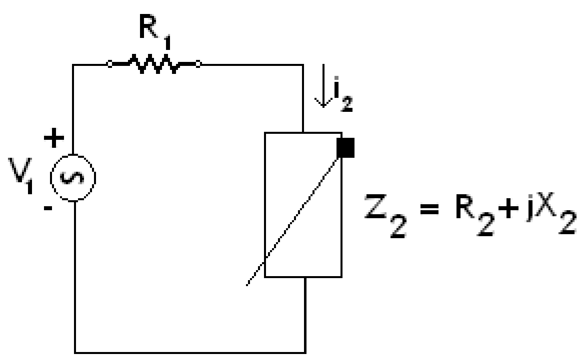


Fig.2.2

The power 'P' dissipated in the load is $P = |I_2|^2.R_2$ where I_2 is the load current flowing in the circuit and it is given by

$$I_2 = \frac{V_1}{R_1 + (R_2 + jX_2)} = \frac{V_1}{(R_1 + R_2) + jX_2}$$

Therefore, $|I_2|^2 = \dfrac{V_1^2}{(R_1 + R_2)^2 + X_2^2}$

Hence, the Power 'P' dissipated in the load is

$$P = |I_2|^2.R_2 = \frac{V_1^2.R_2}{(R_1 + R_2)^2 + X_2^2}$$

If the load reactance X_2 is fixed and the Power 'P' is maximised by varying the load resistance R_2 the condition for maximum Power transfer is

$$\frac{dP}{dR_2} = 0$$

OR $(R_1 + R_2)^2 + X_2{}^2 - R_2.2(R_1 + R_2) = 0$

OR $R_2{}^2 = R_1{}^2 + X_2{}^2$

OR $R_2 = \sqrt{R_1{}^2 + X_2{}^2}$

Therefore, the value of R_2 at which the Power delivered to the load is maximum, when

$$R_2 = \sqrt{R_1{}^2 + X_2{}^2}$$

Q.3 In the circuit shown in Fig.3 below, $v_3(t) = 2\sin 2t$. Using the corresponding phasor as the reference, draw a phasor diagram showing all voltage and current phasors. Also find $v_1(t)$ and $v_2(t)$. **(14)**

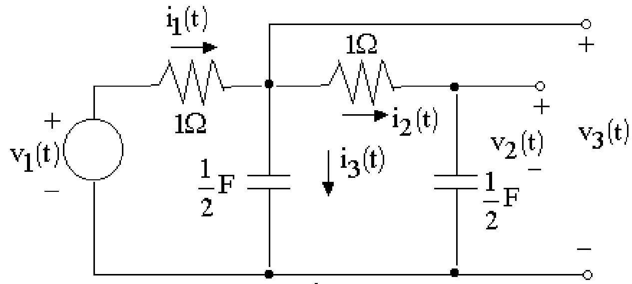


Fig.3

Ans:

Given that $V_3(t) = 2\sin 2t$ as a reference phase and w = 2.

Therefore, $V_3 = 2\underline{|\,0^0}$ ------------------ (1)

The given circuit is redrawn as shown in Fig.3.1.

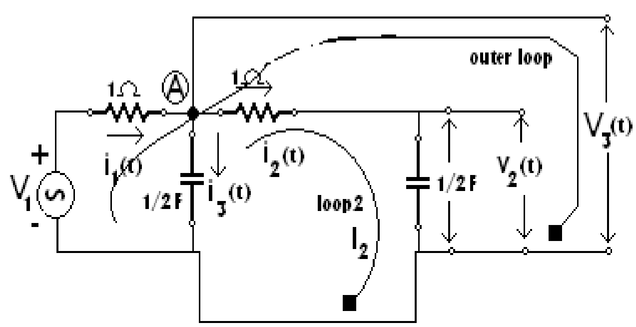


Fig.3.1

From Fig.3.1. $Z_C = \dfrac{1}{jwc} = \dfrac{1}{j\left(2\times\dfrac{1}{2}\right)} = \dfrac{1}{j} = -j$ ------------------ (2)

$\because \quad w = 2 \quad \& \quad c = \dfrac{1}{2}F$

Now the voltage V_3 from fig.3.1. is

$V_3 = I_2 + (-j)I_2 = 2\underline{|0^0} \quad (\because V_3 = 2\underline{|0^0})$

OR $I_2(1-j) = 2$

OR $I_2 = \dfrac{2}{1-j} = \sqrt{2}(1+j) = 2\underline{|45^0}$ ----------------- (3)

OR $I_2 = 2\underline{|45^0}$ ----------------- (4)

Also the voltage V_2 from Fig.3.1 is

$V_2 = I_2(-j)$

$= \sqrt{2}(1+j)(-j)$ $\quad [\because I_2 = \sqrt{2}(1+j)$ from equation (3)]

$= \sqrt{2}(1-j) = 2\underline{|-45^0}$

Therefore $V_2 = 2\underline{|-45^0}$

OR $V_2(t) = 2\sin\left(2t - \pi/4\right)$

Then, by applying KVL in loop 2, we obtain

$-jI_3 - I_2 - (-j)I_2 = 0$

OR $-jI_3 + I_2(1-j) = 0$

OR $jI_3 = \sqrt{2}(1+j) - 1 + j)$ $\quad \left[\because I_2 = \sqrt{2}(1+j)\right]$

OR $jI_3 = -2\sqrt{2}$

OR $I_3 = 2\sqrt{2}\,j \equiv 2\sqrt{2}\underline{|90^0}$

$\because \quad I_3 = 2\sqrt{2}\underline{|90^0}$

OR $I_3(t) = 2\sqrt{2}\sin\left(2t + \pi/2\right)$

By applying KCL at node A, we obtain

$I_1 = I_2 + I_3$

$= \sqrt{2}(1+j) + 2\sqrt{2}\,j$ $\quad \left[\because \quad I_2 = \sqrt{2}(1+j) \,\&\, I_3 = 2\sqrt{2}\,j\right]$

$= \sqrt{2}(1+3j)$

$= \sqrt{2}.\sqrt{10}\,\underline{|\tan^{-1}(3)}$

$\therefore \quad I_1 = \sqrt{20}\,\underline{|71.56^0}$

OR $I_1(t) = \sqrt{20}\sin\left(2t + 71.56^0\right)$

By applying KVL in the outer loop of Fig.3.1 we obtain

$V_1 - I_1 - V_3 = 0$

OR $V_1 - I_1 - 2 = 0$ $\quad (\because V_3 = 2)$

OR $V_1 = I_1 + 2$

$= \sqrt{2}(1+3j) + 2$ $\quad \left[\because I_1 = \sqrt{2}(1+3j)\right]$

OR $V_1 = \left(2+\sqrt{2}\right) + 3\sqrt{2}\,j$

$V_1 = 5.44\,\underline{|51.17^0}$

$\therefore \quad V_1 = 5.44\,\underline{|51.17^0}$

OR $V_1(t) = 5.44\sin\left(2t + 51.17^0\right)$

Therefore, the resultant voltage $V_1(t)$ and $V_2(t)$ are

$V_1(t) = 10.89\sin\left(2t + 51.17^0\right)$ and $V_2(t) = 2\sin\left(2t - 45^0\right)$

In order to draw the Phasor diagram, take V_3 as the reference Phasor. The resultant voltages and currents are listed below for drawing the Phasor diagram.

Voltages	Currents
$V_1 = 5.44\,\angle 51.17^0$	$I_1 = \sqrt{20}\,\angle 71.56^0 = 4.47\,\angle 71.56^0$
$V_2 = 2\,\angle -45^0$	$I_2 = 2\,\angle 45^0$
$V_3 = 2\,\angle 0^0$	$I_3 = 2\sqrt{2} = 2.82\,\angle 90^0$

The resultant Phasor diagram for voltages V_1, V_2, I_1, I_2 and I_3 & V_3 as a reference Phasor is shown in Fig.3.2.

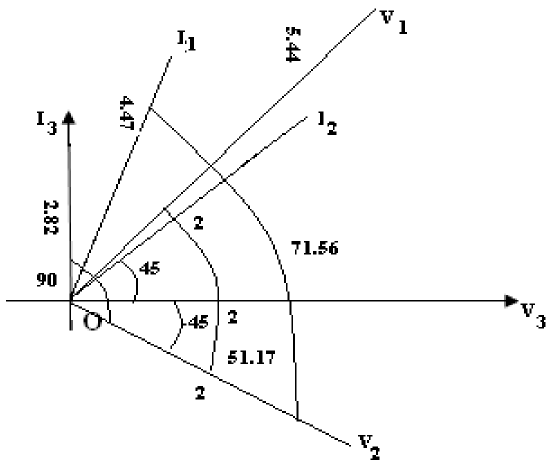


Fig.3.2.

Q.4 In the circuit shown in Fig.4 below, $v_1(t) = 2\cos t, C = 1F,\ L_1 = L_2 = 1H$ and $M = \frac{1}{4}H$. Find the voltage $v_a(t)$. **(7)**

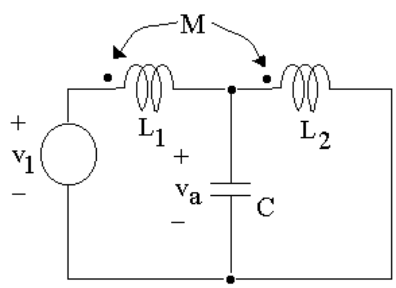


Fig.4

Ans:

Determination of $v_a(t)$

Given data

$V_1(t) = 2\cos t$

$L_1 = L_2 = 1H$

$M = \frac{1}{4}H$ & $C = 1F$

By transform the given data into Laplace transform. We have $V_1 = 2$, $X_{L_1} = X_{L_2} = 1$. Mutual Inductance $X_m = \frac{1}{4} = 0.2$ & $X_c = 1$. The Laplace transformed equivalent of the given diagram is shown in Fig.4.1.

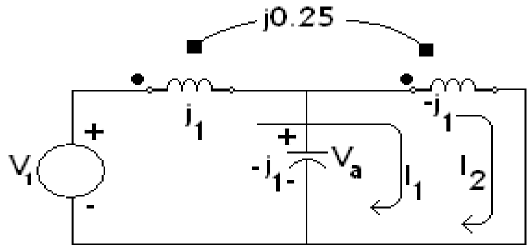


Fig.4.1

Writing loop equations for I_1 and I_2 we have

$2 = j0.25I_1 + j1.25I_2$ --------------- (1)

and $0 = j1.25I_1$ --------------- (2)

From equation(2) $I_1 = \dfrac{0}{j1.25} = 0$ --------------- (3)

By substituting the value of I_1 from equation(3) into equation(1), we get

$2 = j0.25(0) + j1.25I_2$

$$I_2 = \frac{2}{j1.25} = \frac{1}{j}1.6 = -j1.6$$

OR

Hence the voltage across the capacitor $V_a(t)$ given by

$V_a(t) = 1.6\cos t$

$[\because \quad V_a(t) = -(I_2)\times 1]$

Q.5 Determine the equivalent Norton network at the terminals a and b of the circuit shown in Fig.5 below. **(7)**

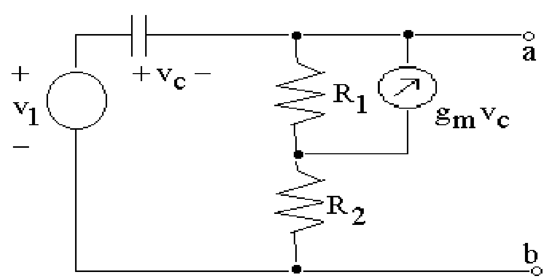


Fig.5

Ans:

Determination of the equivalent Norton network for the diagram shown in Fig.5.1

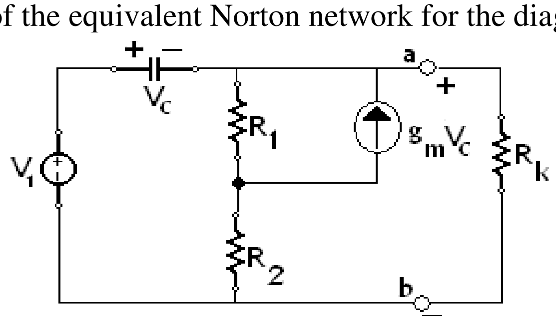


Fig.5.1

The simplified circuit for the diagram of Fig.5.1 is given in Fig.5.2

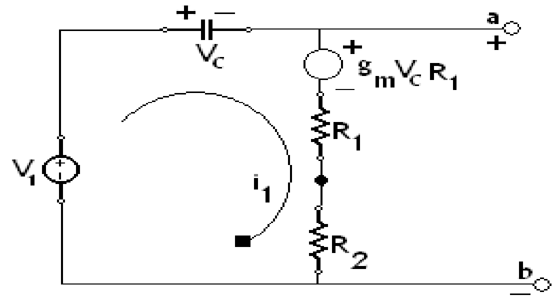


Fig.5.2

By writing loop equations for the circuit shown in Fig.5.2, we have

$$-V_1 + V_c + g_m V_c R_1 + i_1(R_1 + R_2) = 0$$

OR $\quad i_1(R_1 + R_2) = V_1 - V_c(1 + g_m R_1)$

OR $\quad i_1 = \dfrac{V_1 - V_c(1 + g_m R_1)}{(R_1 + R_2)}$

Thevenin's Impedance for the circuit is given by

$$Z_{Th} = \frac{R_1 + R_2}{1 + SC(R_1 + R_2) + g_m R_1}$$

OR $\quad I_{SC} = V_1(S).\dfrac{[SC(R_1 + R_2) + g_m R_1]}{R_1 + R_2}$

The equivalent Norton Network at the terminals a and b is shown in Fig.5.3

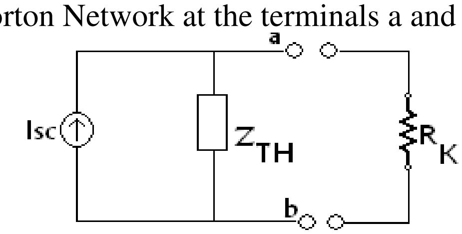


Fig.5.3

Q.6 In the network shown in Fig.6 below, $C_1 = C_2 = 1F$ and $R_1 = R_2 = 1\Omega$. The capacitor C_1 is charged to $V_0 = 1V$ and connected across the $R_1 - R_2 - C_2$ network at t = 0. C_2 is initially uncharged. Find an expression for $v_2(t)$. **(14)**

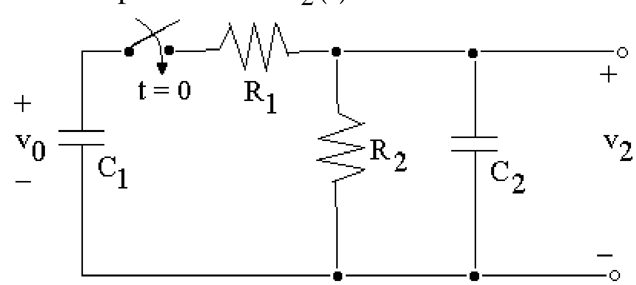


Fig.6

Ans:

The capacitor C_1 is changed to $V_0 = 1V$ prior to closing the switch S. Hence this capacitor C_1 can be replaced by a voltage source of value 1V in series with a capacitor $C_1 = 1F$ and connected across the $R_1 - R_2 - C_2$ network at t = 0. The resulting network is shown in Fig.6.1.

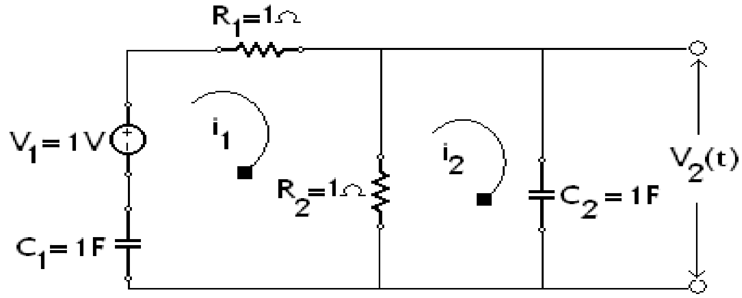


Fig.6.1

By applying KVL for the Fig.6.1, we have

$$\frac{1}{C_1}\int i_1 dt + R_1 i_1 + R_2(i_1 - i_2) = V_0 \quad \text{------------------- (1)}$$

and $R_2(i_2 - i_1) + \frac{1}{C_2}\int i_2 dt = 0$ ------------------- (2)

At $t = 0^+$ the capacitors C_1 and C_2 behave as short circuits.
Therefore,

$$\frac{1}{C_1}\int i_1 dt = 0 \quad \text{and} \quad \frac{1}{C_2}\int i_2 dt = 0 \quad \text{----------------- (3)}$$

By substituting equation (3) in equations (1) and (2), we get

$0 + R_1 i_1 + R_2(i_1 - i_2) = V_0$ ----------------- (4)

and $R_2(i_2 - i_1) + 0 = 0$ ----------------- (5)

From equation (5), we have

$R_2 i_2 - R_2 i_1 = 0$ OR

$R_2 i_2 = R_2 i_1$ OR $i_1(0^+) = \frac{R_2 i_2}{R_2} = i_2$

$i_2 = i_1$

By substituting the value of i_2 in equation (4), we have

$R_1 i_1 + R_2(i_2 - i_1) = V_0$ ---------------- (6) $\because\ i_2 = i_1$

OR $R_1 i_1 + 0 = V_0$ OR

$i_1(0^+) = \frac{V_0}{R_1} = \frac{1}{1} = 1$ Amp and

$i_1(0^+) = i_1 = 1$ Amp.

The voltage across capacitor C_2 is given by

$$V_2(t) = \frac{q_2}{C_2}$$

Where q_2 is the charge across capacitor C_2 and it is given by

$$q_2 = \int i_2 dt + K_2 = \int \frac{V_0}{C} e^{-C/Rt} dt + K_2$$

$$= -\frac{V_0}{C} e^{-C/Rt} + K_2$$

Therefore, voltage across is given by

$$V_2(t) = \frac{q_2}{C_2} = -\frac{V_0}{CC_2}\left(1 - e^{-C/_{R't}}\right)$$

Q.7 The switch K (Fig.7) is in the steady state in position a for $-\infty < t < 0$. At t = 0, it is connected to position b. Find $i_L(t), t \geq 0$. **(7)**

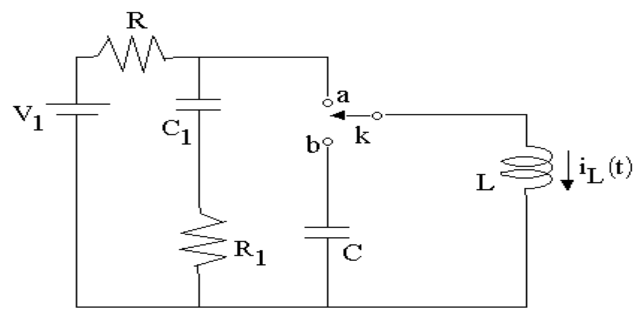


Fig.7

Ans:

Determination of $i_L(t)$:-

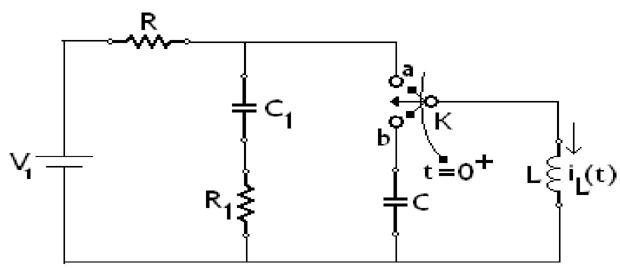


Fig.6.1

At position a, the steady state current $i_L(0^-)$ from Fig.6.1 is given by

$$i_L(0^-) = \frac{V_1}{(R + R_1)}$$

------------------ (1)

Now, when the switch K is moved to position b, the equivalent circuit is shown in Fig.6.2
By applying KVL for the circuit of Fig.6.2, we have

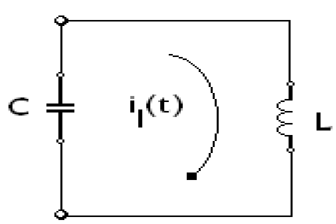


Fig.6.2

$$L\frac{di_L}{dt} + \frac{1}{C}\int_0^t i_L(t)dt = 0$$

------------------ (2)

Taking Laplace Transform for the equation (2), we get

$$L\left[SI_L(S) - i_L(0^+)\right] + \frac{1}{C}\frac{I_L(S)}{S} = 0$$ OR

$$\left(LS+\frac{1}{CS}\right)I_L(S)=L.i_L\left(O^-\right)$$

OR

$$\left(LS+\frac{1}{CS}\right)I_L(S)=L.\frac{V_1}{(R+R_1)} \qquad \left(\because \ i_L\left(O^-\right)=\frac{V_1}{(R+R_1)}\right)$$

OR $\quad I_L(S)=\frac{L.V_1}{(R+R_1)}.\frac{CS}{1+LCS^2}$

OR $\quad I_L(S)=\frac{V_1}{(R+R_1)}.\frac{S}{S^2+\frac{1}{LC}}$ ------------------ (3)

By taking Inverse Laplace Transform for the equation (3), we get

$$I_L(S)=\frac{V_1}{(R+R_1)}.COS\left(\frac{t}{\sqrt{LC}}\right)$$

Q.8 A battery of voltage v is connected at t = 0 to a series RC circuit in which the capacitor is relaxed at t = $0-$. Determine the ratio of the energy delivered to the capacitor to the total energy supplied by the source at the instant of time t. **(7)**

Ans:

The charging voltage across the capacitor in a series RC circuit excited by a voltage source V is given as

$V_C(t)=V\left(1-e^{-t/RC}\right)$ and

the current in the circuit is

$$i(t)=\frac{V}{R}e^{-t/RC}$$

Now, the total energy supplied by the source is

$$W_T=V.i(t).t$$

$$W_T=V.\frac{V}{R}.e^{-t/RC}.t=\frac{V^2t}{R}e^{-t/RC}$$

And the energy delivered to the capacitor is

$$W_C=\frac{1}{2}q_c(t).V_c(t)$$

$$W_C=\frac{1}{2}i(t).t.V_c(t)=\frac{1}{2}\frac{V^2t}{R}\left(1-e^{-t/RC}\right).e^{-t/RC}$$

Therefore, the Ratio of the energy delivered to capacitor to the total energy supplied by the source at the instant of time t is

$$\frac{W_C}{W_T}=\frac{\frac{1}{2}\frac{V^2t}{R}\left(1-e^{-t/RC}\right).e^{-t/RC}}{\frac{V^2t}{R}e^{-t/RC}} \qquad \text{OR}$$

$$\frac{W_C}{W_T}=\frac{1}{2}\left(1-e^{-t/RC}\right)$$

Q.9 Determine the condition for which the function $F(s)=\dfrac{s^2+a_1s+a_0}{s^2+b_1s+b_0}$ is positive real. It is given that a_0, b_0 a_1 and b_1 are real and positive. **(10)**

Ans:

The given function is $F(s)=\dfrac{s^2+a_1s+a_0}{s^2+b_1s+b_0}$ ----------------- (1)

Test whether F(s) is positive real by testing each requirement as given below:-

(i) The first condition is, if the coefficients of the denominator b_1 and b_0 are positive, then the denominator must be Hurwitz. For the given F(s), a_0, b_0, a_1 and b_1 are real and positive.

(ii) The second condition is, if b_1 is positive, then F(s) has no poles on the jw axis. Therefore, for the given F(s). We may then ignore the second requirement.

(iii) The third condition can be checked by first finding the even part of F(s), which is

$$Ev[F(s)]=\frac{(s^2+a_0)(s^2+b_0)-a_1b_1s^2}{(s^2+b_0)^2-b_1{}^2s^2}$$

$$=\frac{s^4+[(a_0+b_0)-a_1b_1]s^2+a_0b_0}{(s^2+b_0)^2-b_1{}^2s^2} \qquad \text{---------------- (2)}$$

The Real Part of F(jω) is then

$$\mathrm{Re}[F(j\omega)]=\frac{\omega^4-[(a_0+b_0)-a_1b_1]\omega^2+a_0b_0}{(-\omega^2+b_0)^2+b_1{}^2\omega^2}$$

----------------- (3)

From equation (3), we see that the denominator of Re[F(jω)] is truly always positive, so it remains for us to determine whether the numerator of Re[F(jω)] ever goes negative. By factoring the numerator, we obtain

$$\omega_1^2.2=\frac{(a_0+b_0)-a_1b_1}{2}\pm\frac{1}{2}\sqrt{[(a_0+b_0)-a_1b_1]^2-4a_0b_0}$$

------------ (4)

There are two situations in which Re[F(jω)] does not have a simple real root.

(i) The first situation is, when the quantity under the radical sign of equation (4) is zero[double, real root] or negative(complex roots). In other words

$[(a_0+b_0)-a_1b_1]^2-4a_0b_0\le 0$

OR $[(a_0+b_0)-a_1b_1]^2\le 4a_0b_0$ ----------------- (5)

If (i) $(a_0+b_0)-a_1b_1\ge 0$

Then $(a_0+b_0)-a_1b_1\le 2\sqrt{a_0b_0}$

Or $a_1b_1\ge\left(\sqrt{a_0}-\sqrt{b_0}\right)^2$ ----------------- (6)

If (ii) $(a_0+b_0)-a_1b_1<0$

Then $a_1b_1-(a_0+b_0)\le 2\sqrt{a_0b_0}$

But $(a_0+b_0)-a_1b_1<0<a_1b_1-(a_0+b_0)$

So again $a_1 b_1 \geq \left(\sqrt{a_0} - \sqrt{b_0}\right)^2$ ----------------- (7)

(ii) The second situation in which Re[F(jω)] does not have a simple real root in which $\omega^2_{1,2}$in equation (4) is negative, so that the roots are imaginary. This situation occurs when

$[(a_0 + b_0) - a_1 b_1]^2 - 4a_0 b_0 > 0$ ----------------- (8)

and $(a_0 + b_0) - a_1 b_1 < 0$ ----------------- (9)

From equation (8), we have

$$a_1 b_1 - (a_0 + b_0) > 2\sqrt{a_0 b_0} > (a_0 + b_0) - a_1 b_1$$

Thus $a_1 b_1 > \left(\sqrt{a_0} - \sqrt{b_0}\right)^2$

Therefore, the necessary and sufficient condition for a biquadratic function

$$F(s) = \frac{s^2 + a_1 S + a_0}{s^2 + b_1 S + b_0}$$

to be Positive Real is $a_1 b_1 \geq \left(\sqrt{a_0} - \sqrt{b_0}\right)^2$.

Q.10 Determine the common factor between the even and odd part of the polynomial $2s^6 + s^5 + 13s^4 + 6s^3 + 56s^2 + 25s + 25$. **(4)**

Ans:

The given polynomial P(s) is

$$P(s) = 2s^6 + s^5 + 13s^4 + 6s^3 + 56s^2 + 25s + 25$$

The even part of the polynomial P(s) is

$M(s) = 2s^6 + 13s^4 + 56s^2 + 25$ and the

The odd part of the polynomial P(s) is

$$N(s) = s^5 + 6s^3 + 25s$$

Therefore, the polynomial P(s) is

$$P(s) = \frac{\textit{even part of the polynomial}}{\textit{odd part of the polynomial}} = \frac{M(s)}{N(s)}$$

The common factor between the even and odd part of the polynomial P(s) is obtained by continued fraction method i.e.

$$\begin{array}{llll}
s^5 + 6s^3 + 25s \,\big) & 2s^6 + 13s^4 + 56s^2 + 25 & \big(\, 2s & \\
 & 2s^6 + 12s^4 + 50s^2 & & \\
 & - \quad - \quad - & & \\
\hline
 & s^4 + 6s^2 + 25 \,\big) & s^5 + 6s^3 + 25s & \big(\, s \\
 & & s^5 + 6s^3 + 25s & \\
 & & - \quad - \quad - & \\
\hline
 & & 0 &
\end{array}$$

The continued fraction ends here abruptly. Obviously this is due to the presence of the common ($s^4 + 6s^2 + 25$) between M(s) and N(s).

Hence the common factor between the even and odd part of the polynomial is $s^4 + 6s^2 + 25$.

Q.11 In the network shown in Fig.9 below, find V_2/V_1 if $Z_a Z_b = R$. **(8)**

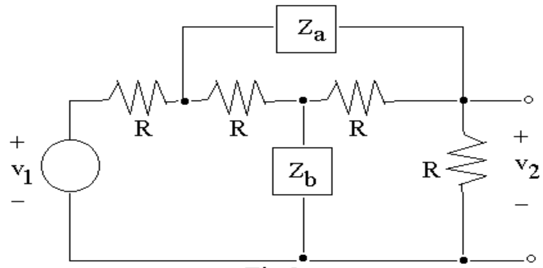


Fig.9

Ans:

The given network is a constant resistance bridge – T circuit. This circuit is called constant resistance, because the impedance looking in at either Port is a constant resistance R when the other (output) Post is terminated in the same resistance R as shown in Fig.9.

Finding of Voltage Transfer Function $\left(\frac{V_2}{V_1}\right)$ for the network of Fig.9:-

The circuit of fig.9 is redrawn as shown in fig.9.1.

Let the voltage at node B be 'V'

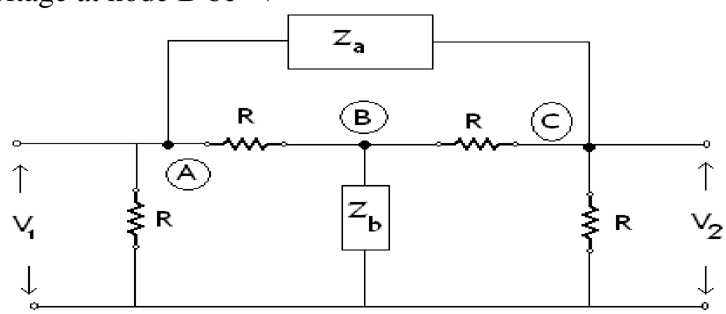


Fig.9.1

By applying KCL at node A gives,

$$\frac{V_1}{R} = \frac{V_1 - V}{R} + \frac{V_1 - V_2}{Z_a}$$

$$\frac{V_1}{R} - \frac{V_1}{R} + \frac{V}{R} - \frac{V_1}{Z_a} + \frac{V_2}{Z_a} = 0$$

OR

$$\left(\frac{1}{R} - \frac{1}{R} - \frac{1}{Z_a}\right)V_1 + \frac{V}{R} + \frac{V_2}{Z_a} = 0$$

OR -------------------- (1)

At node B, applying KCL gives

$$\frac{V_1 - V}{R} = \frac{V}{Z_b} + \frac{V - V_2}{R}$$

$$\frac{V_1}{R} - \frac{V}{R} - \frac{V}{Z_b} - \frac{V}{R} + \frac{V_2}{R} = 0$$

$$-\left(\frac{1}{R} + \frac{1}{Z_b} + \frac{1}{R}\right)V = -\frac{V_1}{R}$$

$$-\left(\frac{Z_b + R + Z_b}{RZ_b}\right)V = -\frac{V_1}{R} + \frac{V_2}{R}$$

$$\therefore \quad -\left(\frac{Z_b + R + Z_b}{RZ_b}\right)V = -\left(\frac{V_1}{R} + \frac{V_2}{R}\right)$$

OR $$\left(\frac{2Z_b + R}{RZ_b}\right)V = \frac{V_1}{R} + \frac{V_2}{R}$$

OR $$V = \frac{\left[\frac{V_1}{R} + \frac{V_2}{R}\right]}{\left(\frac{2Z_b + R}{RZ_b}\right)} = \left[\frac{V_1}{R} + \frac{V_2}{R}\right]\left(\frac{RZ_b}{2Z_b + R}\right)$$

OR $$V = \left[\left(\frac{\cancel{R}Z_b}{2Z_b + R}\right)\frac{V_1}{\cancel{R}}\right] + \left[\left(\frac{\cancel{R}Z_b}{2Z_b + R}\right)\frac{V_2}{\cancel{R}}\right]$$

Therefore $$V = \frac{Z_b}{2Z_b + R}V_1 + \frac{Z_b}{2Z_b + R}V_2 \qquad \text{------------------- (2)}$$

At node C, applying KCL gives,

$$\frac{V_1 - V_2}{Z_a} + \frac{V - V_2}{R} = \frac{V_2}{R}$$

$$\frac{V_1}{Z_a} - \frac{V_2}{Z_a} + \frac{V}{R} - \frac{V_2}{R} - \frac{V_2}{R} = 0$$

OR $$\frac{V_1}{Z_a} - \left(\frac{1}{Z_a} + \frac{1}{R} + \frac{1}{R}\right)V_2 + \frac{V}{R} = 0$$

OR $$\frac{V_1}{Z_a} - \left(\frac{R + Z_a + Z_a}{Z_a R}\right)V_2 + \frac{V}{R} = 0$$

OR $$\frac{V_1}{Z_a} - \left(\frac{R + 2Z_a}{Z_a R}\right)V_2 + \frac{V}{R} = 0$$

OR $$\frac{V}{R} = -\frac{V_1}{Z_a} + \left(\frac{R + 2Z_a}{Z_a R}\right)V_2 \qquad \text{------------------ (3)}$$

By substituting the value of V from equation (2) in equation (3), we get

$$\frac{1}{2}\left[\frac{Z_b}{2Z_b + R}V_1 + \frac{Z_b}{2Z_b + R}V_2\right] = -\frac{V_1}{Z_a} + \left(\frac{R + 2Z_a}{Z_a R}\right)V_2$$

OR $$\left[\frac{Z_b}{2RZ_b + R^2} + \frac{1}{Z_a}\right]V_1 = \left[\frac{R + 2Z_a}{Z_a R} - \frac{Z_b}{R(2Z_b + R)}\right]V_2$$

OR $$\left[\frac{Z_b}{2RZ_b + R^2} + \frac{1}{Z_a}\right]V_1 = \left[\frac{R + 2Z_a}{Z_a R} - \frac{Z_b}{2RZ_b + R^2}\right]V_2$$

OR $\left[\dfrac{Z_aZ_b+(2RZ_b+R^2)}{(2RZ_b+R^2)Z_a}\right]V_1=\left[\dfrac{(R+2Z_a)(2RZ_b+R^2)-Z_aZ_bR}{Z_aR(2RZ_b+R^2)}\right]V_2$

OR $\left[\dfrac{Z_aZ_b+2RZ_b+R^2}{2RZ_aZ_b+Z_aR^2}\right]V_1=\left[\dfrac{2R^2Z_b+R^3+4RZ_aZ_b+2Z_aR^2-Z_aZ_bR}{R(2RZ_aZ_b+Z_aR^2)}\right]V_2$ ----------- (4)

For the constant resistance bridged – T circuit, $Z_aZ_b=R^2$. By substituting the value of $Z_aZ_b=R^2$ in equation (4) and soLving the equation (4) for $\dfrac{V_2}{V_1}$, we get

$$\frac{V_2}{V_1}=\frac{R}{R+Z_a}=\frac{Z_b}{Z_b+R}.$$

Q.12 Synthesize Z_a and Z_b if $\dfrac{V_2}{V_1}=\dfrac{2s^2+1}{s^3+4s^2+5s+2}$ and R =1. **(6)**

Ans:

The given voltage transfer function $\dfrac{V_2}{V_1}$ is

$$\frac{V_2}{V_1}=\frac{2S^2+1}{C^3+4S^2+5S+2}$$

OR $\dfrac{V_2}{V_1}=\dfrac{2\left(S^2+\frac{1}{2}\right)}{\left(S^2+2S+1\right)\left(S+2\right)}$

OR $\dfrac{V_2}{V_1}=\dfrac{V_a}{V_1}.\dfrac{V_2}{V_a}=\left(\dfrac{2}{S+2}\right)\left(\dfrac{S^2+\frac{1}{2}}{S^2+2S+1}\right)$

Now $\dfrac{V_a}{V_1}=\left(\dfrac{2}{S+2}\right)=\dfrac{1}{1+\left(\frac{S}{2}\right)}=\dfrac{R}{R+Z_a}$

OR $\dfrac{V_a}{V_1}=\dfrac{1}{1+\left(\frac{S}{2}\right)}=\dfrac{1}{1+\left(\frac{Z_a}{1}\right)}$ $\qquad [\because \quad R=1]$

Therefore, $Z_{a_1}=\dfrac{S}{2}$ & R = 1

Since $Z_aZ_b=1$, so that $Z_{b_1}=\dfrac{2}{S}$

Hence $Z_{a_1}=\dfrac{S}{2}$ & $Z_{b_1}=\dfrac{2}{S}$

$$\frac{V_2}{V_a}=\frac{S^2+\frac{1}{2}}{(S^2+1)+2S}=\frac{1}{1+\left(\frac{2S+\frac{1}{2}}{S^2+\frac{1}{2}}\right)}=\frac{1}{1+\left(Z_{a_2}\right)}$$

Therefore, $Z_{a_2}=\dfrac{4S+1}{2S^2+1}$ & R = 1

Since $Z_{a_2}.Z_{b_2} = 1$, so that $Z_{b_2} = \dfrac{2S^2+1}{4S+1}$

Hence $Z_{a_2} = \dfrac{4S+1}{2S^2+1}$ & $Z_{b_2} = \dfrac{2S^2+1}{4S+1}$

Q.13 Two two-port networks N_a and N_b are connected in cascade as shown in Fig.10 below. Let the z – and y parameters of the two networks be distinguished by additional subscripts a and b. Find the z_{12} and y_{12} parameters of the overall network. **(10)**

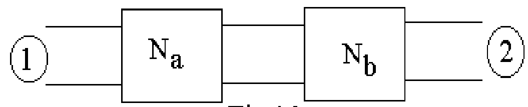


Fig.10

Ans:

Writing the Z-parameter equations

$V_1 = Z_{11}I_1 + Z_{12}I_2$ and

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{when\ I_1 = 0} = \left.\frac{V_{1a}}{I_{2b}}\right|_{I_{1a} = 0}$$

Now we know that

If $I_1 = 0$, then we can have from the network, 'b' load as Z_{22a} again

$V_{1b} = -Z_{12a}I_1 = Z_{11b}I_{1b} + Z_{12b}I_{2b}$

Hence $I_{1b} = \dfrac{-Z_{22b}.I_{2b}}{Z_{11b}+Z_{22a}}$

Also $V_{1a} = Z_{12a}.I_{2a} = -Z_{12a}.I_{1b} = \dfrac{Z_{12a}.Z_{12b}}{Z_{11b}+Z_{22a}}.I_2$

Therefore $Z_{12} = \dfrac{Z_{12a}.Z_{12b}}{Z_{11b}+Z_{22a}}$

Again from Y-parameter equations

$I_1 = Y_{11}V_1 + Y_{12}V_2$ and

$I_2 = Y_{21}V_1 + Y_{22}V_2$

We know by short circuiting Port (1), we have

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0} = \left.\frac{I_{1a}}{V_{2b}}\right|_{V_{1a} = 0}$$

Now when $V_2 = 0$, then the load on network 'b' is equal to Y_{22a}.

Hence $I_{1b} = -Y_{12a}V_{1b} = Y_{11b}.V_{1b} + Y_{12b}.V_{2b}$

Therefore, $V_{1b} = \dfrac{-Y_{12b}.V_{2b}}{Y_{11b}+Y_{22b}}$

Also $I_{2b} = Y_{12b}V_{2b} + Y_{12a}V_{1b}$

$$= \frac{-Y_{12a}Y_{12b}.V_{2b}}{Y_{11b}+Y_{22a}}$$

Hence $Y_{12} = \dfrac{-Y_{12a}Y_{12b}}{Y_{11b}+Y_{22a}}$

Therefore $Z_{12} = \dfrac{Z_{12a}Z_{12b}}{Z_{11b}+Z_{22a}}$ &

$Y_{12} = \dfrac{-Y_{12a}.Y_{12b}}{Y_{11b}+Y_{22a}}$

Q.14 Determine the z-parameters of the network shown in Fig.11 below. **(4)**

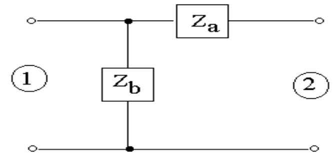


Fig.11

Ans:
The loop equations for the network of Fig.11.1 as

$V_1 = Z_b(I_1+I_2) = Z_bI_1 + Z_bI_2$

$\therefore\ V_1 = Z_bI_1 + Z_bI_2$ ------------ (1) and

$V_2 = Z_aI_2 + Z_b(I_1+I_2)$

OR $V_2 = Z_aI_2 + Z_bI_1 + Z_bI_2$

$V_2 = Z_bI_1 + I_2(Z_a+Z_b)$ ----------- (2)

We know that the Z-parameter equations are

$V_1 = Z_{11}I_1 + Z_{12}I_2$ ----------- (3) and

$V_2 = Z_{21}I_1 + Z_{22}I_2$ ----------- (4)

By comparing the equation (1), with equation (3),
We obtain

$Z_{11} = Z_b$ and $Z_{12} = Z_b$

Next by comparing the equation (2), with equation (4), we obtain

$Z_{21} = Z_b$ and $Z_{22} = (Z_a + Z_b)$

Q.15 Simplify the network, shown in Fig.1, using source transformations: **(8)**

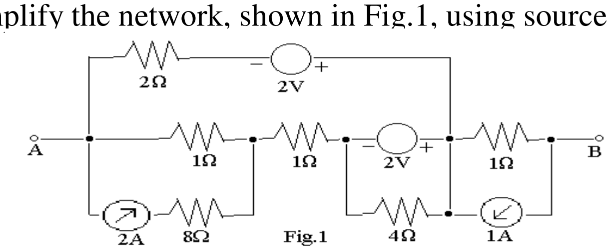


Fig.1

Ans:
In the given network of Fig.1, the resistance of 8Ω which is in series with the 2A current source is short-circuited and the resistance of 4Ω, which is in parallel with the 2V voltage source is open circuited. Therefore, 8Ω and 4Ω resistances are neglected and the given circuit is redrawn as shown in Fig.1.1(a)

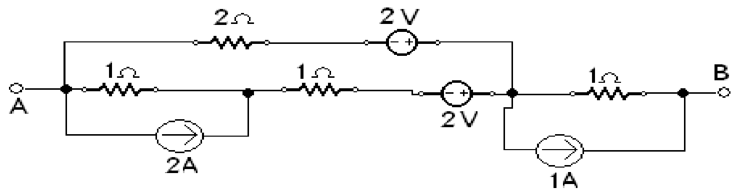


Fig.1.1(a)

Now convert the current source of 2A in parallel with 1Ω resistance into voltage source V=IR=2×1=2V, in series with 1Ω resistance and convert the current source of 1A in parallel with 1Ω resistance into voltage source $V = IR = 1\times1 = 1V$ in series 1Ω with resistance. The resultant diagram is shown in Fig.1.1(b).

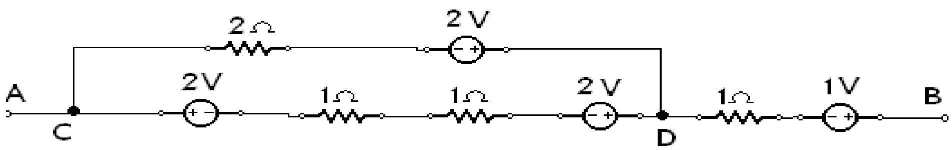


Fig.1.1(b)

The resistances, 1Ω and 1Ω between CD are in series, then the effective resistance is 1Ω + 1Ω = 2Ω & the voltage source of 2V is in opposite polarity with another 2V, then the net voltage of +2V & -2V becomes Zero. The resultant circuit is shown in Fig.1.1(c).

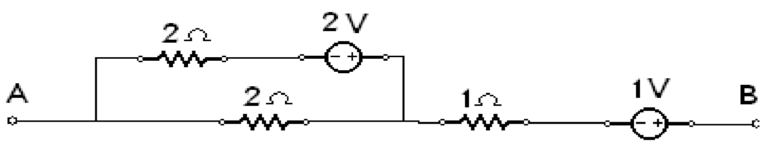


Fig.1.1(c)

Next convert the voltage source of 2V in series with 2Ω resistance into current source $I = \frac{V}{R} = \frac{2V}{2\Omega} 1\,amp$ in Parallel with 2Ω resistance, and the resistance of 2Ω becomes in parallel with another 2Ω, then the effective resistance is $2 \parallel 2 = \frac{2\times2}{2+2} = 1$. The resultant diagram is shown in Fig.1.1(d).

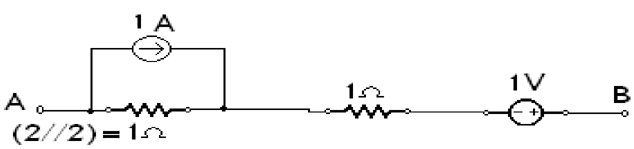


Fig.1.1(d)

Finally, convert the 1A current source into equivalent voltage source $V = 1R = 1A\times1\Omega = 1V$ and the diagram is shown in Fig.1.1(e).

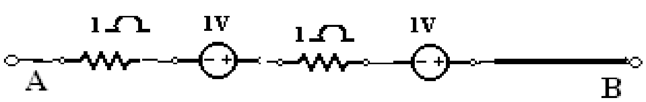


Fig.1.1(e)

Hence the resistance b/w AB is 1Ω in series with 1Ω equal to 2Ω and 1V is in same polarity with another 1V, which becomes 2V and the final simple circuit is shown in Fig.1.1(f)

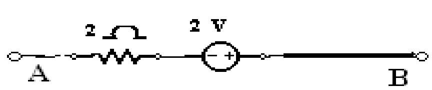


Fig.1.1(f)

Q.16 Using any method, obtain the voltage V_{AB} across terminals A and B in the network, shown in Fig.2: **(8)**

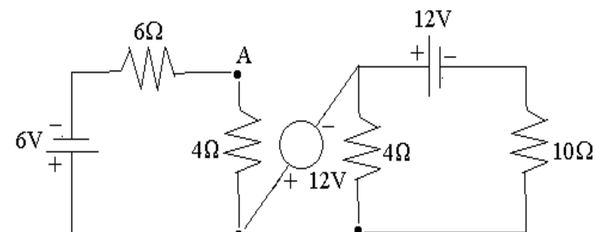


Fig. 2

Ans:

The circuit of Fig.2 can be redrawn as shown in Fig.2.1(a)

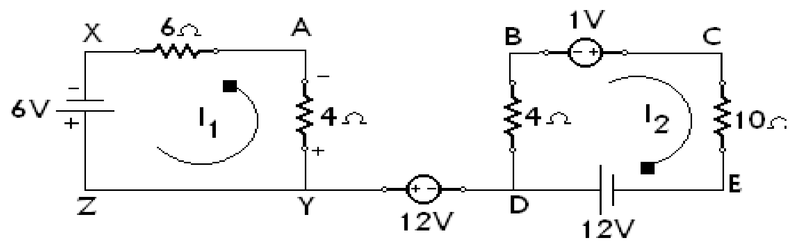


Fig.2.1(a)

In loop XAYZ, loop current I_1 as shown in Fig2.1(a) is

$$I_1 = \frac{6}{6+4} = \frac{6}{10} = 0.6A$$

In loop BCED, loop current I_2 as shown in Fig.2.1(a) is

$$I_2 = \frac{12}{4+10} = \frac{12}{14} = 0.86A$$

V_A = voltage drop across 4Ω resistor is

$$V_A = I_1 \times 4\Omega = 0.6 \times 4 = 2.4V$$

V_B = voltage drop across 4Ω resistor is

$$V_B = I_2 \times 4\Omega = 0.86 \times 4 = 3.44V$$

Therefore,

The voltage between points A and B is the sum of voltages as shown in Fig.2.1(b)

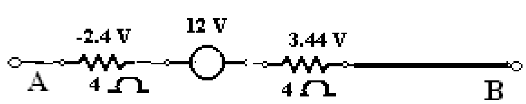


Fig.2.1(b)

Hence, $V_{AB} = -2.4 + 12 + 3.44 = 13.04V$

Q.17 For the network shown in Fig.3, the switch is closed at t = 0. If the current in L and voltage across C are 0 for t < 0, find $i(0^+), \left.\frac{di(t)}{dt}\right|_{t=0+}, \left.\frac{d^2i(t)}{dt^2}\right|_{t=0+}$. **(8)**

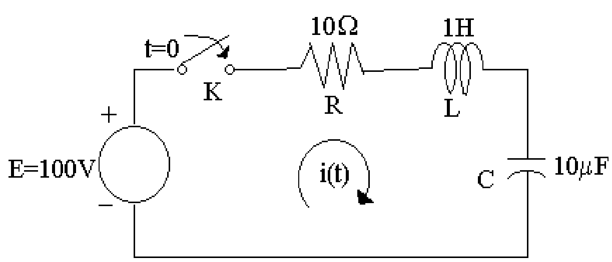


Fig.3

Ans:

(i) Since, before $t = 0^+$, the switch is open, then $i(0^-) = 0$ that is $i(0^+) = i(0^-) = 0$ as the current through the inductor cannot change instantaneously.
Also $V_c(0^-) = 0$ and therefore $V_c(0^+) = 0$.

(ii) Determination of $\left.\frac{di(t)}{dt}\right|_{t=0^+}$:

After the switch is closed at $t = 0^+$.
Writing Kirchoff's Voltage Law to the network shown in Fig.3 we get

$$100 = 10i + 1\frac{di}{dt} + 10i\int dt \qquad \text{--------------------} \ (1)$$

$$100 = 10i(0^+) + 1\left.\frac{di}{dt}\right|_{t=0^+} + 10\left[\int idt\right]_{t=0^+}$$

Since $\left[\int idt\right]_{t=0^+} = V_c(0^+) = 0$, we have

$$100 = 10i(0^+) + 1\left.\frac{di}{dt}\right|_{t=0^+} \quad \text{from which}$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = 100 \ \text{Amp/sec} \qquad \left[\because \ i(0^+) = 0\right]$$

(iii) Determination of $\left.\frac{d^2i(t)}{dt^2}\right|_{t=0^+}$:

From eqn(1),

We have $100 = 10i + 1\frac{di}{dt} + 10i\int dt$

Differentiating the above equation with respect to 'i', we have

$$0 = 10\frac{di}{dt} + 1\frac{d^2i}{dt^2} + 10i$$

OR $$\left.\frac{d^2i}{dt^2}\right|_{t=0^+} = -10\left.\frac{di}{dt}\right|_{t=0^+} - 10 \times i(0^+)$$

$$\left.\frac{d^2i}{dt^2}\right|_{t=0^+} = -10 \times 100 - 0 \quad \left[\because \ \left.\frac{di}{dt}\right|_{t=0^+} = 100 \ and \ i(0^+) = 0\right]$$

OR $$\left.\frac{d^2i}{dt^2}\right|_{t=0^+} = -1000 \ \text{Amp/sec}^2.$$

Q.18 Use the Thevenin equivalent of the network shown in Fig.4 to find the value of R which will receive maximum power. Find also this power. **(8)**

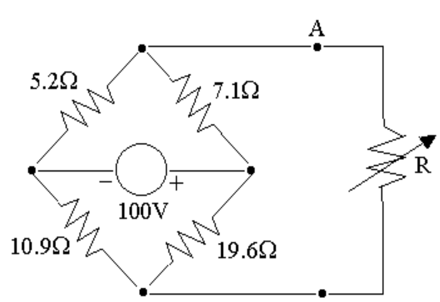


Fig.4

Ans:

Removing R from Fig.4 between points AB reduce the circuit to the simple series-parallel arrangement shown in fig.4.1

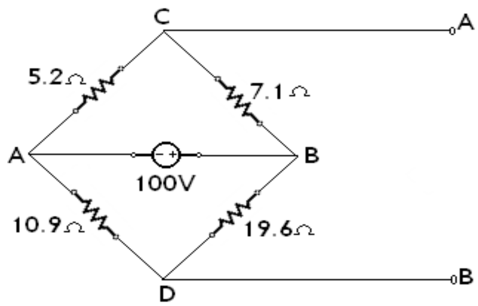


Fig.4.1

Consider the point A (the reference point) at ground potential,

$$V_A = \frac{5.2}{5.2+7.1}\times(100v) = \frac{5.2}{12.3}\times(100v) = 42.27\ volts$$

Similarly, $V_B = \dfrac{10.9}{10.9+19.6}\times(100v) = \dfrac{10.9}{30.5}\times(100v) = 35.73\ volts$

Therefore, $V_{Th} = V_{CD} = 42.27 - 35.73 = 6.53\ volts$

Now, apply the second step of Thevenin's Theorem, to find R_{Th}. For finding of R_{Th}, short the points C & D together, that is replace the voltage generator by its internal resistance (considered here as a short) and measure the resistance between points A and B. This is illustrated in Fig.4.2

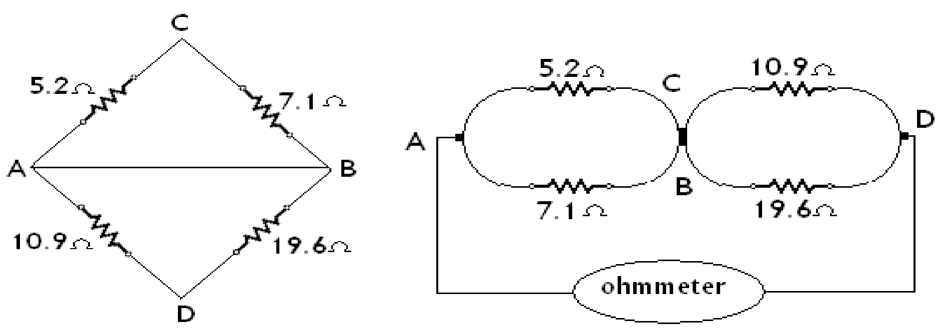


Fig.4.2

From the Fig.4.2, $R_{Th} = (5.2 \parallel 7.1) + (10.9 \parallel 19.6)$

$$= \frac{5.2\times7.1}{5.2+7.1} + \frac{10.9\times19.6}{10.9\times19.6} = \frac{36.92}{12.3} + \frac{213.64}{30.5} = 10\Omega$$

Now, replacing all the circuit (except R) with the Thevenin generator and Thevenin resistance and then replacing 'R' back between points 'AB' results in the circuit of Fig.4.3

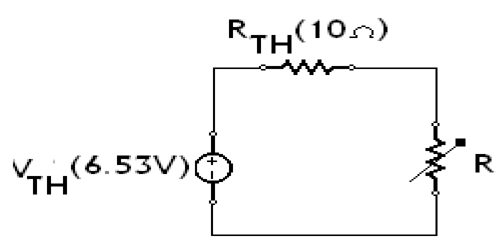


Fig.4.3

Therefore,

(i) The value of R which will receive maximum power is

$$R_{Th} = R = 10\Omega$$

(ii) The Maximum Power is

$$P_{R(Max)} = I_R^2 . R = \frac{V_s^2}{4R^2}(R) = \frac{V_s^2}{4R} = \frac{(6.53)^2}{4 \times 10} = 1.066\, W$$

Q.19 Express the impedance Z (s) for the network shown in Fig.5 in the form: $Z(s) = K\dfrac{N(s)}{D(s)}$. Plot its poles and zeros. From the pole-zero plot, what can you infer about the stability of the system?

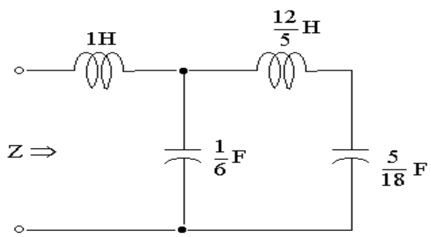


Fig.5

(8)

Ans:

The S-domain equivalent of the network of Fig.5 is shown in Fig.5.1.
The impedance Z(S) for the network shown in Fig.5 is

$$Z(S) = S + \cfrac{1}{\cfrac{S}{6} + \cfrac{1}{\cfrac{12S}{5} + \cfrac{1}{\cfrac{5S}{18}}}} = S + \cfrac{1}{\cfrac{S}{6} + \cfrac{5S}{12S^2 + 18}}$$

$$= S + \frac{6(2S^2 + 3)}{S(2S^2 + 8)} = \frac{S^4 + 10S^2 + 9}{S(S^2 + 4)}$$

$$Z(S) = \frac{(S^2 + 9)(S^2 + 1)}{S(S^2 + 4)} \qquad \text{------------------ (1)}$$

Therefore, the equation (1) is in the form of

(i) $Z(S) = K\dfrac{N(S)}{D(S)}$

Where K = 1

$N(S) = (S^2 + 9)(S^2 + 1)$ and

$D(S) = S(S^2 + 4)$

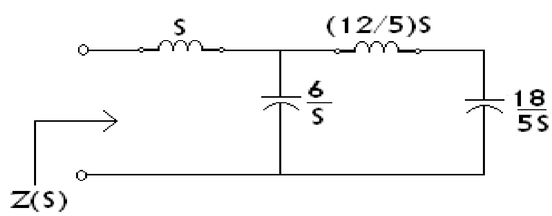


Fig.5.1

$$Z(S) = \frac{(S^2 + 1)(S^2 + 1)}{S(S^2 + 4)} = \frac{(S + j3)(S - j3)(S + j1)(S - j1)}{S(S + j2)(S - j2)}$$

(ii) Poles are given by (when denominator = 0)
S = 0, S = -j2 and S = +j2
Zeros are given by (when numerator = 0)
S = -j3, S = +j3, S = -j1 and S = +j1

(iii) The pole-zero pattern for the network of Fig.5 is shown in Fig.5.2

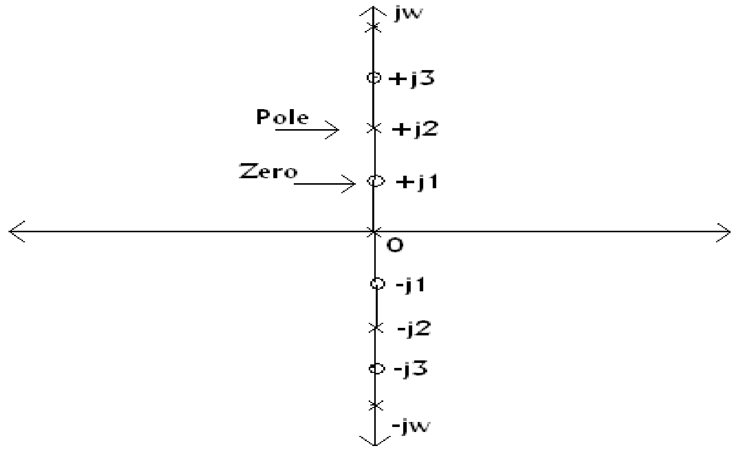


Fig.5.2

a) Condition 1 for checking stability:
The given function Z(S) has three poles at S = 0, -j2 and +j2. Hence Z(S) has one pole i.e., (+j2) on the right-half of jw plane. So, the given function is unstable.

b) Condition 2 for checking stability:
The given function Z(S) has multiple poles i.e., (-j2 & +j2) on the jω-axis. Hence the function Z(S) is unstable.

c) Condition 3 for checking stability:
For the function Z(S), the degree of numerator is 4 is exceeding the degree of the denominator (i.e. 3). Hence the system is unstable.

The three conditions i.e. a, b & c are not satisfied. Hence the given function Z(S) is unstable.

Q.20 Switch K in the circuit shown in Fig.6 is opened at t = 0. Draw the Laplace transformed network for t > 0+ and find the voltages $\upsilon_1(t)$ and $\upsilon_2(t)$, t > 0+. **(8)**

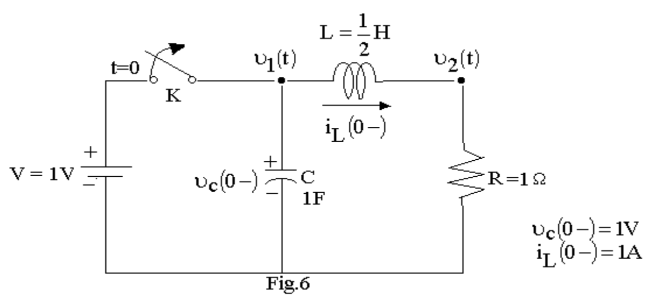


Fig.6

Ans:

The Laplace transformed network for $t > 0^+$ is shown in Fig.6.1

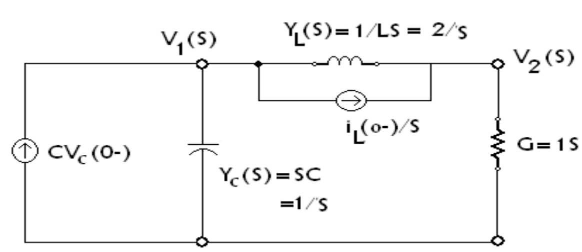


Fig.6.1

The node equations for Fig.6.1 are

Node V_1:

$$\frac{-i_L(0^-)}{S} + CV_C(0^-) = \left(SC + \frac{1}{SL}\right)V_1(S) - \frac{1}{SL}V_2(S) \qquad \text{---------------- (1)}$$

and Node V_2:

$$\frac{i_L(0^-)}{S} = -\frac{1}{SL}V_1(S) + \left(\frac{1}{SL} + G\right) + V_2(S) \qquad \text{---------------- (2)}$$

Since prior to opening of switch the network has been in steady-state, then we have $V_C(0^-) = 1V$ and $I_L(0^-) = 1A$. By substituting the numerical values in eqns (1) & (2) we have

$$1 + \frac{1}{S} = \left(S + \frac{1}{S}\right)V_1(S) - \frac{2}{S}V_2(S) \qquad \text{---------------- (3)}$$

$$\frac{1}{S} = \frac{2}{S}V_1(S) + \left(\frac{2}{S} + 1\right)V_2(S) \qquad \text{---------------- (4)}$$

Solving the equations (3) & (4) for $V_1(S)$ & $V_2(S)$ we have

$$V_1(S) = \frac{S+1}{\left(S^2 + 2S + 2\right)} = \frac{S+1}{(S+1)^2 + 1} \qquad \text{---------------- (5)}$$

$$V_2(S) = \frac{S+2}{\left(S^2 + 2S + 2\right)} = \frac{S+2}{(S+1)^2 + 1}$$

$$V_2(S) = \frac{S+2}{(S+1)^2 + 1} = \frac{S+1}{(S+1)^2 + 1} + \frac{1}{(S+1)^2 + 1} \qquad \text{--------------- (6)}$$

Taking the inverse Laplace Transform of the two eqns (5) & (6), we obtain

$V_1(t) = e^{-t}\cos t$

And $V_2(t) = e^{-t}(\cos t + \sin t)$

Q.21 Given the ABCD parameters of a two-port, determine its z-parameters. **(8)**

Ans:

ABCD parameters of a two-port network are represented by.

$V_1 = AV_2 - BI_2$ ------------------------ (i)

And $I_1 = CV_2 - DI_2$ ------------------------ (ii)

Equation (ii) can be written as

$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2 \qquad \text{------------------------ (iii)}$$

Similarly, from eqn (i) we have

$V_1 = AV_2 - BI_2$ ------------------------ (iv)

By substituting the value of from equation (iii) we have

$$V_1 = \left[\frac{1}{C}.I_1 + \frac{D}{C}.I_2\right]A - BI_2$$

$$V_1 = \frac{A}{C}.I_1 + \frac{AD - BC}{C}.I_2 \qquad \text{----------------------- (v)}$$

Also Z-parameters of a two port network are represented by

$V_1 = Z_{11}I_1 + Z_{12}I_2$ ----------------------- (vi)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ ----------------------- (vii)

By comparing equations (v) and (vi) we have

$Z_{11} = \frac{A}{C}$ and $Z_{12} = \frac{AD - BC}{C}$

By comparing equations (iii) and (vii) we have

$Z_{21} = \frac{1}{C}$ and $Z_{22} = \frac{D}{C}$

Q.22 Find the y-parameters for the network shown in Fig.7. **(8)**

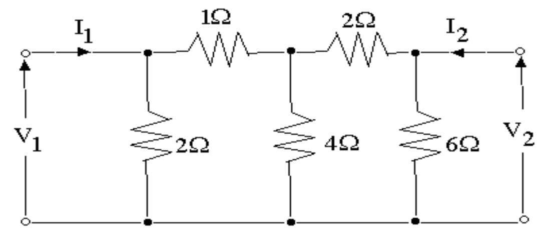


Fig.7

Ans:

With port $2-2^1$ open circuited, the circuit may be redrawn as shown in Fig.5.1, therefore the output voltage $V_2 = I_3 \times 6$ ----------- (1)

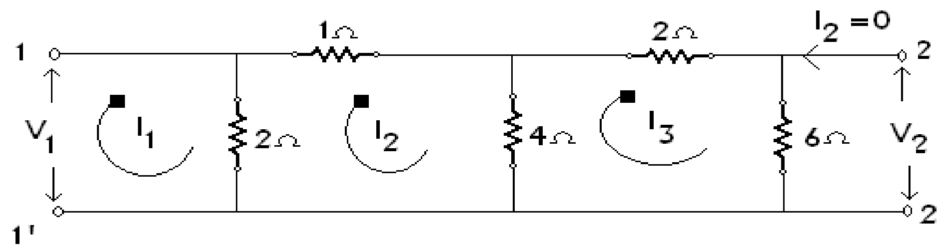


Fig.5.1

For mesh 3 in Fig.5.1, applying KVL, we get

$(I_3 - I_2)4 + (6+2)I_3 = 0$

$4I_3 - 4I_2 + 8I_3 = 0$

Or $12I_3 = 4I_2$ ---------------- (2)

For mesh 2 in Fig.5.1, applying KVL, we get

$2(I_2 - I_1) + 1I_2 + (I_2 - I_3)4 = 0$

$2I_2 - 2I_1 + I_2 + 4I_2 - 4I_3 = 0$

Or $7I_2 - 2I_1 - 4I_3 = 0$ ---------------- (3) and

$V_1 = (I_1 - I_2)2$

Or $V_1 = 2I_1 - 2I_2$ ---------------- (4)

From equation (2), we have $12I_3 = 4I_2$

Or $I_3 = \frac{4}{12}I_2 = \frac{1}{3}I_2$ ----------- (5)

By substituting the value of I_3 in equation (3), we get

$7I_2 - 2I_1 - 4\left(\frac{1}{3}I_2\right) = 0$ $\left[\because \quad I_3 = \frac{I_2}{3}\right]$

Or $\quad 7I_2 - \frac{4}{3}I_2 = 2I_1$

Or $\quad \left(\frac{17}{3}\right)I_2 = 2I_1$

Or $\quad I_2 = \frac{6}{17}I_1$ ---------- (6)

From equation (4), we have

$2I_1 - 2I_2 = V_1$

$2I_1 - 2\left(\frac{6}{17}I_1\right) = V_1 \qquad \left(\because\ I_2 = \frac{6}{17}I_1\right)$

Or $\quad \left(\frac{34-12}{17}\right)I_1 = V_1$

Or $\quad Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{17}{22} = 0.7727$

From equation (1), we have $V_2 = I_3 \times 6$ and

From equation (2), we have $I_2 = \frac{12}{4}I_3 = 3I_3$

From equation (3), we have

$7I_2 - 2I_1 - 4I_3 = 0$

$7(3I_3) - 2I_1 - 4I_3 = 0$

$21I_3 - 4I_3 = 2I_1$

Or $\quad I_3 = \frac{2}{17}I_1$

By substituting the value of I_3 in equation (1), we have

$V_2 = I_3 \times 6$

$= \left(\frac{2}{17}I_1\right)6 = \frac{12}{17}I_1$

Or $\quad Y_{21} = \left.\frac{I_1}{V_2}\right|_{V_2=0} = \frac{17}{12} = 1.4166$

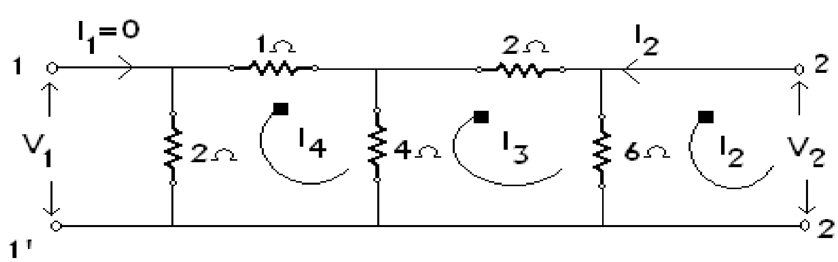


Fig.5.2

With port $1-I^1$, open circuited, the circuit may be redrawn as shown in Fig.5.2, therefore

$(I_2 - I_3)6 = V_2$

Or $\quad 6I_2 - 6I_3 = V_2$ ----------------- (1)

For mesh (4), in Fig.5.2 by applying K.V.L., we get

$$(I_4 - I_3)4 + (1+2)I_4 = 0$$

Or $\quad 4I_4 - 4I_3 + 3I_4 = 0$

Or $\quad 7I_4 - 4I_3 = 0$

Or $\quad I_4 = \frac{4}{7}I_3$ ----------------- (2)

For mesh (3) in Fig.5.2, by applying KVL, we get

$$(I_3 - I_2)6 + 2I_3 + (I_3 - I_4)4 = 0$$

Or $\quad 12I_3 - 6I_2 - 4I_4 = 0$ ----------------- (3)

$$12I_3 - 6I_2 - 4\left(\frac{4}{7}I_3\right) = 0 \qquad \left[\because \quad I_4 = \frac{4}{7}I_3\right]$$

Or $\quad \left(12 - \frac{16}{7}\right)I_3 = 6I_2$

Or $\quad \left(\frac{68}{7}\right)I_3 = 6I_2$

Or $\quad I_3 = \frac{42}{68}I_2$ ----------------- (4)

From equation (1), we have

$$6I_2 - 6I_3 = V_2$$

Or $\quad 6I_2 - 6\left(\frac{42}{68}I_2\right) = V_2 \qquad \left[\because \quad I_3 = \frac{42}{68}I_2\right]$

Or $\quad \left(\frac{408-252}{68}\right)I_2 = V_2$

Or $\quad \left(\frac{156}{68}\right)I_2 = V_2$

Or $\quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{68}{156} = 0.436$

From Fig.5.2, since $I_1 = 0$ so that

$$V_1 = I_4 \times 2 \qquad \text{------------------ (5)}$$

Now from equation (2), we have

$$7I_4 = 4I_3$$

Or $\quad I_3 = \frac{7}{4}I_4$

From equation (3), we have

$$12I_3 - 6I_2 - 4I_4 = 0$$

$$12\left(\frac{7}{3}I_4\right) - 4I_4 = 6I_2 \qquad \left(\because \quad I_3 = \frac{7}{3}I_4\right)$$

Or $\quad 28I_4 - 4I_4 = 6I_2$

Or $\quad I_4 = \frac{6}{24}I_2$

Now substituting the value of I_4 in equation (5), we get

$$V_1 = I_4 \times 2$$
$$= \frac{6}{24} I_2 \times 2$$
$$V_1 = \frac{12}{24} I_2$$

Or

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_1 = 0} = \frac{24}{12} = 2$$

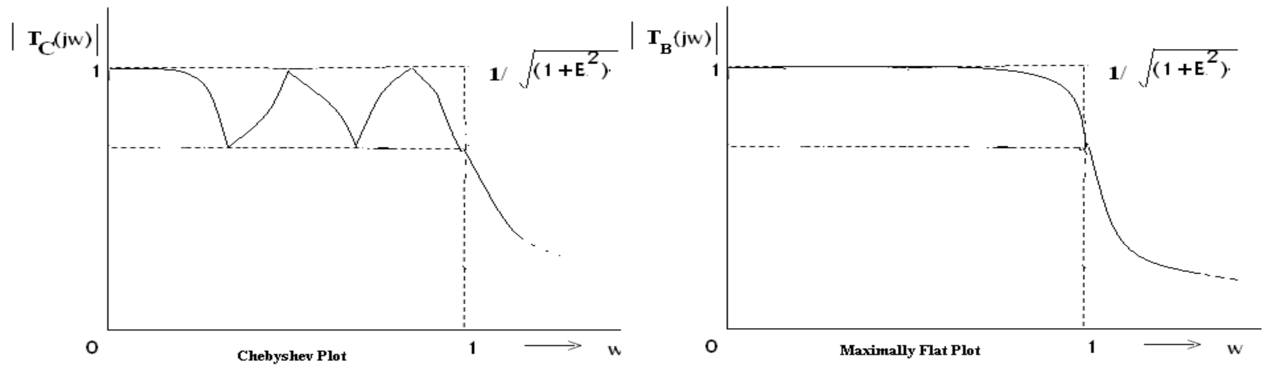


Fig.6

C_n approaches the value $C_n(\omega) \approx 2^{n-1}\omega^n$ for $\omega \to \infty$ and further assume $\in^2 \omega^2 >> 1$. Then

$\alpha_B \approx 20\log\left(\in \omega^n\right)$ and

$\alpha_C \approx 20\log\left(2^{n-1} \in \omega^n\right) = 2\log(n-1)\log 2 + 20\log(\in \omega^n)$

That is, for large ω, $\alpha_C = 6.02(n-1) + \alpha_B$

This is a substantial increase in stop band attenuation over the maximally flat approximations. For example for a fifth-order filter, the Chebyshev attenuation is 24dB larger than in the Maximally Flat case.

Q.23 Distinguish between Chebyshev approximation and maximally flat approximation as applicable to low pass filters. What is the purpose of magnitude and frequency scaling in low pass filter design? **(8)**

Ans:

i) Maximally Flat Function defined by

$|T_B(J\omega)|^2 = \dfrac{1}{1+\in^2 \omega^{2n}}$ and comparing its performance with Chebyshev function defined by $|T_B(J\omega)|^2 = \dfrac{1}{1+\in^2 C_n^2(\omega)}$

The subscripts B and C respectively to label the maximally flat (or Butterworth) and the Chebyshev functions.

ii) Both the functions have same specifications for LPF over the Passband:

$\alpha_{\max} = 10\log\left(1+\in^2\right)$ in $0 \le \omega \le 1$

iii) A plot of the Maximally Flat (or Butterworth) and Chebyshev functions is shown in Fig.6. At high frequencies

$\alpha_B = 10\log(1+\in^2 \omega^{2n})$ for Butterworth and

$\alpha_C = 10\log\left[1+\left(2^{n-1} \in \omega^n\right)^2\right]$

iv) The value of Q in Chebyshev approximation is always larger (due to increased attenuation) than the Maximally flat approximation. The higher values of Q in Chebyshev response leading to more difficult circuit realizations and less linear phase characteristics than maximally flat response.

v) Increased attenuation in the Chebyshev case is a phase characteristic of increased nonlinearity than the Maximally flat case.

Purpose of Magnitude and frequency scaling in low Pass Filter design:

i) It avoids the need to use very small or very large component values, such as PF capacitors and MΩ resistors.

ii) It permits us to design filters whose critical specifications are on the frequency axis "in the neighbourhood" of ω = 1 rad/sec.

iii) It permits us to deal with only dimension less specifications and components without having to be concerned with units, such as HZ, Ω, F, OR H.

iv) Much of the work of filter designers is base on the use of design tables. In the tables so called "Prototype" lowpass transfer functions are assumed to have passband along the normalized frequency ω in $0 \le \omega \le \omega_p = 1$ and a stop band in $1 < \omega_s < \omega < \infty$. These Prototype filters are designed with normalized dimensionless elements.

Q.24 Show that the voltage-ratio transfer-function of the ladder network shown in Fig.8 is given by: $\dfrac{V_2(s)}{V_1(s)} = \dfrac{8s^2}{12s^2+12s+1}$. **(8)**

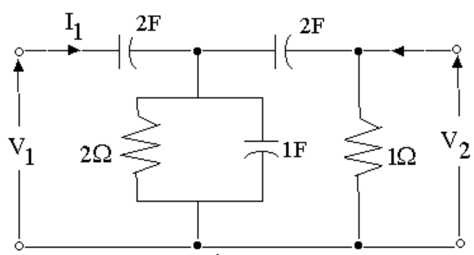


Fig.8

Ans:

The Laplace Transform of the given network of Fig.8 is shown in Fig.8.1

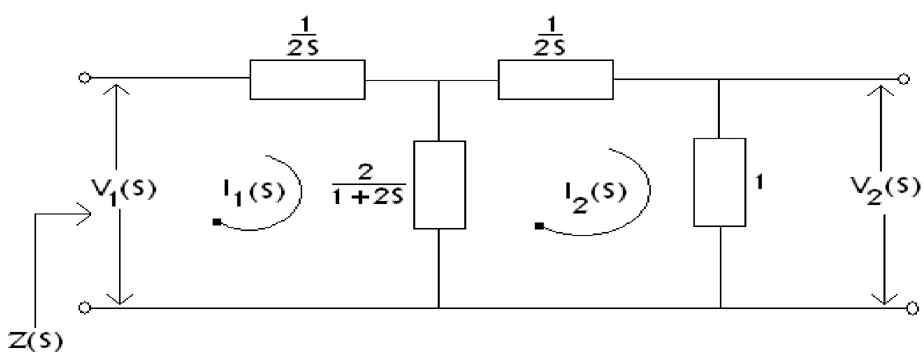


Fig.8.1

From Fig.8.1, the equivalent impedance Z(S) is given by

$$Z(S) = \frac{1}{2S} + \left[\left(\frac{2}{1+2S}\right) \Bigg\| \left(1+\frac{1}{2S}\right)\right]$$

$$= \frac{1}{2S} + \frac{\left\{\frac{2}{1+2S}\left(1+\frac{1}{2S}\right)\right\}}{\left\{\frac{2}{1+2S}+1+\frac{1}{2S}\right\}}$$

$$= \frac{1}{2S} + \left[\frac{\frac{2}{1+2S}+\frac{2}{2S(1+2S)}}{\frac{2}{1+2S}+1+\frac{1}{2S}}\right]$$

$$Z(S) = \frac{1}{2S} + \frac{4S+2}{4S^2+8S+1} = \frac{12S^2+12S+1}{2S(4S^2+8S+1)}$$

As we know that $I_1(S) = \frac{V_1(S)}{Z(S)}$ and

$$I_2(S) = I_1(S).\frac{\frac{2}{1+2S}}{\frac{2}{1+2S}+\frac{1}{2S}+1}$$

$$I_2(S) = \frac{4SI_1(S)}{4S+1+2S+2S(1+2S)} = \frac{4SI_1(S)}{4S^2+8S+1}$$

Therefore,

$$I_2(S) = \frac{4SI_1(S)}{4S^2+8S+1} = \frac{4SV_1(S)}{ZS(4S^2+8S+1)} \qquad \left[\because \quad I_1(S) = \frac{V_1(S)}{Z(S)}\right]$$

Also from Fig.8.1

$$V_2(S) = 1 \times I_2(S)$$

$$= 1 \times \frac{4S.V_1(S)}{ZS[4S^2+8S+1]} = \frac{4SV_1(S)}{(4S^2+8S+1)}.\frac{1}{Z(S)}$$

$$V_2(S) = \frac{4S.V_1(S)}{(4S^2+8S+1)}.\frac{25(4S^2+8S+1)}{12S^2+12S+1} \qquad \left[\because \quad Z(S) = \frac{12S^2+12S+1}{2S(4S^2+8S+1)}\right]$$

$$V_2(S) = \frac{8S^2}{12S^2+12S+1}.V_1(S)$$

Therefore, the voltage transfer ration for the given network is

$$\frac{V_2(S)}{V_1(S)} = \frac{8S^2}{12S^2+12S+1}$$ Hence proved.

Q.25 Explain the following:
(i) Phasor. (ii) Resonance. (iii) Damping coefficient. **(8)**

Ans:

(i) <u>Phasor</u>: A phasor 'S' is a complex number characterized by a magnitude and a phase angle. This is represented by $S(t) = Ae^{jwt}$, where A is the magnitude and jwt is the phase angle. The angular frequency 'w' of the phasor can be thought of as a velocity at the end of the phasor. In particular, the velocity w is always at right angles to the phasor as shown in Fig.9.1

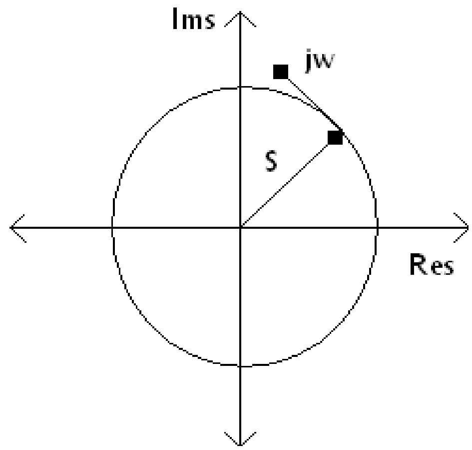


Fig.9.1

Therefore, the generalized sinusoidal signal is $S(t) = Ae^{st} = Ae^{(\sigma jw)t}$ describes the growth and decay of the amplitudes in addition to angular frequencies.

(ii) <u>Resonance</u>: The property of cancellation of reactance when inductive and capacitive reactance are in series, or cancellation of susceptance when in parallel, is called RESONANCE. Resonant circuits are formed by the combinations of inductances and capacitances which may be connected in series or in parallel giving rise to series resonant and parallel resonant circuits respectively. The cancellation of reactance when in series and cancellation of susceptance when in parallel leads to operation of reactive circuits under unity power factor conditions, or with current and voltage in phase.

<u>Types of Resonant Circuits</u>

a) Series resonance circuits

b) Parallel resonance circuits

The resonance circuits are also known as Tuned circuits.

<u>Applications</u>: Resonance circuits are used in Radio Recievers to vary various broadcasting stations.

(iii) <u>Damping Coefficient:</u> The characteristic equation of a parallel RLC circuit is

$$S^2 + \frac{1}{RC}S + \frac{1}{LC} = 0$$

The two roots of the characteristic equation are

$$S_1 = -\frac{1}{2RC} + \left[\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}\right]^{1/2} \quad \text{and}$$

$$S_2 = -\frac{1}{2RC} - \left[\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}\right]^{1/2}$$

The roots of the characteristic equation may be written as $S_1 = -\alpha \pm \sqrt{\alpha^2 - w_O^2}$ where w_o is the resonance frequency.

Where $\alpha = \dfrac{1}{2RC}$ is called the damping coefficient and it determines how quickly the oscillations in a circuit subside.

There are three possible conditions in a parallel RLC circuit as

a) Two real and distinct roots when $\alpha^2 > w_o^2$ [over damped]

b) Two real equal roots when $\alpha^2 = w_o^2$ [critically damped]

c) Two complex roots when $\alpha^2 < w_o^2$ [under damped]

Q.26 Determine the Thevenin equivalent circuit of the network shown in Fig.9. **(8)**

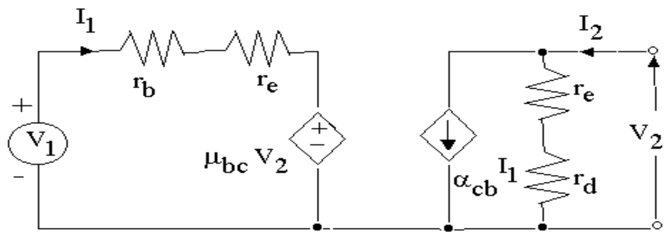


Fig.9

Ans:

By applying KVL to the left side mesh of Fig.9, we get

$(r_b + r_e)I_1 + \mu_{bc}V_2 = V_1$ ------------------------ (1)

The parallel circuit on right hand side of the network of Fig.9 gives

$V_2 = (r_e + r_d)(-\alpha_{cb}I_1)$

Or $V_2 = -(r_e + r_d)(\alpha_{cb}I_1)$

Or $I_1 = -\dfrac{V_2}{(r_e + r_d)\alpha_{cb}}$ ------------------------ (2)

By substituting the value of I_1 from equation (2) in equation (1) we have

$$(r_b + r_e)\left(-\frac{V_2}{(r_e + r_d)\alpha_{cb}}\right) + \mu_{bc}V_2 = V_1$$

$$-\left[\frac{r_b + r_e}{(r_e + r_d)\alpha_{cb}} - \mu_{bc}\right]V_2 = V_1$$

Or $$V_2 = -\left[\frac{V_1}{\dfrac{(r_b + r_e) - (r_e + r_d)\mu_{bc}\alpha_{cb}}{(r_e + r_d)\alpha_{cb}}}\right]$$

Or $$V_2 = -\left[\frac{V_1(r_e + r_d)\alpha_{cb}}{(r_b + r_e) - (r_e + r_d)\mu_{bc}\alpha_{cb}}\right] = V_{oc}$$

To find Z_{TH}:

Next, we short circuit the terminals A and B as shown in fig.10.1

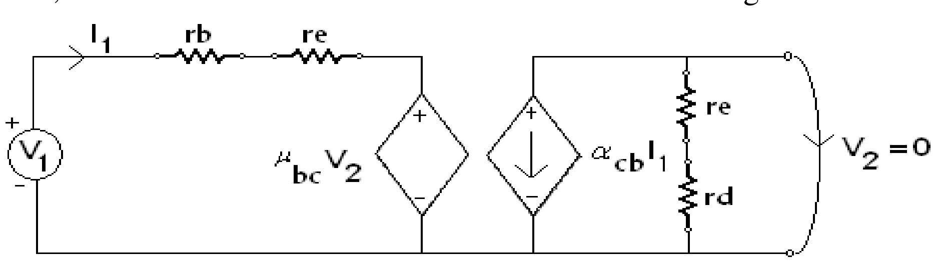


Fig.10.1

Therefore, from Fig.10.1, $V_2 = 0$ ------------------ (3)

From equations (1) and (3), we have

$(r_b + r_e)I_1 + O = V_1$

Or $I_1 = \dfrac{V_1}{(r_b + r_e)}$ ------------------ (4)

Also, from Fig.10.1, $I_{sc} = -\alpha_{cb}.I_1$ ------------------ (5)

By substituting the value of I_1 from equation (4) in equation (5), we have

$$I_{sc} = -\alpha_{cb}\left(\frac{V_1}{r_b + r_e}\right) = -\left(\frac{\alpha_{cb}V_1}{r_b + r_e}\right)\text{Amp}$$

Therefore, the Thevenin's equivalent impedance Z_{Th} is

$$Z_{Th} = \frac{V_{oc}}{I_{sc}} = -\left(\frac{\cancel{V_1}(r_e + r_d)\cancel{\alpha}_{cb}}{(r_b + r_e) - (r_e + r_d)\mu_{bc}\alpha_{cb}}\right) - \left(\frac{(r_b + r_e)}{\cancel{\alpha}_{cb}\cancel{V_1}}\right)$$

$$Z_{Th} = \left(\frac{(r_e + r_d)(r_b + r_e)}{(r_b + r_e) - (r_e + r_d)\mu_{bc}\alpha_{cb}}\right)$$

The resultant Thevenin's equivalent network is shown in Fig.10.2.

$$Z_{Th} = \left(\frac{(r_e + r_d)(r_b + r_e)}{(r_b + r_e) - (r_e + r_d)\mu_{bc}\alpha_{cb}}\right)$$

.

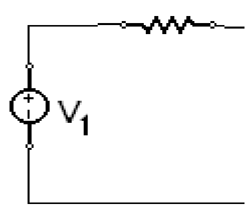


Fig.10.2

$$V_{oc} = -\left(\frac{\cancel{V_1}(r_e + r_d)\cancel{\alpha}_{cb}}{(r_b + r_e) - (r_e + r_d)\mu_{bc}\alpha_{cb}}\right)$$

Q.27 Test whether:

(i) the polynomial $F_1(s) = s^4 + s^3 + 2s^2 + 3s + 2$ is Hurwitz; and

(ii) the function $F_2(s) = \dfrac{Ks}{s^2 + \alpha}$ is positive real, where α and K are positive constants. **(8)**

Ans:

(i) The even part e(s) and odd part o(s) of the given $F_1(S)$ are

$e(S) = S^4 + 2S^2 + 2$

And $o(S) = S^3 + 3S$

Continued fraction expansion. $F_1(S) = \dfrac{e(S)}{o(S)}$ can be obtained by dividing e(S) by o(S) and then investing and dividing again as follows:-

$$
\begin{array}{l}
S^3+3S\;\big)\;S^4+2S^2+2\;\big(\;S \\
\qquad\qquad\;\; \underline{S^4+3S^2} \\
\qquad\qquad\qquad -S^2+2\;\big)\;S^3+3S\;\big(\;-S \\
\qquad\qquad\qquad\qquad\qquad\;\; \underline{S^3-2S} \\
\qquad\qquad\qquad\qquad\qquad\qquad 5S\;\big)\;-S^2+2\;\big(\;-\frac{S}{5} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\;\; \underline{-S^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2\;\big)\;5S\;\big(\;\frac{5}{2}S \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\;\; \underline{5S} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times
\end{array}
$$

Hence continued expansion $F_1(S)$ is

$$F_1(S)=\frac{e(S)}{o(S)}=S+\cfrac{1}{-S+\cfrac{1}{-\frac{S}{5}+\cfrac{1}{\frac{5}{2}S}}}$$

Since two quotient terms -1 and $-\frac{1}{5}$ out of the total quotient terms 1, -1, $-\frac{1}{5}$ and $-\frac{5}{2}$ are negative. Therefore, $F_1(S)$ is not Hurwitz.

(ii) Given that $F_2(s)=\dfrac{K(s)}{s^2+\alpha}$ $\qquad \alpha, K \ge 0$

$F_2(s)$ can be written as $F_2(s)=\dfrac{1}{\frac{S}{K}+\frac{\alpha}{KS}}$ -------------- (1)

The terms in equation (1) are $\frac{S}{K}$ and $\frac{\alpha}{KS}$. These two terms are Positive Real, because α and K are positive constants. Therefore, the sum of the two terms $\frac{S}{K}$ and $\frac{\alpha}{KS}$ must be Positive Real. Since the reciprocal of a Positive Real function is also Positive Real.
Hence the function $F_2(s)$ is also Positive Real.

Q.28 A system admittance function Y(s) has two zeros at $s=-2,-3$ and two poles at $s=-1,-4$, with system constant = 1. Synthesise the admittance in the form of three parallel branches: R_1, R_2-L_2 in series, and R_3-C_3 in series. **(8)**

Ans:
The given admittance function Y(s) is

$$Y(s)=\frac{(s+2)(s+3)}{(s+1)(s+4)} \qquad [\because \text{ Two zeros at s = -2, -3 \& Two poles at s=-1, -4}]$$

The partial fraction expansion for Y(s) is

$$Y(s)=\left.\frac{(s+2)(s+3)}{(s+4)}\right|_{s=-1}=\frac{(-1+2)(-1+3)}{(-1+4)}=\frac{1\times 2}{3}=\frac{2}{3}$$

$$Y(s) = \left.\frac{(s+2)(s+3)}{(s+1)}\right|_{s=-4} = \frac{(-4+2)(-4+3)}{(-4+1)} = \frac{(-2)(-1)}{(-3)} = \frac{-2}{3}$$

Therefore, the partial fraction expansion for Y(s) is

$$Y(s) = 1 + \frac{2/3}{s+1} + \frac{-2/3}{s+4} \qquad \text{---------------- (1)}$$

Since one of the residues in equation is negative, this equation cannot be used for synthesis. An alternative method would be to expand $\frac{Y(s)}{s}$ and then multiply the whole expansion by s.

Hence $\frac{Y(s)}{s} = \frac{(s+2)(s+3)}{s(s+1)(s+4)}$

Partial Fractions are:

$$\left.\frac{(s+2)(s+3)}{(s+1)(s+4)}\right|_{s=0} = \frac{(0+2)(0+3)}{(0+1)(0+4)} = \frac{2\times 3}{1\times 4} = \frac{6}{4} = \frac{3}{2}$$

$$\left.\frac{(s+2)(s+3)}{s(s+4)}\right|_{s=-1} = \frac{(-1+2)(-1+3)}{(-1)(-1+4)} = \frac{(1)(2)}{(-1)(3)} = -\frac{2}{3}$$

$$\left.\frac{(S+2)(S+3)}{S(S+1)}\right|_{S=-4} = \frac{(-4+2)(-4+3)}{(-4)(-4+1)} = \frac{(-2)(-1)}{(-4)(-3)} = \frac{2}{12} = \frac{1}{6}$$

Hence the Partial Fraction expansion for $\frac{Y(S)}{S}$ is

$$\frac{Y(S)}{S} = \frac{\frac{3}{2}}{S} - \frac{\frac{2}{3}}{S+1} + \frac{\frac{1}{6}}{S+4} \qquad \text{----------------------- (2)}$$

By multiplying the equation (2) by S, we obtain

$$\frac{Y(S)}{S} = \frac{3}{2} - \frac{\frac{2}{3}S}{S+1} + \frac{\frac{1}{6}S}{S+4} \qquad \text{----------------------- (3)}$$

In equation (3), Y(S) also has a negative term $\left(\frac{-\frac{2}{3}S}{S+1}\right)$.

This negative term can be eliminated by dividing the denominator of negative tern $\left(-\frac{1}{S+1}\right)$ into the numerator, we obtain i.e.

$$\frac{\frac{2}{3}(S+1) - \frac{2}{3}}{S+1} = \frac{2}{3} - \frac{\frac{2}{3}}{S+1}$$

Therefore, Admittance Function Y(S) will now become as

$$Y(S)=\frac{3}{2}-\left(\frac{2}{3}-\frac{\frac{2}{3}}{S+1}\right)+\frac{\frac{1}{6}S}{S+4}$$

$$Y(S)=\frac{5}{6}+\frac{\frac{2}{3}}{S+1}+\frac{\frac{1}{6}S}{S+4} \qquad \text{-------------------- (4)}$$

The resultant admittance function of eqn (4) consists of

(i) one resistor $R_1=\frac{6}{5}\Omega$ (ii) series combination of R_2 of $\frac{3}{2}\Omega$ & inductance L_2 of 3/2 Henrys (iii) $R_3(6\Omega)$ & $C_3\left(\frac{1}{24}F\right)$ series combination.

This is shown in Fig.11.

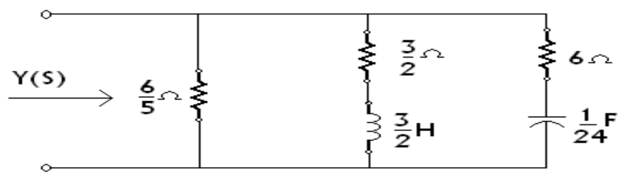


Fig.11

Q.29 Explain the meaning of "zeros of transmission". Determine the circuit elements of the constant-resistance bridged- T circuit, shown in Fig.10, that provides the voltage-ratio:

$\frac{V_2(s)}{V_1(s)}=\frac{s^2+1}{s^2+2s+1}$ .Assume R=1 Ω. **(8)**

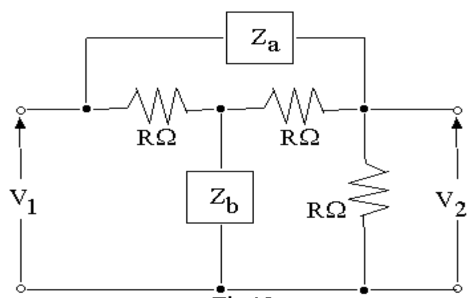


Fig.10

Ans:

<u>Zeros of Transmission</u> : A zero of transmission is a zero of a Transfer function. At a zero of transmission there is zero output for an input of the same frequency. For the network in fig.12.1, the capacitor is an open circuit at S=0, so there is a zero of transmission at S=0. For the networks in Figs.12.2 & 12.3, the zero of transmission occurs at $S=\pm j/\sqrt{LC}$. For the network in Fig.12.4, the zero of transmission occurs at $S=-\frac{1}{RC}$.

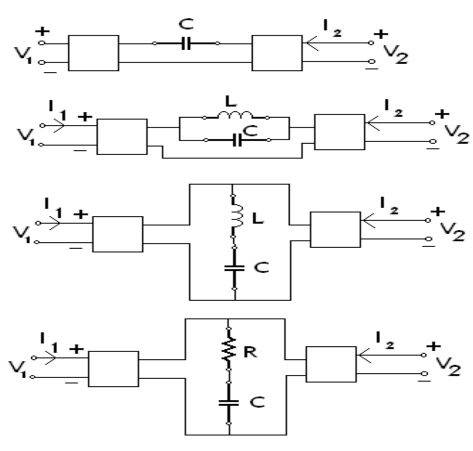


fig.12.1 to fig.12.4

The transfer functions that have zeros of transmission only on the jw axis (or) in the left-half plane are called Minimum Phase functions. If the function has one or more zeros in the right-half plane, then the function is Non-minimum phase function.

Given voltage Ratio is

$$\frac{V_2(S)}{V_1(S)} = \frac{S^2+1}{S^2+2S+1} \qquad \text{------------------------------ (1)}$$

The equation (1) can be written as

$$\frac{V_2(S)}{V_1(S)} = \frac{1}{1+\left[\dfrac{2S}{(S^2+1)}\right]} = \frac{R}{R+Z_a} = \frac{Z_b}{Z_b+R}$$

So that $Z_a = \dfrac{2S}{S^2+1}$ and $Z_b = \dfrac{S^2+1}{2S}$

It can be recognized as Z_a is a Parallel L-C Tank Circuit and Z_b is a Series L-C Tank Circuit. The resultant final network is shown in Fig.12.5.

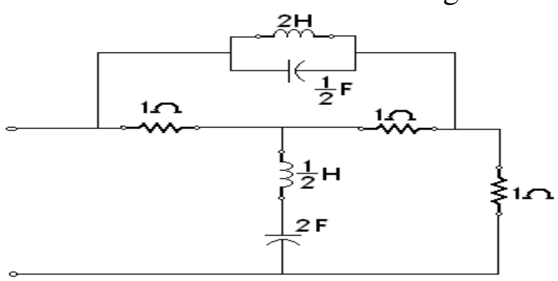


Fig.12.5

Q.30 Synthesise a ladder network whose driving-point impedance function is given by

$$Z(s) = \frac{2s^5+12s^3+16s}{s^4+4s^2+3}. \qquad \textbf{(8)}$$

Ans:

The continued fraction expansion for Z(S) is given as follows:

$$S^4+4S^2+3 \;\big)\; 2S^5+12S^3+16S \;\big(\; 2S \leftrightarrow Z$$
$$\underline{2S^5+8S^3+6S}$$
$$4S^3+10S \;\big)\; S^4+4S^2+3 \;\big(\; \tfrac{1}{4}S \leftrightarrow Y$$
$$\underline{S^4+\tfrac{5}{2}S^2}$$
$$\tfrac{3}{2}S^2+3 \;\big)\; 4S^3+10S \;\big(\; \tfrac{8}{3}S \leftrightarrow Z$$
$$\underline{4S^3+8S}$$
$$2S \;\big)\; \tfrac{3}{2}S^2+3 \;\big(\; \tfrac{3}{4}S \leftrightarrow Y$$
$$\underline{\tfrac{3}{2}S^2}$$
$$3 \;\big)\; 2S \;\big(\; \tfrac{2}{3}S \leftrightarrow Z$$
$$\underline{2S}$$
$$0$$

The final network for the impedance function Z(S) is shown in Fig.12.6.

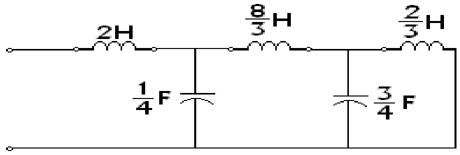


Fig.12.6

Q.31 Using nodal analysis, find the power dissipated in the 4Ω resistor of the network shown in Fig.2. **(8)**

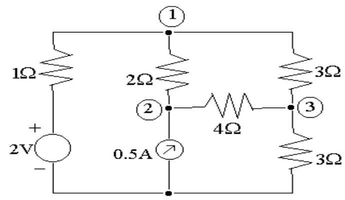


Fig.2

Ans.

Assume voltages V_1, V_2 and V_3 at nodes 1,2 and 3 respectively as shown in Fig.2.1.

At node 1, using nodal method we have $\frac{V_1 - 2}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{3} = 0$.

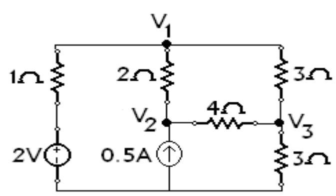


Fig.2.1.

$$V_1 - 2 + \frac{V_1}{2} - \frac{V_2}{2} + \frac{V_1}{3} - \frac{V_3}{3} = 0$$

$$V_1 + \frac{V_1}{2} - \frac{V_2}{2} + \frac{V_1}{3} - \frac{V_3}{3} = 2$$

$$1.83V_1 - 0.5V_2 - 0.33V_3 = 2 \qquad \text{-------------------- (1)}$$

At node 2, nodal equations are

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{4} = 0.5A$$

$$\frac{V_2}{2} - \frac{V_1}{2} + \frac{V_2}{4} - \frac{V_3}{4} = 0.5A$$

$$0.75V_2 - 0.5V_1 - 0.25V_3 = 0.5$$

$$-0.5V_1 + 0.75V_2 - 0.25V_3 = 0.5 \qquad \text{------------------- (2)}$$

At node 3, nodal equations are

$$\frac{V_3 - V_1}{3} + \frac{V_3 - V_2}{4} + \frac{V_3}{3} = 0$$

$$\frac{V_3}{3} + \frac{V_3}{4} + \frac{V_3}{3} - \frac{V_1}{3} - \frac{V_2}{4} = 0$$

$$-0.33V_1 - 0.25V_2 + 0.9167V_3 = 0 \qquad \text{------------------- (3)}$$

Applying Cramer's rule to equations (1), (2) and (3), we have

$$V_2 = \frac{\Delta_2}{\Delta}$$

Where

$$\Delta = \begin{vmatrix} 1.83 & -0.5 & -0.33 \\ -0.5 & 0.75 & -0.25 \\ -0.33 & -0.25 & 0.9167 \end{vmatrix}$$

= 1.83[(0.75)(0.9167)-(0.25)(0.25)] + 0.5[-(0.5)(0.9167) – (0.25)(0.33)]
-0.33[(0.5)(0.25) + (0.33)(0.75)]
= 1.83[0.688 – 0.0625] + 0.5[-0.458 – 0.0825] - 0.33[0.125 + 0.2475]
= 1.83[0.6255] + 0.5[-0.5405] – 0.33[0.3725]
= 1.14467 – 0.2702 – 0.1229
= 1.14467 – 0.393125 = 0.7515

$$\Delta_2 = \begin{vmatrix} 1.83 & 2 & -0.33 \\ -0.5 & 0.5 & -0.25 \\ -0.33 & 0 & 0.9167 \end{vmatrix}$$

= 1.83[(0.5)(0.9617) + 0] – 2[(-0.5)(0.9617) – (0.25)(0.33)] – 0.33[(0.33)(0.5)]
= 1.83[0.4583] -2[-0.45835 – 0.0825] -0.33[0.165]
= 0.83868 -2[-0.54085] – 0.33[0.165]
= 0.83868 + 1.0817 – 0.05445 = 1.8659

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{1.8659}{0.7515} = 2.4829\,V$$

Similarly,

$$V_3 = \frac{\Delta_3}{\Delta}$$

Where $\Delta_3 = \begin{vmatrix} 1.83 & -0.5 & 2 \\ -0.5 & 0.75 & 0.5 \\ -0.33 & -0.25 & 0 \end{vmatrix}$

= 1.83[(0.75)0 + (0.25)(0.5) + 0.5[0+(0.33)(0.5)] + 2[(0.5)(0.25) + (0.33)(0.75)]
= 1.83[0.125] + 0.5[0.165] + 2[0.125 + 0.2475]
= 1.83[0.125] + 0.5[0.165] + 2[0.3725]
= 0.22875 + 0.0825 + 0.745 = 1.056

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{1.056}{0.7515} = 1.405\,V$$

The current in the 4Ω resistor is

$$I_4 = \frac{V_2 - V_3}{4} = \frac{2.4829 - 1.0586}{4} = 0.3567\ A$$

$\therefore$ The power dissipated in the 4Ω resistor is

$$I_4^2 R_4 = (0.3567)^2 \times 4 = 0.508\text{ W}$$

Q.32 The switch S in the circuit shown in Fig.3 is closed at t = 0. Obtain an expression for $v_c(t)$, $t > 0$. **(8)**

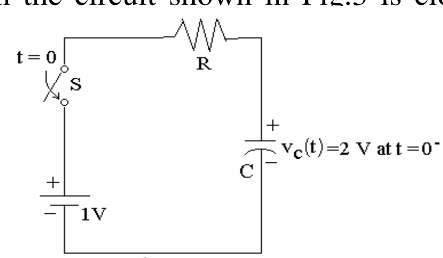


Fig.3

Ans:

Let the current in the circuit is i(t). Applying KVL in the circuit shown in Fig.3. We have

$I = Ri(t) + V_c(t)$

And $V_c(t) = 2 + \frac{1}{c}\int_0^t i(t)dt$

Therefore $1 = Ri(t) + 2 + \frac{1}{c}\int_0^t i(t)dt$

Or $Ri(t) + \frac{1}{c}\int_0^t i(t)dt = -1$

On differentiating the above equation, we get

$R\frac{di(t)}{dt} + \frac{1}{c}i(t) = 0$

Or $\frac{di(t)}{dt} + \frac{1}{Rc}i(t) = 0$

The general solution of the above differential equation is

$i(t) = K.e^{-\frac{1}{Rc}t}$

Since initial voltage across the capacitor is 2V, therefore the initial current is

$i(o^+) = \frac{V}{R} = \frac{1-2}{R} = -\frac{1}{R} = Ke^0$

$K = -\frac{1}{R}$

So, the value of the current i(t) is

$i(t) = -1/\mathrm{R}e^{-\frac{-1}{Rc^t}}$

Then $V_c(t) = 2 + \frac{1}{c}\int_0^t i(t)dt$

$V_c(t) = 2 + \frac{1}{c}\int_0^t -\frac{1}{\mathrm{R}}\,\mathrm{e}^{\frac{-1}{Rc}}\,dt$

$V_c(t) = 2 - \frac{1}{Rc}\int_0^t e^{-\frac{1}{Rc^t}}dt$

$= 2 - \frac{1}{Rc}\left[\frac{e^{-\frac{1}{Rc^t}}}{-\frac{1}{Rc}}\right]\Big|_0^t$

$= 2 - \frac{1}{R¢}\times -R¢\left[e^{-\frac{1}{Rc^t}}\right]\Big|_0^t\Big|$

$= 2 + \left(e^{-\frac{1}{Rc^t}} - 1\right)$

$$V_c(t) = 1 + e^{-\frac{1}{Rc^t}} V$$

Q.33 A sinusoidal excitation $x(t) = 3\cos\left(200t + \frac{\pi}{6}\right)$ is given to a network defined by the input [x(t)] – output [y(t)] relation: $y(t) = x^2(t)$. Determine the spectrum of the output. **(8)**

Ans:
The given sinusoidal excitation x(t) is

$$x(t) = 3\cos\left(200t + \pi/6\right)$$

Thus the output is Y(t) is given by

$$Y(t) = x^2(t)$$

$$= \left[3\cos\left(200t + \pi/6\right)\right]^2$$

Or $Y(t) = \left[9\cos^2\left(200t + \pi/6\right)\right]$

Or $Y(t) = 9\left[\frac{\cos 2\left(200t + \pi/6\right) + 1}{2}\right]$ $\left[\because \quad \cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)\right]$

Or $Y(t) = \frac{9}{2}\left[1 + \cos\left(400t + \pi/3\right)\right]$

Or $Y(t) = \left[\frac{9}{2} + \frac{9}{2}\cos\left(400t + \pi/3\right)\right]$ ------------------------- (1)

The output Y(t) or equation (1) consists of

(i) The first term is of $\left(\frac{9}{2}\right)$ d.c. component.

(ii) The second term is a sinusoidal component of amplitude $\left(\frac{9}{2}\right)$ and frequency $\omega = 400$ or $2\pi f = 400$ or $f = \frac{400}{2\pi}$ or $f = \frac{200}{\pi}$. The spectrum of the output is shown in Fig.3.1.

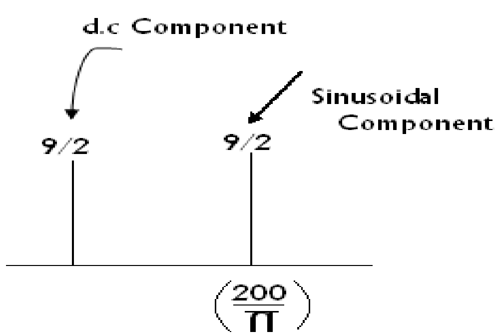


Fig.3.1.

Q.34 Using Kirchhoff's laws to the network shown in Fig.4, determine the values of v_6 and i_5. Verify that the network satisfies Tellegen's theorem. **(8)**

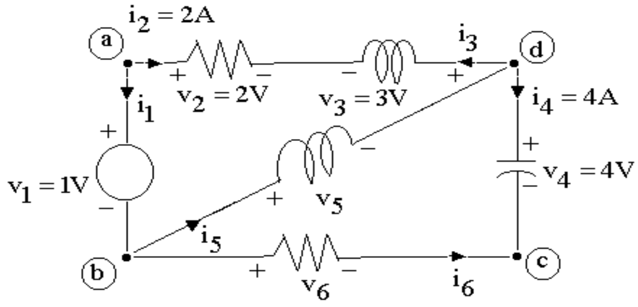


Fig.4

Ans:

(i) Applying KVL in loop (a) (d) (b),

$V_5 = -V_1 + V_2 - V_3$

= -1 + 2 -3 = -2V

(ii) Applying KVL in loop (b) (d) (c),

$V_6 = V_5 + V_4$

= -2 + 4 = 2V.

$V_6 = 2V$

Determination of current i_5:

In the loop (a), (d), $i_3 = -i_2 = -2A$

Applying KCL at node (a), we get

$i_1 = -i_2 = -2A$

Applying KCL at node (c), we get

$i_6 = -i_4 = -4A$

Applying KCL at node (b), we get

$i_5 = i_1 - i_6 = -2 + 4 = 2A$

Applying KCL at node (d), we get

$i_5 = i_3 + i_4 = -2 + 4 = 2A$

Therefore summarising the voltages and currents

$V_1 = 1V$, $V_2 = 2V$, $V_3 = 3V$, $V_4 = 4V$, $V_5 = -2V$, $V_6 = 2V$

$i_1 = -2A$, $i_2 = 2A$, $i_3 = -2A$, $i_4 = 4A$, $i_5 = 2A$, $i_6 = -4A$

Applying Tellegaris Theorem to the voltages and currents, we get

$$\sum_{k=1}^{6} V_k i_k = (1\times -2) + (2\times 2) + (3\times -2) + (4\times 4) + (-2\times 2) + (2\times -4) = 0$$

Hence proved.

Q.35 Allowing transients to die out with switch S in position 'a', the switch is then moved to position 'b' at t=0, as shown in Fig.5. Find expressions for $v_c(t)$ and $v_R(t)$ for t > 0. **(8)**

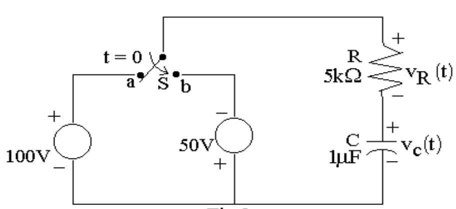


Fig.5

Ans:

When the switch is at Position 'a', steady state is reached and the capacitor acquires 100V potential, current being zero, as shown in Fig.4.1.

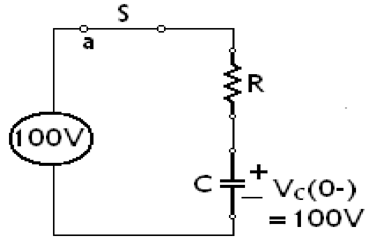


Fig.4.1

$\therefore \quad V_c(0) = 100V; \qquad i(0^-) = 0$

However, just after the switching the potential current will increase but the capacitor voltage will not change initially.

$\therefore \quad V_c(0^+) = 100V$

Also, the polarity of the capacitor is such that the 50V source becomes additive with $V_c(0^+)$ but drives the current in opposite direction, as shown in Fig.4.2.

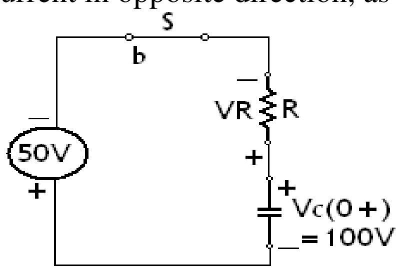


Fig.4.2.

The total voltage in the circuits is thus $\therefore \quad V_c(0^+) = 50 \ V$

=100V+50V=150V

RC being equal to 5000x1x10^{-6} = 5x10^{-3} sec the transient part of Vc is given by $150.e^{-2\times10^2 t}$

However, the steady state voltage will be (-50V) as this opposite current i will now charge the capacitor in the opposite direction and will raise the potential to negative value in time t = α. Hence the total solution becomes

$V_c = V_{c\,transient} + V_{c\,steady\ state} = \left(150.e^{-2\times10^2 t} - 50\right)V.$

However, at time t>0, when the switch 'S' is at position 'b', KVL application gives

$V_R - V_C - 50 = 0$ (V_R being the resistive drop)

$\therefore V_R = 50 + V_C = 50 + 150.e^{-2\times10^2 t} - 50$

$V_R = 150.e^{-2\times10^2 t}.V$

Q.36 The only information known about the system in the black-box in Fig.6 is that:

(i) it is an initially relaxed linear system,

(ii) when $v_i(t) = \delta(t)$, output is $v_o(t) = \left(e^{-2t} + e^{-3t}\right)u(t)$.

Determine the system excitation $v_i(t)$ required to produce a response $v_o(t) = t.e^{-2t}.u(t)$. **(8)**

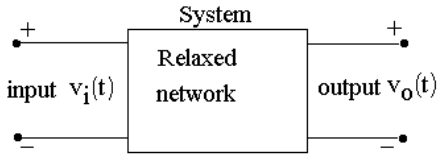


Fig.6

Ans:

The Transfer Function T(s) of the network shown in Fig.4.3 is given by

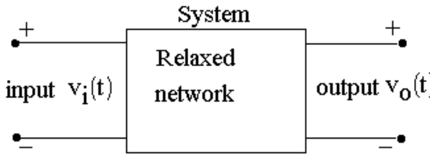


Fig.6

Fig.4.3

$$T(s) = \delta\{(e^{-2t} + e^{-3t})u(t)\}$$

$$= \frac{1}{S+2} + \frac{1}{S+3} = \frac{(S+3)+(S+2)}{(S+2)(S+3)}$$

Or $T(s) = \dfrac{(2S+5)}{(S+2)(S+3)}$

Again, $V_0(t) = t.e^{-2t}u(t)$ [given]

$$V_0(s) = \frac{1}{(S+2)^2}\{\therefore \delta V_0(t) = V_0(s) = \delta[t.e^{-2t}u(t)]\}$$

But $V_0(s) = T(s).V_I(s)$

$$\therefore V_I(s) = \frac{V_0(s)}{T(s)} = \frac{1}{(S+2)^2} \times \frac{(S+2)(S+3)}{(2S+5)}$$

$$V_I(s) = \frac{S+3}{(S+2)(2S+5)} = \frac{K_1}{S+2} + \frac{K_2}{2S+5}$$

Using Partial Fraction method

$K_1 = -0.5; K_2 = 1$

$$\therefore V_I(s) = \left[\frac{(-0.5)}{S+2} + \frac{1}{2S+5}\right] = \left[\frac{(1/2)}{S+2.5} - \frac{0.5}{S+2}\right]$$

i.e. $V_i(t) = [0.5e^{-2.5t} - 0.5e^{-2t}]u(t)$

Therefore, the System Excitation $V_i(t)$ becomes

$V_i(t) = [e^{-25t} - e^{-2t}]u(t)$

Q.37 For the symmetrical 2-port network shown in Fig.7, find the z-and ABCD-parameters. **(8)**

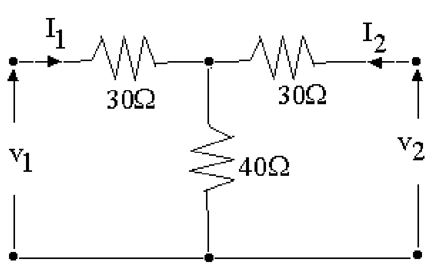


Fig.7

Ans:

Using KVL for loops ABCD and CDEF, the loop equations for Fig.7 are given by

$V_1 = 30I_1 + 40(I_1 + I_2)$

Or $\quad V_1 = 70I_1 + 40I_2$ ---------------- (i)

Or $\quad V_2 = 30I_2 + 40(I_1 + I_2)$

Or $\quad V_2 = 40I_1 + 70I_2$ ---------------- (ii)

<u>Z-parameters:</u>

<u>Case I:</u> When $I_2 = 0$, From equations (i) and (ii)

$V_1 = 70I_1$, and $V_2 = 40I_1$

Or $\quad I_1 = \frac{V_1}{70}$

Therefore,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = \frac{V_1}{\left(\frac{V_1}{70}\right)} = 70\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = \frac{40I_1}{I_1} = 40\Omega$$

<u>Case II:</u> When $I_1 = 0$, from equations (i) and (ii)

$V_1 = 40I_2$ and $V_2 = 70I_2$ or $I_2 = \left(\frac{V_2}{70}\right)$

Therefore,

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = \frac{40I_2}{I_2} = 40\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = \frac{V_2}{\left(\frac{V_2}{70}\right)} = 70\Omega$$

Condition of Symmetry $Z_{11} = Z_{22}$ is satisfied ABCD parameters:

$$A = \frac{Z_{11}}{Z_{21}} = \frac{70\Omega}{40\Omega} = 1.75$$

$$B = \frac{Z_{22}Z_{11} - Z_{12}Z_{21}}{Z_{21}} = \frac{70 \times 70 - 40 \times 40}{40}$$

$$B = \frac{4900 - 1600}{40} = \frac{3300}{40} = 82.5\Omega$$

$$C = \frac{1}{Z_{21}} = \frac{1}{40} = 0.025\,\text{mho}$$

$$\mathrm{D} = \frac{\mathrm{Z}_{22}}{\mathrm{Z}_{21}} = \frac{70}{40} = 1.75$$

Condition of Symmetry A = D is satisfied.

Q.38 Determine the y-parameters for the network shown in Fig.8. **(8)**

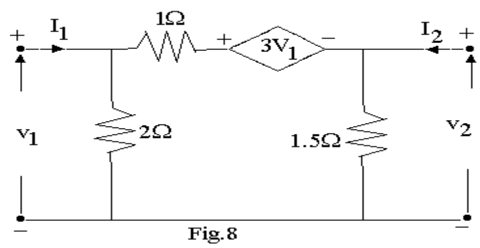


Fig.8

Ans:

Let I_3 be the current in middle loop.

By applying KVL, we get

From the Figure 5.1, we have $V_1 = 2(I_1 - I_3)$ -------------------- (i)

$2(I_3 - I_1) + 1.I_3 + 3V_1 + 1.5(I_3 + I_2) = 0$

Or $4.5I_3 = 2I_1 - 1.5I_2 - 3V_1$ --------------------- (ii)

And $V_2 = 1.5(I_2 + I_3)$ --------------------- (iii)

From equations (i), (ii), and (iii), we have

$$V_1 = 2I_1 - 2\left[\frac{2I_1}{4.5} - \frac{1.5I_2}{4.5} - \frac{3V_1}{4.5}\right]$$

$$= 2I_1 - 2[0.44I_1 - 0.33I_2 - 0.666V_1]$$

$$= 2I_1 - 0.888I_1 - 0.666I_2 - 1.333V_1$$

$$V_1 = 1.112I_1 - 0.666I_2 - 1.333V_1$$

$$V_1 + 1.333V_1 = 1.112I_1 - 0.666I_2$$

$$2.333V_1 = 1.112I_1 - 0.666I_2$$

Or $V_1 = 0.476I_1 - 0.285I_2$ -------------------- (iv)

And

$$V_2 = 1.5I_2 + 1.5\left[\frac{2I_1}{4.5} - \frac{1.5I_2}{4.5} - \frac{3V_1}{4.5}\right]$$

$$V_2 = 1.5I_2 + 0.666I_1 - 0.499I_2 - 1.00V_1$$

$$V_2 = 0.666I_1 + 1.00I_2 - 1.00V_1$$

$$= 0.666I_1 + 1.00I_2 - 1.00[0.476I_1 - 0.285I_2]$$

$$\left[\because V_1 = 0.476I_1 - 0.285I_2 \quad \textit{from equation}(iv)\right]$$

$V_2 = 0.19I_1 + 1.285I_2$ ------------------ (v)

Therefore, from equations (iv) and (v), we have

$$Z = \begin{bmatrix} 0.476 & -0.285 \\ 0.19 & 1.285 \end{bmatrix}$$

Now Y-Parameters can be found out by Z-Parameters.

$$[Y] = [Z]^{-1} = \begin{bmatrix} 0.476 & -0.285 \\ 0.19 & 1.285 \end{bmatrix}^{-1} = \begin{bmatrix} -1.285 & -0.285 \\ 0.19 & -0.476 \end{bmatrix}$$

Q.39 Given the network function $F(s) = \dfrac{s+1}{s^2 + 2s + 5}$, plot the zero and poles on s-plane. Obtain the amplitude and phase for $F(j3)$ from the plot. **(8)**

Ans:

The zeros are given by the roots of P(s) = s + 1 => s = -1 and hence the zero is at s = -1. The poles are given by the roots of the equation

$$Q(s) = s^2 + 2s + 5 = 0$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4 \times 5}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4j}{2}$$

$$s = -1 \pm 2j$$

Hence the roots of the given equation are at s = -1 + 2j and -1 -2j and the poles are at $s = (+1 + 2j)$ and $s = (+1 - 2j)$.

<u>Finding of Amplitude and Phase for F(j3) from the POLE-ZERO Plot:-</u>

The Pole-Zero Plot on jω axis is shown in Fig.6.1. From the Poles and Zeros of F(s), we draw Vectors to the Point w=3, as shown in the figure.

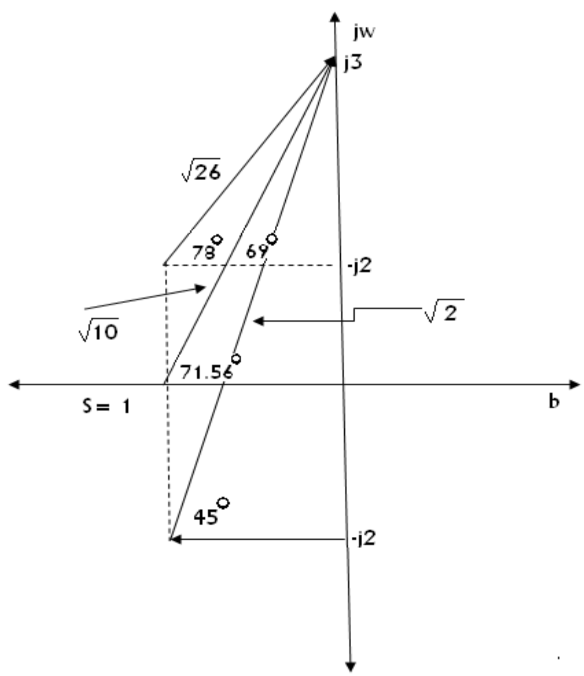


Fig.6.1.

Now F(s) may be represented as

$$F(j\omega) = |M(j\omega)| e^{\Phi(j\omega)}$$

$$F(j3) = \frac{j3+1}{(j3+1+2j)(j3+1-2j)} = \frac{1+j3}{(1+5j)(1+j)}$$

Conversion of Rectangular form into Polar from

$$\begin{array}{l|l|l} 1+j3 & 1+5j & 1+j \\ R=\sqrt{1^2+3^2}=\sqrt{10} & R=\sqrt{1^2+5^2}=\sqrt{26} & R=\sqrt{1^2+1^2}=\sqrt{2} \\ \Phi = Tan^{-1}(3) & \Phi = Tan^{-1}\left(\frac{5}{1}\right) & \Phi = Tan^{-1}\left(\frac{1}{1}\right) \\ \quad = 71.56^0 & \quad = 78.69^0 & \quad = 45^0 \\ Hence\ 1+j3=\sqrt{10}\underline{|71.56^0} & Hence\ 1+5j=\sqrt{26}\underline{|78.69^0} & Hence\ 1+j=\sqrt{2}\underline{|45^0} \end{array}$$

$$F(j3)=\frac{\sqrt{10}\underline{|71.56^0}}{\sqrt{26}\underline{|78.69^0}\sqrt{2}\underline{|45^0}}$$

Now $F(j3)=|M(j3)|\Phi(j3)$

Where $|M(j3)|=\dfrac{\sqrt{10}}{\sqrt{26}.\sqrt{2}}=\dfrac{3.1622}{5.099\times 1.4142}=0.4385$

Therefore, the Magnitude of the given function F(3j) is

$|M(j3)|=0.4385$ and

The phase of the given function F(3j) is

$$\Phi(j3)=\frac{71.56^0}{78.69^0+45^0}$$

$$= 71.56^0-78.69^0-45^0$$

$\Phi(j3)=-52.13^0$

The phase of the given function F(3j) is

$\Phi(j3)=-52.13^0$

Q.40 The pole configuration shown in Fig.9 refers to the system function H(s) of a single-tuned circuit. Construct the peaking circle and show the locations of the half power frequencies. **(8)**

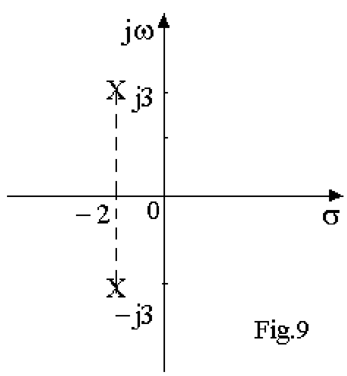


Fig.9

Ans:

From the pole configuration of Fig.9, the system function H(s) of a single-tuned circuit is given by

$$H(s)=\frac{1}{(s+2+j3)(s+2-j3)}$$

Peaking Circle: The Poles of H(s) is shown in Fig.6.4. We next draw the peaking circle with the center at s = -2 and the radius equal to 3. At the point where the circle intersects the jw axis, we see that $\omega_{\max}^2 = \beta^2 - \alpha^2$. To check this result, the equation

$$\omega_{\max} = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5} = 2.236$$

$$[\because \quad \beta = 3 \,\&\, \alpha = 2]$$

$\therefore \quad \omega_{\max} = 2.236$

The amplitude $|H(j\omega_{\max})|$ is then

$$|H(j2.236)| = \frac{1}{(j2.236+2+j3)(j2.236+2-j3)} = \frac{1}{(2+5.23j)(2-0.76j)}$$

Or $|H(j(2.236))| = \dfrac{1}{(2+5.23j)(2-0.76j)} = \dfrac{1}{11.979} = 0.0834$

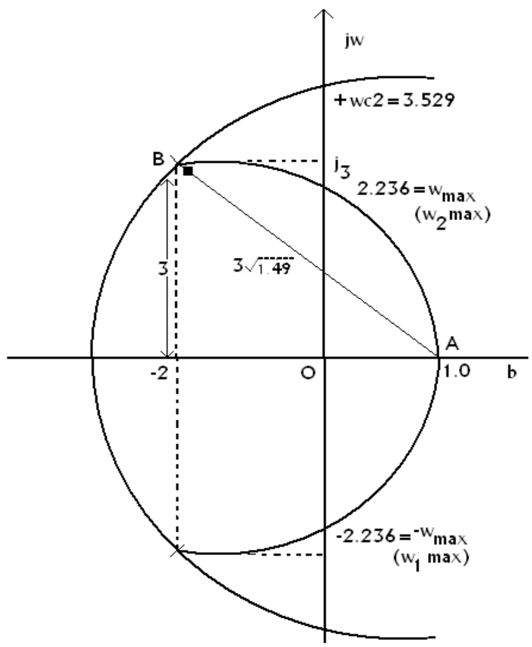


Fig.6.4

Half-Power Frequencies:- The point A at which the Peaking Circle intersects the positive real axis is located at s = 1.0. With the center at A, we draw a circle of radius AB (equal to $3\sqrt{1.49}$), shown in Fig.6.4. At point C, where this new circle intersects the jw axis, we have ω_c & at point D where this new circle intersects the –jw axis, we have $-\omega_c$.

By measurement, we find $\omega_c \simeq$ 3.529 & $-\omega_c \simeq$ -3.529.

Q.41 Find the driving-point impedance $Z(s) = K \cdot \dfrac{N(s)}{D(s)}$, for the network shown in Fig.10. Verify that Z(s) is positive real and that the polynomial D(s) + K.N(s) is Hurwitz. **(8)**

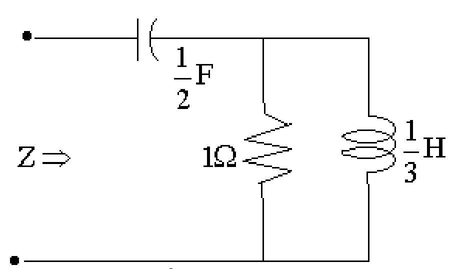


Fig.10

Ans:

The Laplace Transform equivalent of the given network is shown in Fig.7.2.

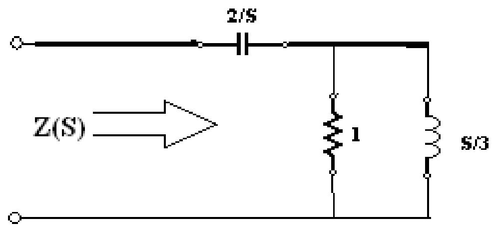


Fig.7.2

The driving-point impedance for the network of Fig.7.2 is

$$Z(s) = \frac{2}{s} + \left(1 \middle\| \frac{s}{3}\right) = \frac{2}{s} + \left(\frac{1\times\frac{s}{3}}{1+\frac{s}{3}}\right) = \frac{2}{s} + \left(\frac{\frac{s}{\cancel{3}}}{\frac{3+s}{\cancel{3}}}\right)$$

$$= \frac{2}{s} + \frac{s}{s+3} = \frac{2(s+3)+s^2}{s(s+3)} = \frac{s^2+2s+6}{s(s+3)} = \frac{s^2+2s+6}{s^2+3s}$$

Therefore, $Z(s) = \dfrac{s^2+2s+6}{s^2+3s}$ ------------------------- (1)

I. **<u>Testing for Positive Real:</u>**

The driving point impdance Z(s) is in the form

$Z(s) = \dfrac{s^2+a_1 s+a_0}{s^2+b_1 s+b_0}$ ------------------------- (2)

By comparing equations(1) and (2), we have

$a_1 = 2, \quad a_0 = 6$

$b_1 = 3, \quad b_0 = 0$.

(i) <u>Test I for Positive Real:</u>

All the coefficient $[a_1 a_0, b_1 \,\&\, b_0]$ of Z(s) are real and Positive Constants.

(ii) <u>Test II for Positive Real:</u>

$a_1 b_1 \geq \left(\sqrt{a_0} - \sqrt{b_0}\right)^2$

Here $a_1 = 2 \,\&\, b_1 = 3$

Hence $a_1 b_1 = 6$ and

$\left(\sqrt{a_0} - \sqrt{b_0}\right) = \left(\sqrt{6} - \sqrt{0}\right)^2 \qquad (\because\ a_0 = 6 \,\&\, b_0 = 0)$

$= \left(\sqrt{6}\right)^2 = 6$

Therefore, $a_1 b_1 = 6 = \left(\sqrt{a_0} - \sqrt{b_0}\right)^2 = 6$

So, the driving point impedance Z(s) is <u>Positive Real Function.</u>

II. **<u>Testing for Hurwitz:</u>**

Now, we write the driving point impedance of the form

$Z(s) = K\dfrac{N(s)}{D(s)} = 1.\dfrac{s^2+2s+6}{s^2+3s}$

Where K = 1. $D(s) = s^2+3s$ & $N(s) = s^2+2s+6$

Hence $D(s) + KN(s) = s^2+3s+1(s^2+2s+6)$

$$= s^2+3s+s^2+2s+6 = 2s^2+5s+6$$

Or $D(s)+K.N(s)=2s^2+5s+6$ ------------------- (3)

Test (i):- All the coefficient of the polynomial $2s^2+5s+6$ are positive and real. Hence it is Hurwitz.

Test(ii):- The given polynomial is Hurwitz because it is a quadratic. With no missing term and all its coefficients are of positive sign.

Q.42 Design a one-port L-C circuit that contains only two elements and has the same driving-point impedance as that of the network shown in Fig.11. **(8)**

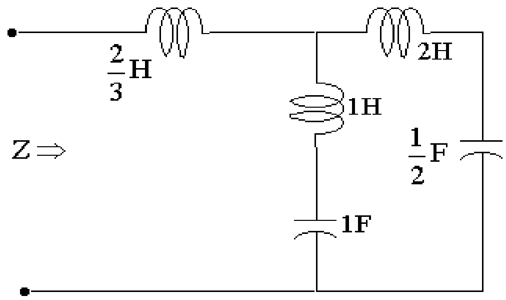


Fig.11

Ans:

The Laplace Transform equivalent of the given network is shown in Fig.7.4

The driving point impedance for the network of Fig.7.4 is

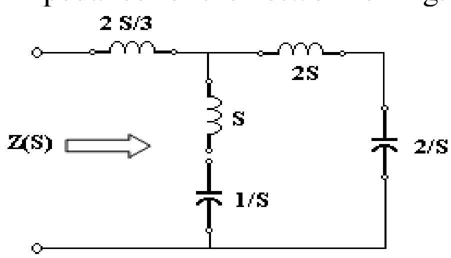


Fig.7.4

$$Z(s)=\frac{2s}{3}+\left[\left(s+\frac{1}{s}\right)\middle\|\left(2s+\frac{2}{s}\right)\right]$$

$$=\frac{2s}{3}+\left[\frac{\left(s+\frac{1}{s}\right)\left(2s+\frac{2}{s}\right)}{s+\frac{1}{s}+2s+\frac{2}{s}}\right] = \frac{2s}{3}+\left[\frac{\left(\frac{s^2+1}{s}\right)\left(\frac{2s^2+2}{s}\right)}{3s+\frac{3}{s}}\right]$$

$$=\frac{2s}{3}+\left[\frac{\left(s+\frac{1}{s}\right)\left(2s+\frac{2}{s}\right)}{\frac{3s^2+3}{s}}\right] = \frac{2s}{3}+\left[\frac{2s^2+\frac{2s}{s}+\frac{2s}{s}+\frac{2}{s^2}}{\frac{3s^2+3}{s}}\right]$$

$$=\frac{2s}{3}+\left[\frac{2s^2+2+2+\frac{2}{s^2}}{\left(\frac{3s^2+3}{s}\right)}\right] = \frac{2s}{3}+\left[\frac{2s^4+2s^2+2s^2+2}{s^2}\times\frac{\cancel{s}}{3s^2+3}\right]$$

$$= \frac{2s}{3} + \left[\frac{2s^4 + 4s^2 + 2}{s(3s^2 + 3)}\right] = \left[\frac{2s(3s^3 + 3s) + 3(2s^4 + 4s^2 + 2)}{3s(3s^2 + 3)}\right]$$

$$= \frac{6s^4 + 6s^2 + 6s^4 + 12s^2 + 6}{9s^3 + 9s} = \frac{12s^4 + 18s^2 + 6}{9s^3 + 9s} = \frac{\cancel{3}(4s^4 + 6s^2 + 2)}{\cancel{3}(3s^3 + 3s)}$$

Therefore, the driving point impedance for the given network is

$$Z(s) = \frac{4s^4 + 6s^2 + 2}{3s^3 + 3s}$$

Now the driving point impedance $Z(s) = \dfrac{4s^4 + 6s^2 + 2}{3s^3 + 3s}$ is synthesized in CAUER Form-I i.e. by continued fractions method.

$$3s^3 + 3s \;)\; 4s^4 + 6s^2 + 2 \;(\; \frac{4s}{3} \leftrightarrow Z_1 = L = \frac{4}{3}H$$
$$\underline{-4s^4 - 4s^2}$$
$$2s^2 + 2 \;)\; 3s^3 + 3s \;(\; \frac{3s}{2} \leftrightarrow Y_1 = C = \frac{3}{2}F$$
$$\underline{-3s^3 - 3s}$$
$$0$$

Therefore, the synthesized network of Z(s) is shown in Fig.7.5 and it contains only two elements L & C. Hence it is proved.

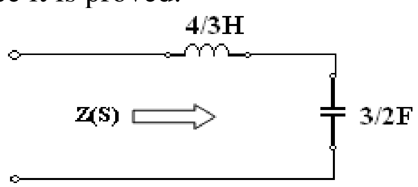


Fig.7.5

Q.43 Synthesise the voltage-ratio $\dfrac{V_2(s)}{V_1(s)} = \dfrac{(s+2)(s+4)}{(s+3)(3s+4)}$ using constant-resistance bridged-T circuits. **(8)**

Ans:

The given voltage ratio $\dfrac{V_2(s)}{V_1(s)}$ for constant-resistance bridged-T circuits is

$$\frac{V_2(s)}{V_1(s)} = \frac{(s+2)(s+4)}{(s+3)(3s+4)} \qquad \text{----------------------- (1)}$$

At first, we break up the voltage ratio in equation (1) into two separate voltage ratios i.e.,

$$\frac{V_2(s)}{V_1(s)} = \frac{V_a(s)}{V_1(s)} \cdot \frac{V_2(s)}{V_a(s)} \qquad \text{----------------------- (2)}$$

By company equation (2) with equation (1), we have

$$\frac{V_a(s)}{V_1(s)} = \frac{s+2}{s+3} \qquad \text{----------------------- (3)}$$

And $\dfrac{V_2(s)}{V_a(s)} = \dfrac{s+4}{3s+4}$ ----------------------- (4)

For a constant-resistance bridged-T circuit, the voltage-ratio transfer function is given as

$$\frac{V_2(s)}{V_1(s)} = \frac{R}{R+Z_a} = \frac{Z_b}{Z_b+R}$$

Or $\dfrac{V_2(s)}{V_1(s)} = \dfrac{1}{1+Z_a} = \dfrac{Z_b}{Z_b+1}$ ---------------------- (5)

[$\because$ for constant-resistance R = 1]

By comparing equation (3) with equation (5), the voltage ratio $\dfrac{V_a(s)}{V_1(s)}$ becomes

$$\frac{V_a(s)}{V_1(s)} = \frac{s+2}{s+3} = \frac{Z_{b_1}}{Z_{b_1}+1}$$

So that $Z_{b_1} = s+2$ and

$$Z_{a_1} = \frac{1}{s+2}$$

Also by company the equation (4) with equation (5), the voltage ratio $\dfrac{V_2(s)}{V_a(s)}$ becomes

$$\frac{V_2(s)}{V_a(s)} = \frac{s+4}{3s+4} = \frac{1}{1+Z_{a_2}}$$

From which we find $Z_{a_2} = \dfrac{2s}{s+4}$ and $Z_{b_2} = \dfrac{s+4}{2s}$

The final synthesized network is shown in Fig.8.1.

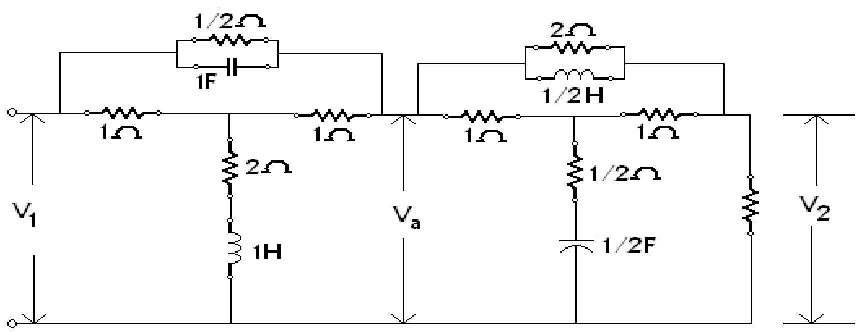


Fig.8.1

Q.44 Design a one-port RL network to realize the driving point function $F(s) = \dfrac{3(s+2)(s+4)}{s(s+3)}$. **(8)**

Ans:

If F(s) is an impedance Z(s), it must be an R-C impedance on the other hand, if F(s) represents on Admittance, it must be on R-L network.

Now the given driving point function F(s) is

$$F(s) = \frac{3(s+2)(s+4)}{s(s+3)} = \frac{3\left(s^2+4s+2s+8\right)}{s^2+3s}$$

Or $F(s) = \dfrac{3\left(s^2+6s+8\right)}{s^2+3s} = \dfrac{3s^2+18s+24}{s^2+3s}$

Therefore, the driving point function F(s) is realized by the continues fraction expansion i.e., (CAUER Form – I)

$$
\begin{array}{l}
s^2+3s \;\big)\; 3s^2+18s+24 \;\big(\; 3 \quad \leftarrow Y_1(s)=\frac{1}{3}\Omega \\
\qquad\qquad\quad \underline{3s^2+9s} \\
\qquad\qquad\qquad 9s+24 \;\big)\; s^2+3s \;\big(\; \frac{1}{9}s \quad \leftarrow Z_2(s)=\frac{1}{9}H \\
\qquad\qquad\qquad\qquad\qquad \underline{s^2+\frac{8}{3}s} \\
\qquad\qquad\qquad\qquad\qquad\quad \frac{1}{3}s \;\big)\; 9s+24 \;\big(\; 27 \quad \leftarrow Y_3(s)=\frac{1}{27}\Omega \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{9s} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 24 \;\big)\; \frac{1}{3}s \;\big(\; \frac{s}{72} \quad \leftarrow Z_4(s)=\frac{1}{72}H \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{\frac{1}{3}s} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \times
\end{array}
$$

The synthesized and designed R L network is shown in Fig.8.2

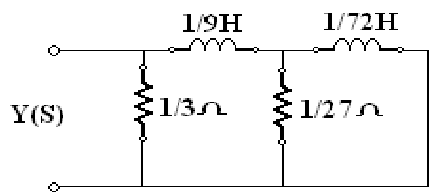


Fig.8.2

Q.45 Synthesise the network that has a transfer admittance $Y_{21}(s)=\dfrac{s^2}{s^3+3s^2+4s+2}$ and a 1Ω termination at the output end. **(8)**

Ans:

The given transfer admittance $Y_{21}(s)$ is

$$Y_{21}(s)=\frac{3s^2+2}{s^3+3s^2+4s+2}$$

The transfer admittance function has two zeros of transmission at s = 0 and one zero at $s=\infty$. Since the numerator is even, we divide by s^3+4s, so that

$$Y_{22}=\frac{3s^2+2}{s^3+4s}$$

Now synthesize Y_{22} to give a zero of transmission at $s=\infty$ and two zeros at s = 0. First, a parallel inductor gives us a zeros of transmission at s = 0. We can remove this parallel inductor by removing the pole at s = 0 of Y_{22} to give

$$Y_1=Y_{22}-\frac{1}{2s}=\frac{5s/2}{s^2+4}$$

If we invert Y_1, we see that we have a series L-C combination, which gives us another transmission zero at s = 0, as represented by the $\frac{5}{8}$-Farad Capacitor and we have the zero of

$=\infty$

transmission at s , also when we remove the inductor of $\frac{2}{5}H$. Therefore, the final realization of $Y_{21}(s)$ is shown in Fig.9.1.

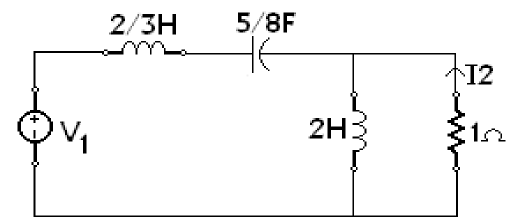


Fig.9.1

Next is to find the order n from 50 decibels at w = 2 specification. The less can be given as approximately.

Loss $= 20\log 0.509 + 6(n-1) + 20n\log 2$

Given that loss is no less than 40db attenuation

Or we assume 50db attenuation, then

$$50 \cong 20\log 0.509 + 6(n-1) + 20n\log 2$$

$$50 \cong -5.8656 + 6n - 6 + 6.020n$$

Or $50 \cong -11.8656 + 12.020n$

Or $50 + 11.8656 - 12.020n = 0$

Or n = 5.14

Therefore, we obtain n = 5.14. Since n must be an integer, we let n = 5.

With the specification of n and ε , the pole locations are completely specified.

Next is to determine these pole locations. First we must find β_k .

β_k for a chebyshev polynomial of LPF is given by

$$\beta_k = \frac{1}{n}\sinh^{-1}\frac{1}{\in}$$

Q.46 Obtain the system function H(s) for a low-pass filter exhibiting Chebyshev characteristics with 1dB ripple in the passband $0 < \omega < 1$ and no less than 40 dB attenuation in the stopband starting at $\omega = 2$. **(8)**

Ans:

We obtain a system function H(s) for a low pass filter exhibiting chebyshev characteristics with 1-decibel ripple in the pass band and no less than 40db attenuation (i.e., we assume 50db attenuation), in the stopband starting at ω=2.

When we design for 1-decibel ripple, we know that at ω=1, $|H(j1)|$ is down 1 decibel so that

$$20\log|H(j1)| = 20\log\frac{1}{\left(1+\in^2\right)^{1/2}} = -1$$

We then obtain $\frac{1}{\left(1+\in^2\right)^{1/2}} = 0.891$

And $\in = 0.509$

Where n is degree of the chebyshev polynomial and $\in$ is the factor controlling ripple width

$\in = 0.509$

Then,

$$\beta_k = \frac{1}{5}\sinh^{-1}\left(\frac{1}{0.509}\right)$$

$$\beta_k = 0.2855$$

In order to find the normalized chebyshev poles from the Butterworth Poles, we must determine $\tanh \beta_k$.

Here we have

$\tanh \beta_k = \tanh 0.2855 = 0.27798$

From the table of Butterworth Polynomial, the n = 5.

Butterworth Poles are $(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$

The roots of the above equation are

$S_1 = -1.0$, $S_{2,3} = -0.309 \pm 0.951j$ and $S_{3,4} = -0.809 \pm 0.5878j$

Multiplying the real parts of these poles by 0.27798, we obtain the normalized chebyshev poles.

$S_1^1 = -0.27798$; $S_{2,3}^1 = -0.0859 \pm 0.951j$ and $S_{4,5}^1 = -0.2249 \pm 0.5878j$

Finally, the denormalized chebyshev poles are obtained by multiplying the normalized ones by $\cosh\beta_k = \cosh(0.2855) = 1.04$, so that the denormalized Poles are

$$S_1 = (-0.27798 \times 1.04)$$

$$S_{2,3} = (-0.309 \pm 0.951j)(1.04) \quad \text{and}$$

$$S_{4,5} = (-0.2249 \pm 0.5878j)(1.04)$$

Therefore,

$$(S_1 = -0.289)$$

$$S_{2,3} = (-0.321 \pm 0.99j)$$

$$S_{4,5} = (-0.234 \pm 0.613j)$$

H(s) for a low-pass filter exhibiting chebyshev characteristics is then

$$H(s) = \frac{1.113}{(s+0.289)(s+0.321+.99j)(s+0.321-0.99j)(s+0.234+0.613j)(s+0.234-0.613j)}$$

Q.47 Draw the dual of the network shown in Fig.2, listing the steps involved. **(8)**

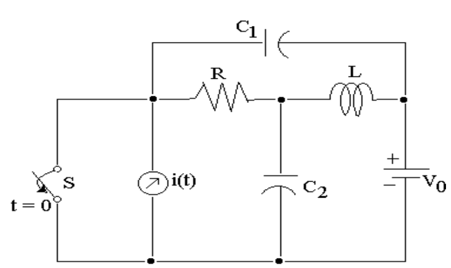


Fig.2

Ans:

The following points are the steps for converting he given network into its equivalent dual network:

(i) There are four meshes in the given network shown in Fig.2.1. Inside each mesh of the given network. Place a dot(node). Assign numbers(1, ,2, 3 & 4) to these dots for convenience. These dots serve as nodes for the dual network. Place an extra dot(node) outside the network.

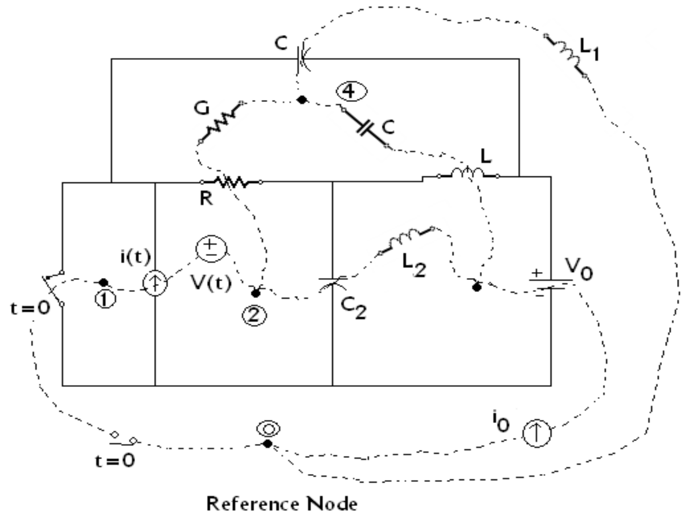
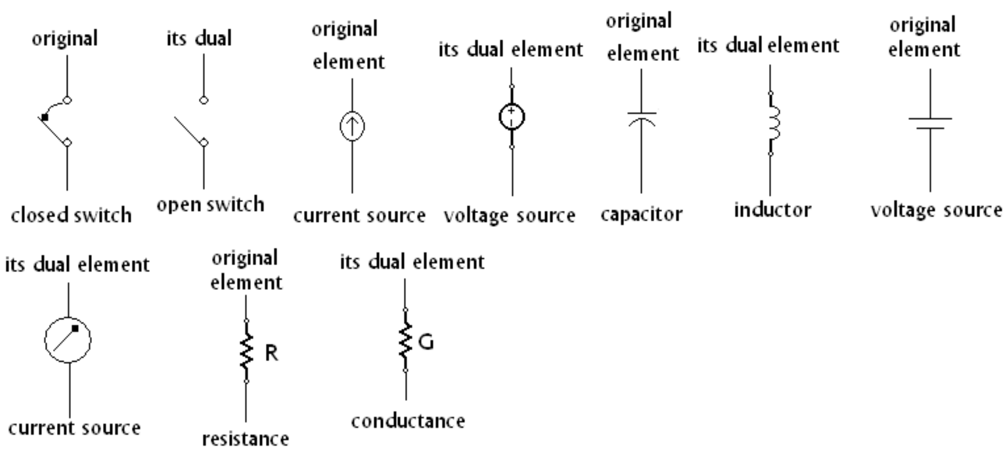


Fig.2.1

This dot is going to be the reference node(O) for the dual network.

(ii) Draw dotted lines from node to node through each element in the original network, traversing only one element at a time is shown in Fig.2.1. Each element thus traversed in the original network, is now replaced by its dual element as given below:-



(iii) Continue this process till all the elements in the original network are accounted for.

(iv) The network constructed in this manner is the dual network. The dual network of the given network is shown in Fig.2.2.

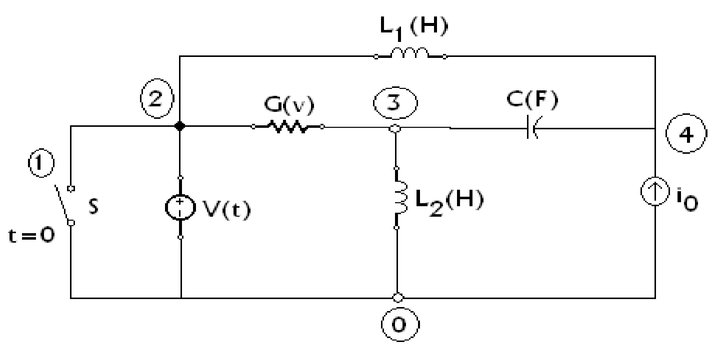


Fig.2.2.

Q.48 Using superposition theorem for the network shown in Fig.3, find the value of i_x. **(8)**

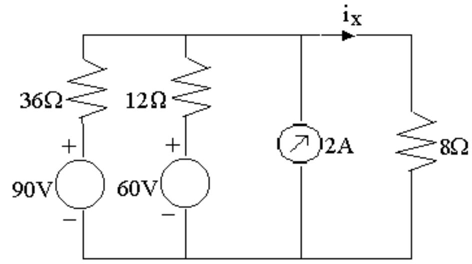


Fig.3

Ans:

First we assume that the 90V source is operating alone and 2A source and 60V sources are suppressed, and the resultant network is shown in Fig.2.4

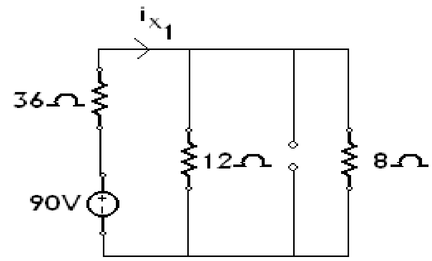


Fig.2.4

The current i_{x_1} given out by the 90V source is

$$i_{x_1} = \frac{90}{36+(12\|8)} = \frac{90}{36+\dfrac{12\times 8}{12+8}} = \frac{90}{36+\dfrac{96}{20}} = \frac{90}{40.8} = 2.20A$$

Next we consider the 60V source and 2A source & 90V sources are suppressed and the resultant network is shown in Fig.2.5.

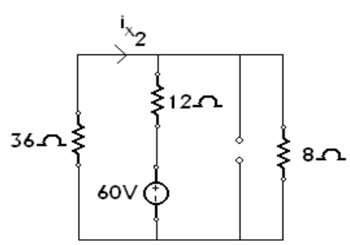


Fig.2.5

The current i_{x_2} given out by 60V source is

$$i_{x_2} = \frac{60}{36+(12\|8)} = \frac{60}{36+\dfrac{96}{20}} = \frac{60}{40.8} = 1.47A$$

Finally, we consider the 2A source alone and 90V & 60V sources are suppressed. The resultant network is shown in Fig.2.6.

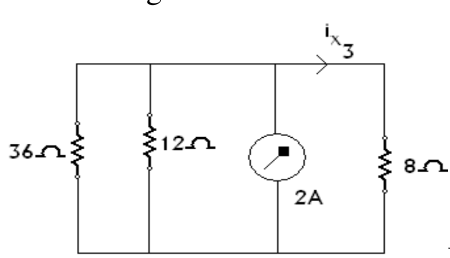


Fig.2.6

The resistance offered by the path for the flow of current i_{x_3} is shown in Fig.2.7

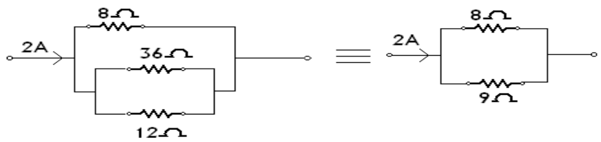


Fig.2.7

$$\frac{36\times12}{36+12}=\frac{432}{48}=9\Omega$$

Therefore, $i_{x_3}=2\times\frac{9}{9+8}=1.05A$

Hence the current $i_x=i_{x_1}+i_{x_2}+i_{x_3}$

$= 2.20 + 1.47 + 1.05 = 4.72\text{A}$

Q.49 Find the transient voltage $v_R(t)$ 40 µs after the switch S is closed at t = 0 in the network shown in Fig.4. **(8)**

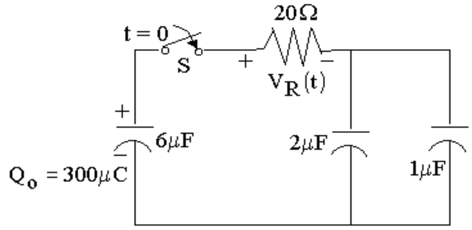


Fig.4

Ans:

The equivalent capacitance of parallel capacitors 2µF & 1µF is 3µF. As soon as the switch S is closed, this equivalent 3µF capacitor is in series with the capacitor C_0 (6µF) will become as

$$\frac{3\mu F\times6\mu F}{3\mu F+6\mu F}=\frac{18\mu F}{9\mu F}=2\mu F$$

Therefore, Time Constant (T) = $RC_t=20\times2\times10^{-6}=40\mu S$

The initial voltage V_0 across capacitor C_0 is given by

$$V_0=\frac{Q_0}{C_0}=\frac{300\mu C}{6\times10^{-6}}=50V$$

With closing of switch S, the capacitor C_0 will start discharging, however at $t=o^+$, there will be no voltage across C_1 and C_2.

Thus, the entire voltage drop will be across R only i.e., V_R at $t=o^+$ time

$\therefore\quad V_R=V_0$ (decaying)

$$V_R=V_0.e^{-t/RC}=50.e^{-t/40\times10^{-6}}$$

$$V_R=50.e^{-25\times10^4t}V$$

Q.50 Obtain the Thevenin equivalent of the network shown in Fig.5. Then draw the Norton's equivalent network by source transformation. **(8)**

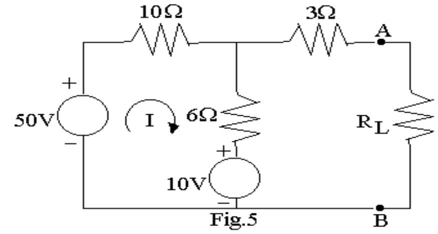


Fig.5

Ans:

First the load resister R_L is removed from terminals A-B and the circuit configuration is shown in Fig.3.3 is obtained.

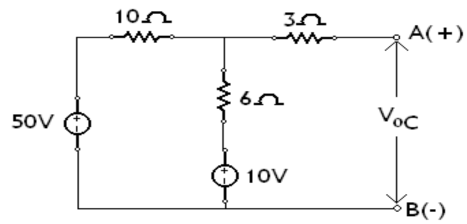


Fig.3.3

Application of KCL at node C in Fig.3.3 results

$$\frac{50-V_{oc}}{10}+\frac{10-V_{oc}}{6}-\frac{V_{oc}}{3}=0$$

$$\frac{50}{10}-\frac{V_{oc}}{10}+\frac{10}{6}-\frac{V_{oc}}{6}-\frac{V_{oc}}{3}=0$$

$$5-0.1V_{oc}+1.67-0.167V_{oc}-0.34V_{oc}=0$$

$$6.67-0.600V_{oc}=0 \Rightarrow 0.600V_{oc}=6.67$$

$$V_{oc}=\frac{6.67}{0.607}=10.99V$$

Q.51 Find the initial conditions $i(0+)$ and $\left.\frac{di(t)}{dt}\right|_{t=0+}$ for the circuit shown in Fig.6, assuming that there is no initial charge on the capacitor. What will be the corresponding initial conditions if an inductor with zero initial current were connected in place of the capacitor? **(8)**

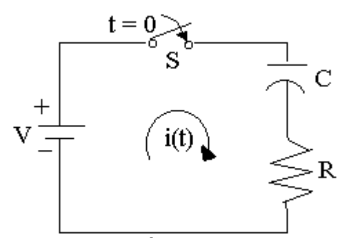


Fig.6

Ans:

Just before the switch S is closed, there is no current in the circuit.

Therefore, $i(o^-)=0$ and also there is no voltage across the capacitor.

Hence, $V_c(o^-)=0$

The circuit for initial conditions is obtained by placing the capacitor by a short circuit as shown in Fig.4.2

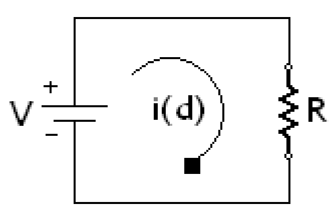


Fig.4.2

Therefore, $i(o^{+}) = \frac{V}{R}$

Now in order to find Thevenin's Resistance (R_{TH}), the independent voltage sources are removed by short circuits shown in Fig.3.4.

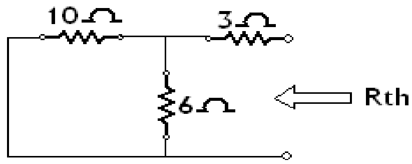


Fig.3.4

$$R_{TH} = [10\|6] + 3 = \frac{10\times 6}{10+6} + 3 = \frac{60}{16} + 3 = 6.75\Omega$$

Now, the Thevenin equivalent of the network for Fig.3.2 is shown n Fig.3.5

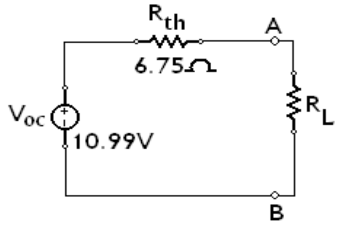


Fig.3.5

Therefore, the Norton's equivalent of the network of Fig.3.5 is shown in Fig.3.6.

$$I_{sc} = \frac{V_{oc}}{R} = \frac{10.99}{6.75} = 1.628A$$

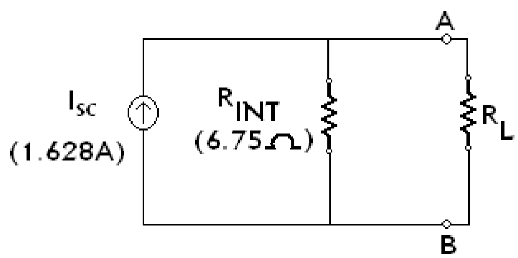


Fig.3.6

<u>Replacing of an Inductor in place of the capacitor:-</u>

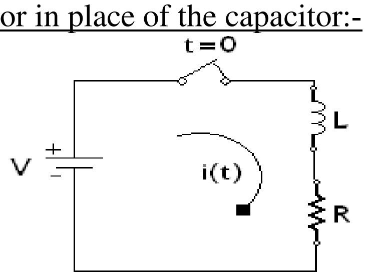


Fig.4.4

The initial condition for the circuit shwn in Fig.4.4, (replacing of capacitor in p;ace of an inductor) is that the current $i(o^{-})$ is zero before the switch is closed and it remains zero just after the switching, because of the presence of inductor 'L', where it places are open circuit shown in Fig.4.5.

Therefore $i(o^{+}) = 0$

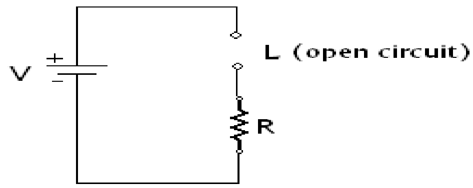


Fig.4.5

$\left.\frac{di}{dt}\right|_{t=o^+}$: To determine the initial values of $\frac{di}{dt}$, write the equation of the network taking the initial value of the voltage of inductance (which is zero) into consideration. Thus the circuit of Fig.4.6 is obtained.

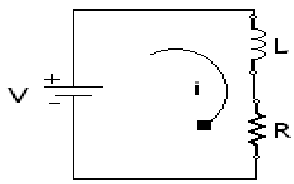


Fig.4.6

By KVL for Fig.4.6, we get

$$V = L\frac{di}{dt} + Ri \qquad \text{---------------------- (1)}$$

$\left.\frac{di}{dt}\right|_{t=o^+}$: To determine the initial values of $\frac{di}{dt}$, write the equation of the network taking the initial value of the voltage of capacitance (which is zero) into consideration. Thus the circuit of Fig.4.3 is obtained

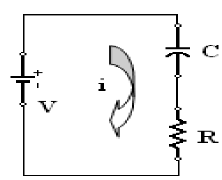


Fig.4.3

By KVL, we get

$$V = Ri + \frac{1}{C}\int idt \qquad \text{-------------------- (i)}$$

Differentiating eqn (1) w.r. to (i), we get

$$O = R\frac{di}{dt} + \frac{i}{C}$$

$$R\frac{di}{dt} = -\frac{i}{C} \Rightarrow R\left.\frac{di}{dt}\right|_{t=o^+} = \frac{-i(o^+)}{C}$$

$$\left.\frac{di}{dt}\right|_{t=o^+} = -\frac{1}{RC}\left[i(o^+)\right] = -\frac{1}{RC}\times\frac{V}{R}$$

$$\left.\frac{di}{dt}\right|_{t=o^+} = -\frac{V}{R^2C}$$

From eqn (1)

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

$$\left.\frac{di}{dt}\right|_{t=o^+} = \frac{V}{L} - \frac{R}{L}\left[i\left(o^+\right)\right]$$

$$\left.\frac{di}{dt}\right|_{t=o^+} = \frac{V}{L} - O \qquad \left[\because \quad i(o^+) = 0\right]$$

$$\left.\frac{di}{dt}\right|_{t=o^+} = \frac{V}{L}$$

Q.52 After steady-state current is established in the R-L circuit shown in Fig.7 with switch S in position 'a', the switch is moved to position 'b' at t = 0. Find $i_L(0+)$ and $i(t)$ for t > 0. What will be the value of i(t) when t = 4 seconds? **(8)**

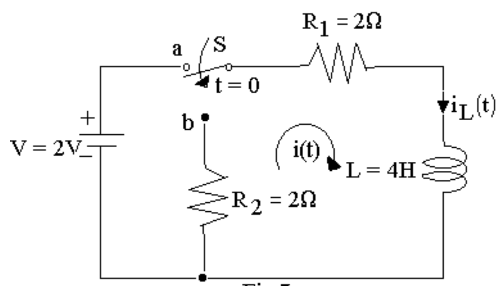


Fig.7

Ans:

When the switch 'S' is in position 'b', Kirchhoff's Voltage Law (KVL) gives

$$L\frac{di}{dt} + R_1 i + R_2 i = 0 \qquad \text{--------------------- (1)}$$

But from the Fig.7, $L = 4H$, $R_1 = 2\Omega$ & $R_2 = 2\Omega$

By substituting these values in equation (1), we get

$$4\frac{di}{dt} + 2i + 2i = 0$$

$$4\frac{di}{dt} + 4i = 0 \quad \Rightarrow \frac{di}{dt} + i = 0 \qquad \text{-------------------- (2)}$$

By applying Laplace Transform to the equation (2), we get

$$SI(s) - i(o^+) + I(s) = 0$$

$$I(s)(s+1) = i(o^+) \qquad \text{-------------------- (3)}$$

Now, the current just before switching to position 'b' is given by (shown in Fig.4.8)

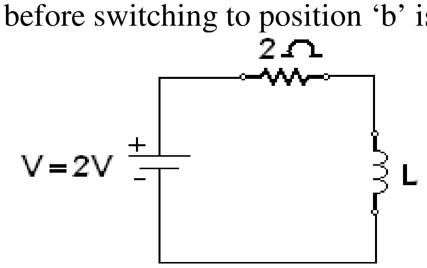


Fig.4.8

$$i(o^-) = \frac{V}{R} = \frac{2}{2} = 1Amp$$

The current $i(o^+)$ after switching to position 'b' must also be 1amp, because of the presence of inductance L in the circuit. Therefore, the equation (3) will become

$$I(s)(s+1) = i(o^+)$$

$$I(s)(s+1)=1 \quad \Rightarrow I(s)=\frac{1}{s+1} \qquad \text{-------------------- (4)}$$

On taking Laplace Transform to equation (4) , we get

$i(t)=e^{-t}$

When t = 4 seconds, finding of i(t) : the value of i(t) will be

$i(t)=e^{-4}$.

Q.53 Determine the amplitude and phase for F(j2) from the pole-zero plot in s-plane for the network function $F(s)=\frac{4s}{s^2+2s+2}$. **(8)**

Ans:

The given network function is $F(s)=\frac{4s}{s^2+2s+2}$

In factored form, F(s) will become as

$$F(s)=\frac{4s}{(s+1+j1)(s+1-j\,1)}$$

F(s) has (i) Zero at s = 0

(ii) Poles are located at (-1+j1) & (-1-j1)

From the poles and zeros of F(s), draw vectors to the point $jw = 2$, as shown in Fig.5.1.

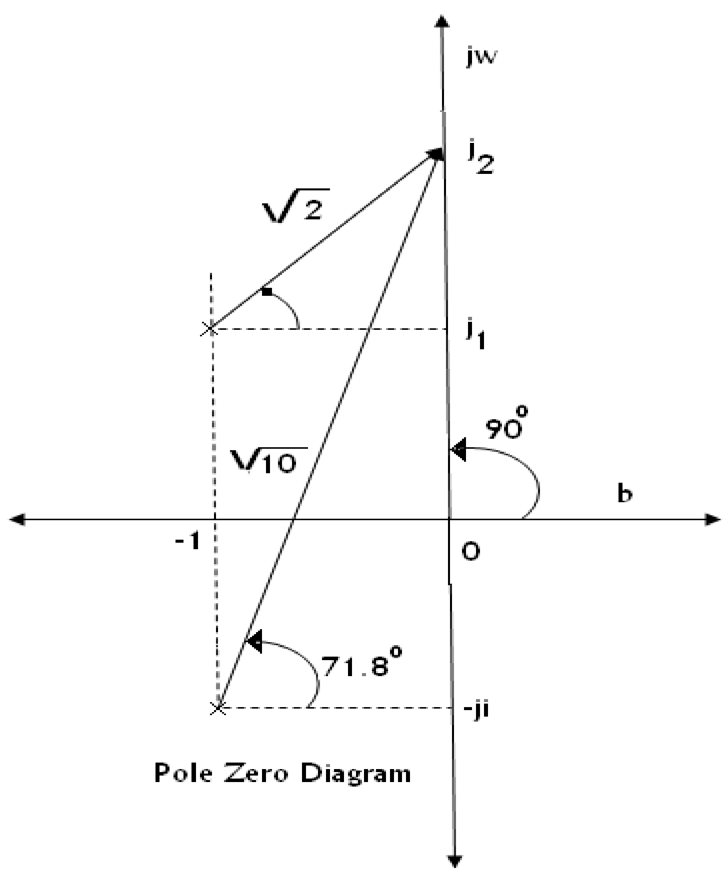


Fig.5.1. Pole-Zero Diagram

From the pole-zero diagram, it is clear that
Magnitude at F(j2) is calculated as

$M(j2) = 4\left(\dfrac{2}{\sqrt{2}\times\sqrt{10}}\right) = 1.78$ and

Phase at F(j2) is calculated as

$\phi(j2) = 90^0 - 45^0 - 71.8^0 = -26.8^0$

Q.54 Determine, by any method, the frequency of maximum response for the transfer function $H(s) = \dfrac{34}{s^2 + 6s + 34}$ of a single-tuned circuit. Find also the half power frequency. **(8)**

Ans:

The given Transfer Function is $H(s) = \dfrac{34}{s^2 + 6s + 34}$

In factored form, H(s) is

$$H(s) = \frac{34}{(s+3+js)(s+3-js)}$$

<u>Construction of Peaking Circle</u> :

H(s) has poles at (-3+js) and (-3-js), as shown in Fig.5.2. Next draw the Peaking Circle with the Center at s = -3 and the radius equal to 5. At the point, where the circle intersects the jw axis is $\omega_{\max} = 4$. To check this result by the Pythagorean Theorem, the equation is

$\omega_{\max}^2 = \beta^2 - \alpha^2$

then

$\omega_{\max}^2 = (5^2 - 3^2) = (25-9)$

$\omega_{\max}^2 = 16 \quad \Rightarrow \omega_{\max} = \sqrt{16} = 4$

Frequency of Maximum Response

$$\omega_{\max} = 4$$

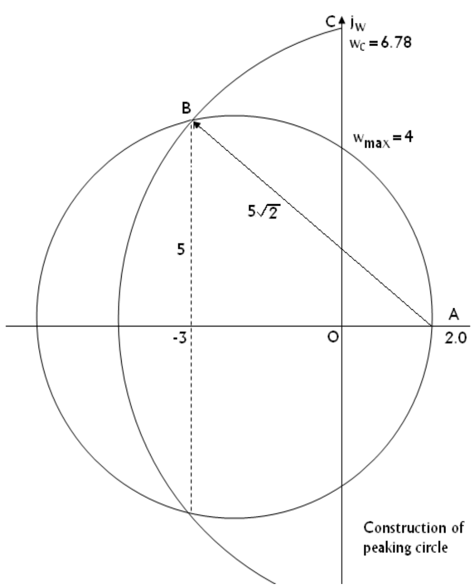


Fig.5. 2

The point A at which the Peaking Circle intersects the positive real axis is located at s=2.0. With the center at A, draw a circle of radius AB (equal to $5\sqrt{2}$). At the point C, where the new circle intersects the jω axis, is called the Half-Power Frequency (ω_c).
By the measurement from the Fig.5.2, we find
$\omega_c \cong 6.78$

or

It can also be calculated as the line segment is of length $5\sqrt{2}$ units long. Then the line segment AO is of length

Then ω_c is given as

$$\omega_c = \sqrt{(AC)^2 - (AO)^2} = \sqrt{\left(5\sqrt{2}\right)^2 - (2)^2} = \sqrt{50-4} = \sqrt{46} = 6.782$$

Q.55 For the resistive 2-port network shown in Fig.8, find v_2/v_1. **(8)**

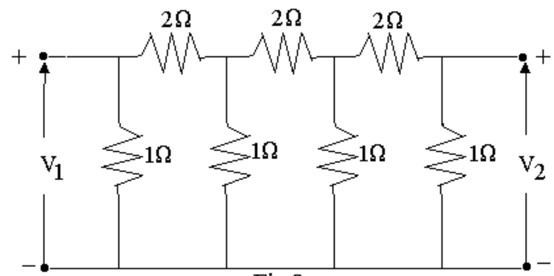


Fig.8

Ans:
The given resistive 2-port network is redrawn with marking of nodes and voltages at the node is shown in Fig.6.2.

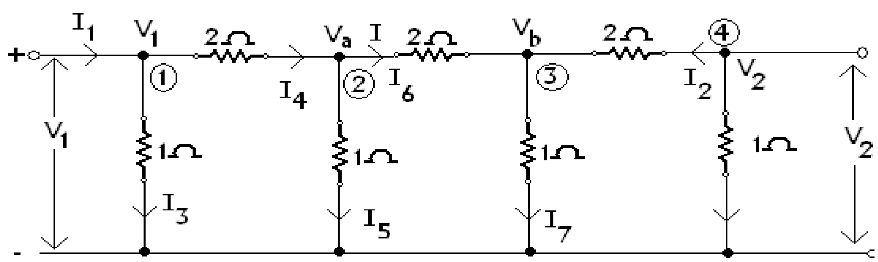


Fig.6.2

At node (1), application of KCL yields

$$I_1 = I_3 + I_4 = \frac{V_1}{1} + \frac{V_1 - V_a}{2} = \frac{2V_1 + V_1}{1} - \frac{V_a}{2} = \frac{3V_1}{2} - \frac{V_a}{2} \qquad \text{------------------ (1)}$$

At node (2), application of KCL yields

$$I_4 = I_5 + I_6 \Rightarrow \frac{V_1 - V_a}{2} = \frac{V_a}{1} + \frac{V_a - V_b}{2}$$

Or $\dfrac{V_1}{2} - \left(\dfrac{V_a + 2V_a + V_a}{2}\right) + \dfrac{V_b}{2}$

Or $\dfrac{V_1}{2} - \dfrac{4V_a}{2} + \dfrac{V_b}{2} = 0$

Or $\dfrac{V_1}{2} - 2V_a + \dfrac{V_b}{2} = 0$ ------------------------ (2)

At node (3), application of KCL yields

$I_6 = I_7 - I_2$ or

$$\frac{V_a - V_b}{2} = \frac{V_b}{1} - \frac{V_2 - V_b}{2}$$

Or $$\frac{V_a}{2} + \left[\frac{-V_b - 2V_b + V_b}{2}\right] + \frac{V_2}{2} = 0$$

$$\frac{V_a}{2} - \frac{3V_b + V_b}{2} + \frac{V_2}{2} = 0$$

Or

Or $$\frac{V_a}{2} - V_b + \frac{V_2}{2} = 0 \qquad \text{------------------------ (3)}$$

At node (4), KCL yields,

Current through the load of 1Ω = current flows between nodes (3) & (4)

i.e. $\frac{V_2}{1} = \frac{V_b - V_2}{2}$ or

$$\frac{2V_2 + V_2}{2} - \frac{V_b}{2} = 0 \quad \text{or}$$

$$\frac{3V_2}{2} - \frac{V_b}{2} = 0 \qquad \text{------------------------ (4)}$$

From equation (4), $V_b = 3V_2$ and

From equation (3), we have

$$\frac{V_a}{2} - V_b + \frac{V_2}{2} = 0 \qquad \text{----------------------- (5)}$$

By substituting the value of V_b in equation (5), we get

$$\frac{V_a}{2} = 3V_2 + \frac{V_2}{2} \quad \text{or}$$

$$\frac{V_a}{2} = \frac{7V_2}{2} \quad \text{or} \quad V_a = 7V_2$$

From equation (2), we have

$$\frac{V_1}{2} - 2V_a + \frac{V_b}{2} = 0$$

$$\frac{V_1}{2} - 2(7V_2) + \frac{V_b}{2} = 0 \qquad [\because \; V_a = 7V_2]$$

Or $$\frac{V_1}{2} - 14V_2 + \frac{3V_2}{2} = 0 \quad (\because \; V_b = 3V_2)$$

Or $$\frac{V_1}{2} = 14V_2 - \frac{3V_2}{2}$$

Or $$\frac{V_1}{2} = \frac{28V_2 - 3V_2}{2}$$

Or $$\frac{V_1}{2} = \frac{25V_2}{2} \quad \text{or} \quad V_1 = 25V_2$$

Or $$\frac{V_2}{V_1} = \frac{1}{25}$$

Q.56 Show that $Z_a \cdot Z_b = R^2$ holds good for both the networks given in Fig.9 if $V_1/I_1=R$. **(8)**

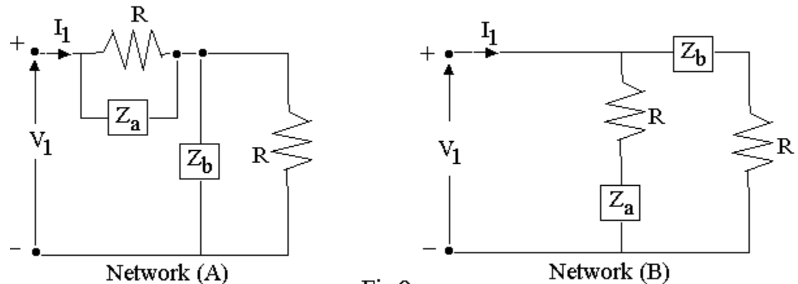


Fig.9

Ans:

(i) From the Fig.9, **Network (A)**

$$Z_{in} = \frac{R\times Z_a}{R+Z_a} + \frac{Z_b \times R}{Z_b + R} = \frac{RZ_a(Z_b+R)+Z_bR(R+Z_a)}{(R+Z_a)(Z_b+R)}$$

$$= \frac{R^2(Z_a+Z_b)+2RZ_aZ_b}{(R+Z_a)(R+Z_b)}$$

$$Z_{in} = \frac{V_1}{I_1} = R$$

$$Z_{in} = R = \frac{R^2(Z_a+Z_b)+2RZ_aZ_b}{(R+Z_a)(R+Z_b)}$$

Or $\left[(R+Z_a)(R+Z_b)\right]R = R^2(Z_a+Z_b)+2RZ_aZ_b$

Or $\left[R^2+RZ_b+RZ_a+Z_aZ_b\right]R = R^2(Z_a+Z_b)+2RZ_bZ_b$

Or $R^2+RZ_b+RZ_a+Z_aZ_b = R(Z_a+Z_b)+2Z_aZ_b$

Or $R^2 = -(\cancel{Z}_a+\cancel{Z}_b)R - Z_aZ_b + R(\cancel{Z}_a+\cancel{Z}_b)+2Z_aZ_b$

Or $R^2 = Z_aZ_b$ Hence proved.

(ii) From the Fig.9, **Network (B)**

$Z_{in} = (R+Z_a)\|(R+Z_b)$ or

$$Z_{in} = \frac{(R+Z_a)(R+Z_b)}{(R+Z_a+R+Z_b)}$$

Given that $Z_{in} = \dfrac{V_1}{R_1} = R$

$$R = \frac{(R+Z_a)(R+Z_b)}{R+Z_a+R+Z_b} \quad \text{or}$$

$(R+Z_a+R+Z_b)R = R^2+Z_bR+Z_aR+Z_aZ_b$

Or $2R^2+(Z_a+Z_b)R = R^2+(Z_a+Z_b)R+Z_aZ_b$

Or $2R^2-R^2 = (\cancel{Z}_a+\cancel{Z}_b)\cancel{R} - (\cancel{Z}_a+\cancel{Z}_b)\cancel{R}+Z_aZ_b$

Or $R^2 = Z_aZ_b$ Hence Proved.

Q.57 Express the driving-point admittance Y(s) in the form $Y(s) = K\dfrac{N(s)}{D(s)}$, for the network shown in Fig.10. Verify that Y(s) is p.r. and that $D(s)+K\cdot N(s)$ is Hurwitz. **(8)**

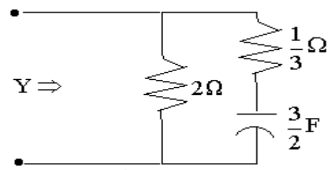


Fig.10

Ans:

The Laplace Transformed network is shown in Fig.7.2.

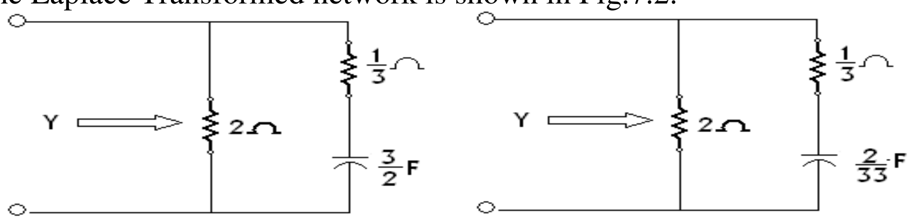


Fig.7.2

Admittance for the network is $Y(s) = \frac{1}{Z(s)}$

Impedance for the network shown in Fig.7.2 is

$$Z(s) = \left[2 \middle\| \left(\frac{1}{3} + \frac{2}{3s}\right)\right] = \left[2 \middle\| \frac{s+2}{3s}\right] = \frac{2 \times \left(\frac{s+2}{3s}\right)}{2 + \left(\frac{s+2}{3s}\right)}$$

$$= \frac{2s+4}{3s} \bigg/ \frac{6s+s+2}{3s} = \frac{2s+4}{\cancel{3s}} \times \frac{\cancel{3s}}{7s+2} = \frac{2s+4}{7s+2}$$

Therefore, $Y(s) = \frac{1}{Z(s)} = \frac{1}{2s+4 \big/ 7s+2} = \frac{7s+2}{2s+4}$

Hence the Admittance Y(s) for the given network is

$$Y(s) = \frac{7s+2}{2s+4}$$

<u>Verification of Y(s) as a Positive Real Function:-</u>

(i) Observation reveals that all the quotient terms of Y(s) are real and hence Y(s) is real if *s* is real.

(ii) It is also evident that the pole of the given function is $\left(-\frac{4}{2}\right) = -2$ lying on the left half of the s-plane and the zero is at $\left(-\frac{2}{7}\right)$, also lying on the left half of the s-plane. Thus, it is needless to state that the denominator is also Hurwitz.

(iii) The real part of Y(jω) can be obtained at

$$\operatorname{Re}[Y(j\omega)] = \operatorname{Re}\left[\frac{7.j\omega+2}{2.j\omega+4}\right]$$

$$\text{Re}\left[\frac{7.j\omega+2}{2.j\omega+4}\right]\left[\frac{-2.j\omega+4}{-2.j\omega+4}\right]=\left[\frac{14\omega^2+28j\omega-4j\omega+8}{4\omega^2+8j\omega-8\omega+16}\right]$$

=

$$\text{Re}[Y(j\omega)]=\left[\frac{8+14\omega^2}{16+4\omega^2}\right]$$

Hence,

$$\text{Re}\,Y(j\omega)\geq 0$$

Therefore, for all values of ω, .

All the above tests certify that the given function is a Positive Real Function.

Proof for Hurwitz :-

Now expressing the driving point admittance Y(s) in the form

$$Y(s)=K\frac{N(s)}{D(s)}=1\frac{7s+2}{2s+4}$$

Where K = 1, N(s) = 7s + 2 & D(s) = 2s + 4

By writing the above function is in the form

D(s) + K.N(s) = 2s + 4 + 1(7s + 2) = 9s + 6

Testing for Hurwitz :-

(i) All the coefficients of the Polynomial(9s + 6) is positive and real.

(ii) There is no power of *s* missing between the highest degree (9s) and lowest degree of the polynomial.

Therefore, by satisfying the above two conditions, the given polynomial 9s + 6 is Horwitz.

Q.58 In Fig. 11, it is required to find Y(s) to satisfy the transfer function $\frac{V_2(s)}{V_0(s)}=\frac{s(s^2+3)}{2s^3+s^2+6s+1}$ Synthesise Y. **(8)**

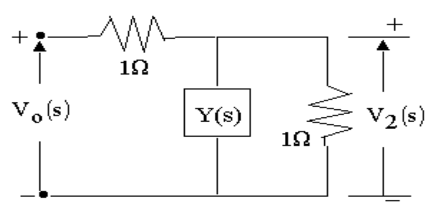


Fig.11

Ans:

Synthesizing of Y(s) :-

The voltage transfer function for the network shown in Fig.11 is

$$\frac{V_2(s)}{V_0(s)}=\frac{1}{2+Y(s)}=\frac{1}{2+\frac{s^2+1}{s(s^2+3)}}$$ which is equivalent to

$$\frac{V_2(s)}{V_0(s)}=\frac{s(s^2+3)}{2s^3+s^2+6s+1}$$

Therefore, $\frac{V_2(s)}{V_0(s)}=\frac{1}{2+\frac{s^2+1}{s(s^2+3)}}=\frac{1}{2+Y(s)}$

Where $Y(s)=\frac{s^2+1}{s(s^2+3)}$

Finding of Partial fractions for the function $Y(s) = \dfrac{s^2+1}{s(s^2+3)}$

$$\therefore\ Y(s) = \frac{s^2+1}{s(s^2+3)} = \frac{A}{s} + \frac{B_s}{s^2+3} \quad \text{------------------------ (1)}$$

$$A = \left.\frac{s^2+1}{s^2+3}\right|_{s=0} = \frac{0+1}{0+3} = \frac{1}{3}$$

Finding of the value of B :-

From equation (1) $s^2+1 = A(s^2+3) + Bs^2$ ------------------- (2)

By comparing s^2 coefficients in equation (2), we have

1 = A + B

$1 = \frac{1}{3} + B \qquad \left[\because\ A = 1/3\right]$

Therefore $B = 1 - 1/3 = \dfrac{3-1}{3} = 2/3$

Hence $A = 1/3$ & $B = 2/3$

By substituting the values of A & B in equation (1) we get

$$Y(s) = \frac{\frac{1}{3}}{s} + \frac{\frac{2}{3}s}{s^2+3}$$

$$\uparrow \qquad \uparrow$$

$3H \quad \frac{3}{2}H\ \&\ \frac{2}{9}F$

So, the synthesized network for the given network is shown in Fig.7.4

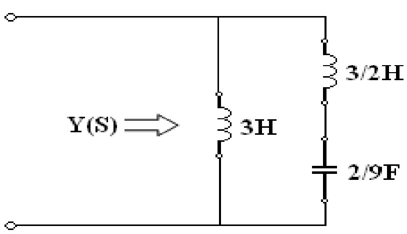


Fig.7.4

Q.59 Synthesise an LC network terminated in 1Ω, given that $Z_{21}(s) = \dfrac{2}{s^3+3s^2+4s+2}$. **(8)**

Ans:

Given that $Z_{21}(s) = \dfrac{2}{s^3+3s^2+4s+2}$

$$Z_{21}(s) = \frac{P(s)}{Q(s)} = \frac{2}{s^3+3s^2+4s+2}$$

Here, all three zeros of transmission at $s = \infty$. Since the numerator P(s) is a constant 2 & ($3s^2+2$) with the odd part of the denominator i.e. s^3+4s as

$Z_{21} = \dfrac{2}{s^3+4s}$ and

$$Z_{22} = \frac{3s^2+2}{s^3+4s}$$

Therefore Z_{21} and Z_{22} have the same poles. Synthesize Z_{22}, so that the resulting network has the transmission zeros of Z_{21} . Synthesize Z_{22} to give LC network structure by the following continued fraction expansion of $\dfrac{1}{Z_{22}}$.

$$3s^2+2\,\big)\, s^3+4s\,\big(\,\tfrac{1}{3}s \leftarrow Y$$
$$s^3+\tfrac{2}{3}s$$
$$\tfrac{10}{3}s\,\big)\, 3s^2+2\,\big(\,\tfrac{9}{10}s \leftarrow Z$$
$$3s^2$$
$$2\,\big)\, \tfrac{10}{3}s\,\big(\,\tfrac{5}{3}s \leftarrow Y$$
$$\tfrac{10}{3}s$$
$$0$$

Since Z_{22} is synthesized from the 1-Ω termination toward the input end, the final network takes the form shown in Fig.8.1

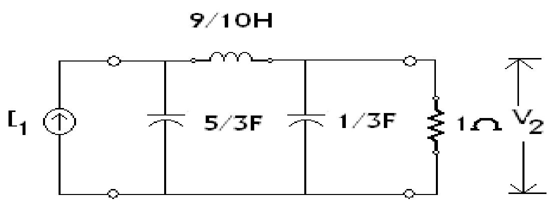


Fig.8.1

Q.60 Find the z-parameters of the network shown in Fig.12. **(8)**

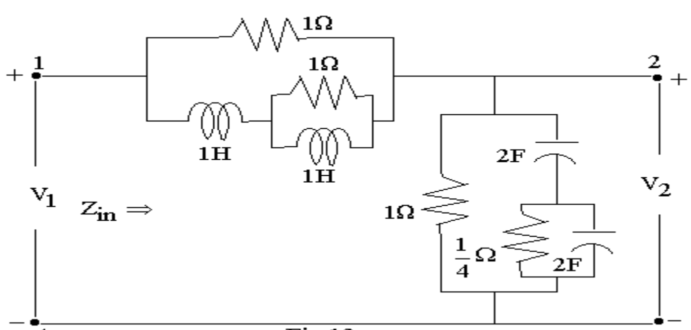


Fig.12

Ans:

Transform the given network shown in Fig.12 into Laplace Transform Domain shown in Fig.8.3.

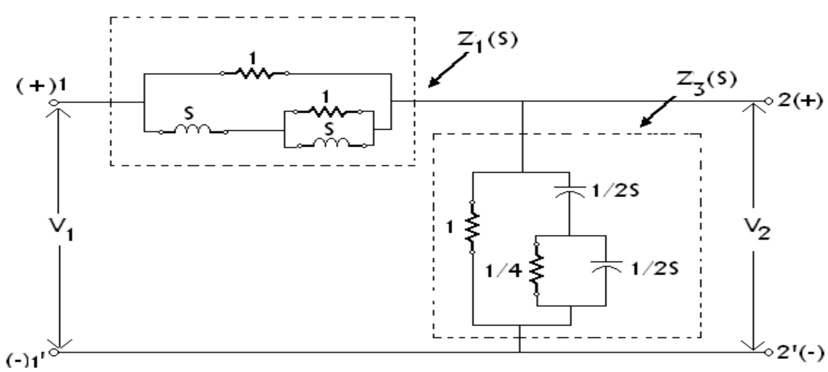


Fig.8.3

$$Z_1(s)=\frac{\left[s+\frac{s\times1}{s+1}\right]\times1}{\left[s+\frac{s\times1}{s+1}\right]+1}=\frac{s^2+2s}{s^2+3s+1} \qquad \text{and}$$

$$Z_3(s)=\frac{\left\{\left(\frac{\frac{1}{4}\times\frac{1}{2s}}{\frac{1}{4}+\frac{1}{2s}}\right)+\frac{1}{2s}\right\}\times1}{\left\{\left(\frac{\frac{1}{4}\times\frac{1}{2s}}{\frac{1}{4}+\frac{1}{2s}}\right)+\frac{1}{2s}\right\}+1}$$

$$=\frac{2s+2s+4}{2s+2s+4+4s^2+8s}=\frac{4s+4}{4s^2+12s+4}$$

$$Z_3(s)=\frac{s+1}{s^2+3s+1}$$

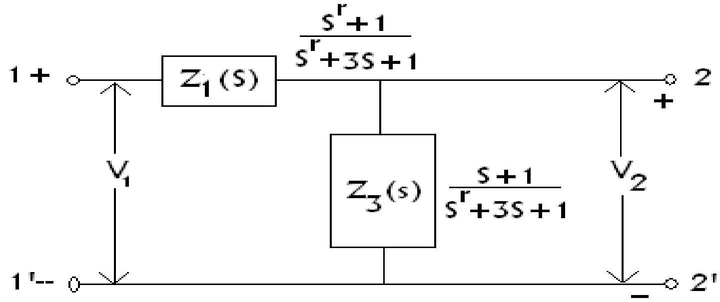


Fig.8.4

<u>Finding of Z-parameters</u> :-

$$Z_{11}(s)=Z_1(s)+Z_3(s)$$

$$Z_{11}(s)=\frac{s^2+2s}{s^2+3s+1}+\frac{s+1}{s^2+3s+1}=\frac{s^2+3s+1}{s^2+3s+1}=1$$

$$Z_{11}(s)=1$$

$$Z_{12}(s)=Z_3(s)=\frac{s+1}{s^2+3s+1}$$

$$Z_{21}(s)=Z_3(s)=\frac{s+1}{s^2+3s+1}$$

$$Z_{22}(s)=Z_2(s)+Z_3(s)=0+\frac{s+1}{s^2+3s+1}$$

$$Z_{22}=\frac{s+1}{s^2+3s+1}$$

Q.61 Consider the system function $Z(s)=\frac{2(s+1)(s+3)}{(s+2)(s+6)}$. Design:

(i) an R-L network.
(ii) an R-C network. **(12)**

Ans:

The given impedance function given by

$$Z(s)=\frac{2(s+1)(s+3)}{(s+2)(s+6)}$$

<u>Design of R-C N/W</u> :-

The partial fraction expansion of Z(s) will yield negative residues at poles s = -2 and s = -6.

Therefore, Z(s) have to expand as $\frac{Z(s)}{s}$ and latter multiply by s.

Hence,

$$\frac{Z(s)}{s}=\frac{a_1}{s}+\frac{a_2}{(s+2)}+\frac{a_3}{(s+6)}$$

Where,

$$a_1=\left.\frac{2(s+1)(s+3)}{(s+2)(s+6)}\right|_{s=0}=\frac{1}{2}$$

$$a_2=\left.\frac{2(s+1)(s+3)}{s(s+6)}\right|_{s=-2}=\frac{1}{4} \qquad \text{and}$$

$$a_3=\left.\frac{2(s+1)(s+3)}{s(s+6)}\right|_{s=-6}=\frac{5}{4}$$

Therefore, $\frac{Z(s)}{s}=\frac{1}{2s}+\frac{1}{4(s+2)}+\frac{5}{4(s+6)}$

Now, it is clear that none of the residues are negative.

Multiplying both the sides by s, we have

$$Z(s)=\frac{1}{2}+\frac{s}{4(s+2)}+\frac{5s}{4(s+6)}$$

$$=\frac{1}{2}+\frac{1}{\frac{1}{1/4}+\frac{1}{s/3}}+\frac{1}{\frac{1}{5/4}+\frac{1}{5s/24}}$$

The resulting R-C network is shown in Fig.9.1.

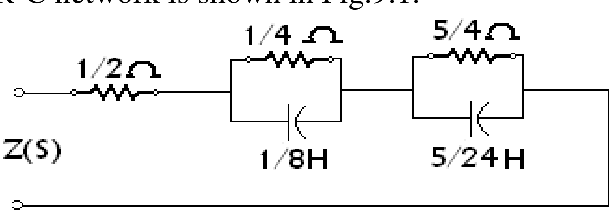


Fig.9.1

<u>Design of R-L network</u> :- it is obtained by repeated removal of poles at s=0 which corresponds to arranging numerator and denominator of Z(s) in ascending powers of s and then find continued fraction expansion. Therefore,

$$Z(s)=\frac{2(s+1)(s+3)}{(s+3)(s+6)}=\frac{2s^2+8s+6}{s^2+8s+12}$$

$$12+8s+s^2 \,\big)\, 6+8s+2s^2 \,\big(\, \frac{1}{2} \to Z$$
$$6+4s+\frac{s^2}{2}$$
$$4s+\frac{3}{2}s^2 \,\big)\, 12+8s+s^2 \,\big(\, \frac{3}{s} \to Y$$
$$12+\frac{9}{2}s$$
$$\frac{7}{2}s+s^2 \,\big)\, 4s+\frac{3}{2}s^2 \,\big(\, \frac{8}{7} \to Z$$
$$4s+\frac{8}{7}s^2$$
$$\frac{5}{14}s^2 \,\big)\, \frac{7}{2}s+s^2 \,\big(\, \frac{49}{5s} \to Y$$
$$\frac{7}{2}s$$
$$s^2 \,\big)\, \frac{5}{14}s^2 \,\big(\, \frac{5}{14} \to Z$$
$$\frac{5}{14}s^2$$
$$0$$

Q.62 Sketch the response of the magnitude function $\dfrac{1}{\sqrt{1+\in^2 c_n^2(\omega)}}$ where C_n is the Chebhyshev polynomial, for n=1, 2 and 3. **(4)**

Ans: The response of the magnitude function $\dfrac{1}{\sqrt{1+\in^2 c_n^2(\omega)}}$ where C_n is the Chebhyshev polynomial, for n=1, 2 and 3 ;can be shown as:

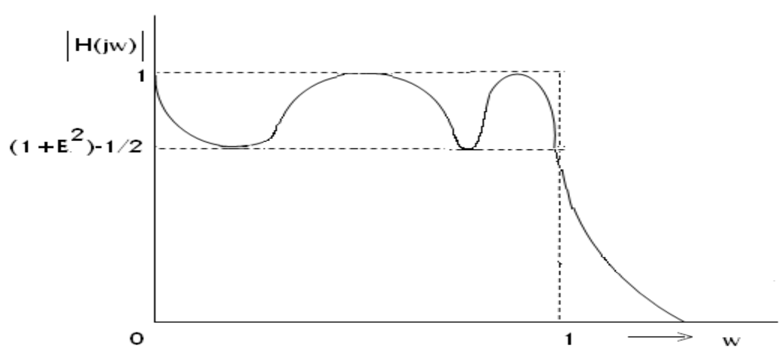


Fig.9.3

Q.63 Write minimum number of integro-differential mesh equations required to solve for all node voltages and branch currents in the network of Fig.Q2.The term v_g should not be present in your equations. **(6)**

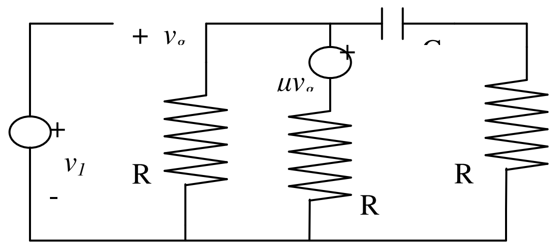


Fig.Q2

Ans:

By converting the voltage source μv_g into equivalent current source and making $(R_1 \| R_2)$, then the resultant network is shown in Fig.3.1.

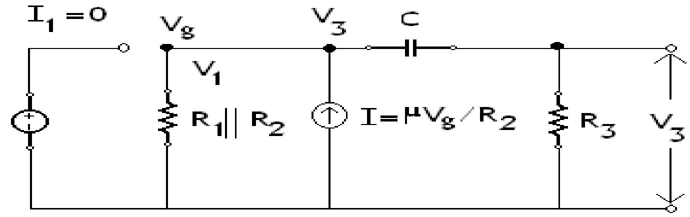


Fig.3.1

At node 1, applying KCL, we have

$$\frac{V_3}{R_1 \| R_2} - \frac{\mu v_g}{R_2} + \frac{d(V_3 - V_2)}{C} = 0$$

Or $\quad \dfrac{V_3}{R_1 R_2} - \dfrac{\mu v_g}{R_2} + \dfrac{d(V_3 - V_2)}{C} = 0$ ---------------- (1)

Also $\; V_2 = \left(\dfrac{\mu v_g}{R_2} + i \right) R_1 \| R_2$

Or $\; V_g = R_2 \left(-i + \dfrac{V_2}{R_2} + \dfrac{V_2}{R_1} \right)$ ---------------- (2)

μ

By substituting the value of v_g from equation (2) in equation (1), we have

$$\frac{V_3(R_1 + R_2)}{R_1 R_2} - \mu \left[\frac{R_2 \left(-i + \frac{V_2}{R_2} + \frac{V_2}{R_1} \right)}{\mu} \right] \frac{1}{R_2} + \frac{d(V_3 - V_2)}{C} = 0$$

Or $\quad \dfrac{V_3(R_1 + R_2)}{R_1 R_2} - \dfrac{\cancel{R_2} \left(-i + \frac{V_2}{R_2} + \frac{V_2}{R_1} \right)}{\cancel{R_2}} + \dfrac{d(V_3 - V_2)}{C} = 0$

<u>On loop basis or mesh basis:-</u>

Let i & i_2 be the currents in Fig.Q.2. So voltage

$V_2 = -\mu v_g + i_2 R_2$

Now applying KVL in loop 1, we have

$iR_1 - \mu v_g + R_2(i - i_2) = 0$

$(R_1 + R_2)i - R_2 i_2 = \mu v_g$ ----------------------- (1)

Now applying KVL in loop, we have

$$\frac{1}{c}\int i_2 dt + R_3 i_2 + R_2(i_2 - i) = -\mu v_g \qquad \text{----------------------- (2)}$$

By solving equations (1) & (2), we have

$$(R_1 + R_2)i - R_2 i_2 = \mu v_g$$

$$\frac{1}{c}\int i_2 dt + R_2 i_2 + R_2 i + R_3 i_2 = -\mu v_g$$

By eqn (1) + eqn (2), we have

$$(R_1 + R_2)i + \frac{1}{c}\int i_2 dt + R_2 i + R_3 i_2 = 0$$

Or $(R_1 + 2R_2)i + \frac{1}{c}\int i_2 dt + R_3 i_2 = 0$

Q.64 Derive the condition for maximum power transfer from a source whose internal impedance is resistive to a load which is a series combination of a resistance and a reactance. **(10)**

Ans:

Any network can be converted into a single voltage source E with series resistance R(Thevenin's equivalent circuit) as shown in Fig.3.2

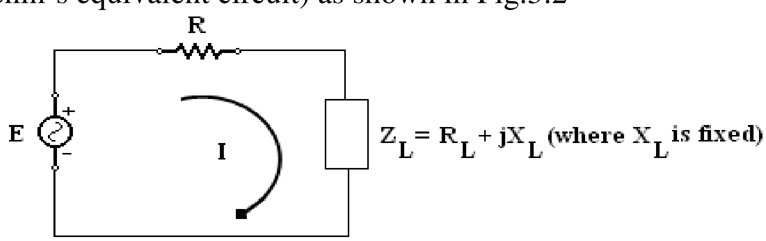


Fig.3.2

The maximum power transfer theorem aims at finding $Z_L = R_L + jX_L$, such that the power dissipated in it is maximum.

From Fig.3.2, we have

$$I = \frac{E}{R + Z_L}$$

Power dissipated in the load is

$$P = |I|^2 R_L$$

Where $Z_L = R_L + jX_L$, then the power dissipated in the load becomes

$$P = |I|^2 R_L = \left(\frac{E}{R + Z_L}\right)^2 . R_L = \frac{E^2 R_L}{R^2 + R_L^2 + X_L^{\,2}}$$

Therefore, $P = \dfrac{E^2 R_L}{(R + R_L)^2 + X_L^{\,2}}$ ----------------------- (1)

If the load reactance X_L is fixed and P is maximised by varying the load resistance R_L, the condition for maximum power transfer is

$$\frac{dP}{dR_L} = 0 \qquad \text{----------------------- (2)}$$

Hence from eqn(1), we have

$$\frac{dP}{dR_L} = \frac{(R+R_L)^2 + {X_L}^2 - R_L.2(R+R_L)}{\left((R+R_L)^2 + {X_L}^2\right)^2} = 0$$

Or $(R+R_L)^2 + {X_L}^2 - R_L.2(R+R_L) = 0$

Or $R_L^2 = R^2 + X_L^2$

Therefore, the condition for maximum power transfer from source where internal impedance is resistance to a load which is series combination of a resistance and a reactance is

$R_L^2 = R^2 + X_L^2$

Q.65 In Fig.Q3, $v_1 = 230\sqrt{2}\cos 2\pi 50t$ and the initial voltage on the capacitor is 50V. If the switch is closed at *t=0*, determine the current through the capacitor and the voltage across the inductor at *t=0+*. **(6)**

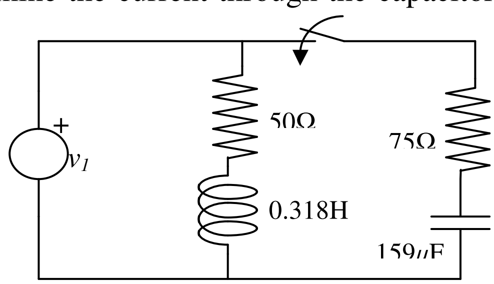


Fig.Q3

Ans:

First, we redraw the circuit for $t = \overline{0}$ (i.e. before the closing of the switch S) by replacing the inductor with a short circuit as shown in Fig.Q.3.1. Then the inductor current i_L is

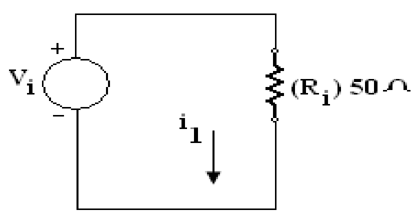


Fig.Q.3.1

$i_L(\overline{0}) = \frac{V_1}{R_1} = \frac{230\sqrt{2}}{50} = 6.5\,\text{Amp}$ -------------------- (1)

And given that $V_C(\overline{0}) = 50V$

Therefore, we have

$i_L(0^+) = i_L(0^-) = 6.5\,\text{Amp}$ -------------------- (2)

And $V_C(0^+) = V_C(0^-) = 50V$ -------------------- (3)

In order to find the current through the capacitor (i_C) and the voltage across the inductor at $t = 0^+$, we throw the switch at t = 0 and redraw the circuit of Fig.Q3 as shown in Fig.3.2.

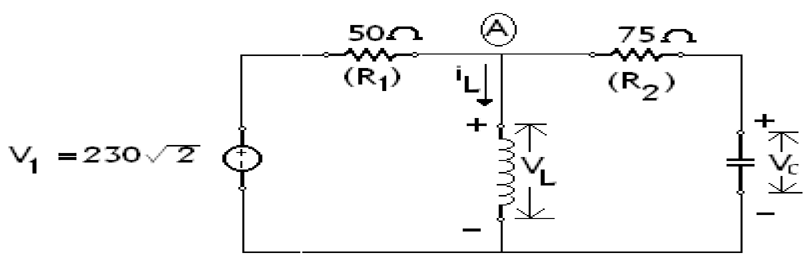


Fig.3.2

Using KVL for the right hand mesh of Fig.3.2, we obtain

$$V_C - V_L + 75 i_L = 0$$

Therefore, at $t = 0^+$

$$-V_L(o^+) = -75 i_L(o^+) + V_C(o^+)$$

From eqn (2) & (3), we have

$i_L(o^+) = 6.5$ Amp and $V_C(o^+) = 50V$ then

$$-V_L(o^+) = -75 \times 6.5 + 50 = -437.50V$$

Or $V_L(o^+) = 437.50V$

Hence the voltage across the inductor at $t = 0^+$ is 437.50V. Similarly, to find $i_C(o^+)$, we write KCL at node (A) to obtain

$$i_L + i_C + \frac{V_C - 230\sqrt{2}}{50} = 0$$

Consequently, at $t = 0^+$, the above equation becomes

$$i_C(o^+) = \frac{230\sqrt{2} - V_C(o^+)}{50} - i_L(o^+)$$

From eqn (2) & (3), we have

$i_L(o^+) = 6.5$ Amp and $V_C(o^+) = 50V$, then $i_C(o^+)$ is

$$i_C(o^+) = \frac{230\sqrt{2} - 50}{50} - 6.5 \text{ Amp}$$

$$= 5.5 - 6.5$$

Or $i_C(o^+) = 1$ Amp

Therefore, the current through the capacitor at $t = 0^+$ is -1 Amp.

Q.66 Assume that the switch in Fig.Q3 has been closed for a long time. Using phasor methods, find the current drawn from the source and the circuit impedance, resistance and reactance. **(10)**

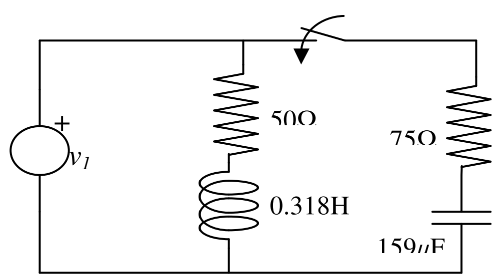


Fig.Q3

Ans:

The given circuit of Fig.Q.3 can be drawn as shown in Fig.Q.4.1.

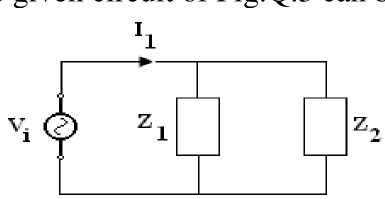


Fig.Q.4.1

Where $Z_1 = R_1 + j\omega L$ and $Z_2 = R_2 + \dfrac{1}{j\omega C}$

Now, the circuit impedance of Fig.Q.4.1 is

$$Z_{e_{ff}} = (Z_1 \| Z_2) = \frac{Z_1.Z_2}{Z_1 + Z_2} = \frac{\left[(R_1 + j\omega L)\left(R_2 + \frac{1}{j\omega C}\right)\right]}{\left[(R_1 + j\omega L) + \left(R_2 + \frac{1}{j\omega C}\right)\right]}$$

$$Z_{e_{ff}} = \frac{\left(R_1R_2 + \frac{R_1}{j\omega C} + R_2\, j\omega L + \frac{j\omega L}{j\omega C}\right)}{\left(R_1 + R_2 + j\omega L + \frac{1}{j\omega C}\right)}$$

Or

$$Z_{e_{ff}} = \left(\frac{R_1R_2 + \frac{L}{C}}{R_1 + R_2}\right) + \left(\frac{-\frac{jR_1}{\omega C} + R_2\, j\omega L}{j\omega L - \frac{j}{\omega C}}\right)$$

Or

$$Z_{e_{ff}} = \left(\frac{R_1R_2 + \frac{L}{C}}{R_1 + R_2}\right) + j\left(\frac{R_2\omega L - \frac{R_1}{\omega C}}{\omega L - \frac{1}{\omega C}}\right)$$

Or ------------------------- (1)

Now, equation (1) is in the form of

$Z_{e_{ff}} = R + jX_C$

Where the resistance of the circuit is

$$R = \left(\frac{R_1R_2 + \frac{L}{C}}{R_1 + R_2}\right) \quad \text{and}$$

$$X_C = \left(\frac{R_2\omega L - \frac{R_1}{\omega C}}{\omega L - \frac{1}{\omega C}}\right)$$

The given data is $R_1 = 50\Omega$, $R_2 = 75\Omega$, $L = 0.318H$ and $C = 159\mu F$

Therefore, the resistance R becomes

$$R = \left(\frac{R_1R_2 + \frac{L}{C}}{R_1 + R_2}\right) = \left(\frac{50\times 75 + \frac{0.318}{159\times 10^{-6}}}{50 + 75}\right)$$

Or $R = \left(\frac{3750 + 2000}{125}\right) = 46\Omega$

Hence the resistance of the circuit R = 46Ω and the X_C reactance is

$$X_C = \left(\frac{R_2 \omega L - \frac{R_1}{\omega C}}{\omega L - \frac{1}{\omega C}} \right)$$

From the given data, $V_1 = 230\sqrt{2}\cos 2\pi 50t$.

$V_1 = V_m \cos \omega t$

By comparing the equation of V_1 with

Where $\omega = 2\pi 50$ or $\omega = 100\pi$

Now the reactance X_C becomes

$$X_C = \frac{\left(75 \times 100\pi \times 0.318 - \frac{50}{100\pi \times 159 \times 10^{-6}}\right)}{\left(100\pi \times 0.318 - \frac{1}{100\pi \times 159 \times 10^{-6}}\right)}$$

Or $X_C = \frac{7488.9 - 1000.98}{99.90 - 20.0194} = \frac{6491.718}{79.88}$

Or $X_C = 81.267\Omega$

Therefore the impedance of the given network is

$Z_{e_{ff}} = (46 + j81.267)\Omega$

By converting the above rectangular form of equation into polar form, we get

$R\cos\phi = 46$ and $R\sin\phi = 81.267$, then

$R = \sqrt{46^2 + 81.267^2} = 93.38$ and

$\phi = Tan^{-1}\left(\frac{81.267}{46}\right) = 60.49^0$ (approx.)

Now the polar form of impedance Z_{eff} becomes

$Z_{eff} = 93.38\angle 60.49^0\ \Omega$

Next, the current drawn from the source is

$$I_1 = \frac{V_1}{Z_{eff}}$$

Where the voltage V_1 is

$V_1 = 230\sqrt{2}(w2\pi 50t + 0^0)$

The rms value of V_1 is

$$V_1 = \frac{V_m}{\sqrt{2}} = \frac{230\sqrt{2}}{\sqrt{2}} = 230V.$$

Therefore, the current drawn from the source is

$$I_1 = \frac{230\angle 0^0}{93.38\angle 60.49^0}$$

Or $I_1 = 2.46\angle -60.49^0$ Amp

Q.67 Define Chebyshev cosine polynomial $C_n(\omega)$. Using recursive formula for $C_n(\omega)$, or otherwise, obtain their values when n = 0 to 4. Plot C_3 and C_4 for $|\omega| \le 1$.

(2+2+4 = 8)

Ans:

$$C_n(\omega) \stackrel{\Delta}{=} \cos\left(n \cos^{-1} \omega\right) \stackrel{\Delta}{=} \cosh\left(n \cosh^{-1} \omega\right)$$

and recursive formula

$$C_n(\omega) = 2\omega C_{n-1}(\omega) - C_{n-2}(\omega)$$

$$n = 0 \Rightarrow C_0(\omega) = 1$$

$$n = 1 \Rightarrow C_1(\omega) = \omega$$

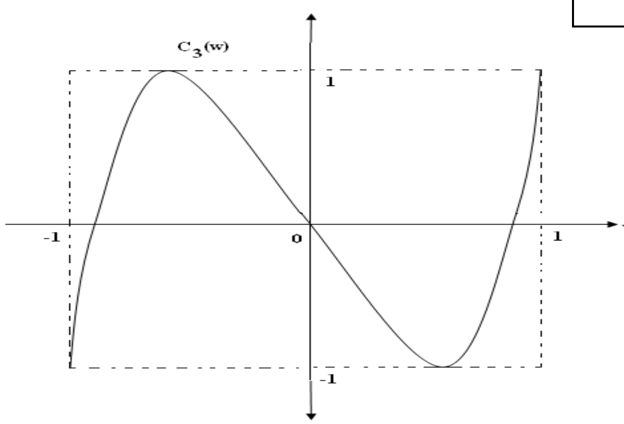
$$n = 2 \Rightarrow C_2(\omega) = 2\omega^2 - 1$$

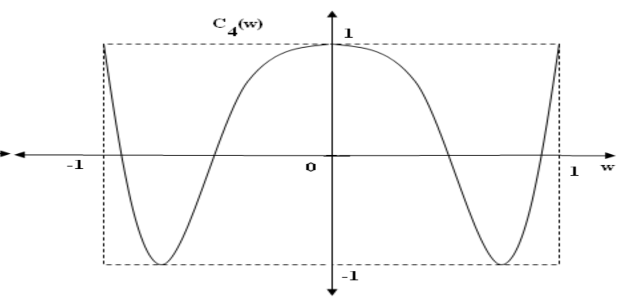
$$n = 3 \Rightarrow C_3(\omega) = 4\omega^3 - 3\omega$$

$$n = 4 \Rightarrow C_4(\omega) = 8\omega^4 - 8\omega^2 + 1$$

$$\Rightarrow C_4(\omega) = 2\omega\left(4\omega^3 - 3\omega\right) - \left(2\omega^2 - 1\right)$$

ω	0	-1	1
$C_3(\omega)$	0	-1	1
$C_4(\omega)$	1	1	1





Q.68 Find the Thevenin's voltage and the Thevenin's equivalent resistance across terminals a-b in Fig.Q4. Assume V_1=10V, R_1=5 Ohms, R_2= 2 Ohms, r= 1 Ohm, I=2A and V=12V. Determine the power drawn from the 12V source when the load is connected. **(8)**

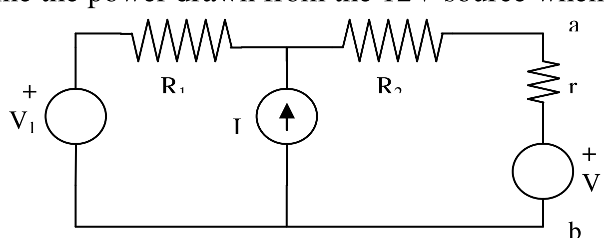


Fig.Q4

Ans:

Let the terminals a-b be open circuited. No current flows through r and the current is shown in Fig.5.1.

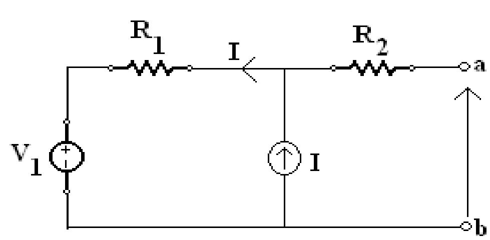


Fig.5.1

Using KVL at left hand loop of Fig.5.1, we have

$V_{a-b} = V_{TH} = V_1 + IR_1$

$= 10 + 2\times 5$

$\therefore \quad V_{TH} = 20V$

Next, all the independent sources of the given circuit are replaced by their internal resistances in order to find the Thevenin's resistance across a-b. The circuit configuration is shown in Fig.5.2. The voltage source V_1 is short circuited and the current source is open circuited.

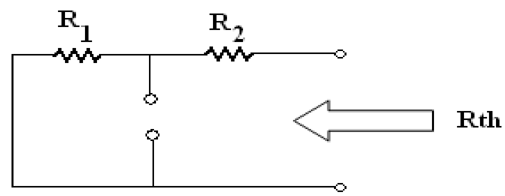


Fig.5.2

$\therefore \quad R_{Th} = R_1 + R_2 = 5 + 2 = 7\Omega$

Thus we obtain $V_{Th} = 20V$ and $R_{Th} = 7\Omega$

Now, the load current through the load is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{20}{7+1} = \frac{20}{8} = 2.5 \text{ Amp}$$

The power drawn from the source 'V' is

$$P = \frac{V^2}{R_L} = \frac{(12-10)^2}{1} = \frac{2^2}{1} = 4W$$

Q.69 Consider a series resonance circuit consisting of a 10 Ohms resistance, a 2mH inductance and a 200nF capacitance. Determine the maximum energy stored , the energy dissipated per cycle and the bandwidth of the circuit. Write the normalized form of the admittance for this circuit. **(8)**

Ans:

The frequency of resonance occurs, when

$X_L = X_C$ i.e., $\omega L = \dfrac{1}{\omega C}$

$$\omega = \frac{1}{\sqrt{LC}} \; rad/\sec$$

Or

Given that $L = 2mH$ and $C = 200\times 10^{-9} F$

$= L = 2\times 10^{-3} H$

$$\omega = \frac{1}{\sqrt{2\times 10^{-3}\times 200\times 10^{-9}}} = 50000 \; rad/\sec$$

Or $f_r = \dfrac{1}{2\pi}\times(\omega) = 795.74 \; HZ$

Or $f_r = 7.958 \; KHZ \; approx$

Assume that the source voltage 'V' is

$V = 50V \; rms$

Now the current through the circuit at resonance becomes maximum, i.e.,

$$I_{\max} = \frac{V}{R} = \frac{50}{10} = 5\ Amp \quad (\because\ R = 10\Omega)$$

Maximum energy stored:

A series RLC circuit at resonance stores a constant amount of energy. Since when the capacitor voltage is maximum, the inductor current is zero and vice-versa i.e.,

Maximum energy stored in a RLC series circuit at resonance is the maximum energy stored in inductor is equal to the maximum energy stored in a capacitor i,e.,

$$W = \frac{1}{2}LI_{\max}^2 = \frac{1}{2}CV_{\max}^2$$

$$W = \frac{2\times10^{-3}(5)^2}{2} = 0.025$$

Or $W = 25.25mJ$

Energy dissipated per cycle:-

The total energy dissipated per cycle is equivalent to the energy dissipated by the inductor and the energy dissipated by the capacitor i.e.,

$$\frac{I^2R}{2}\times\frac{1}{f}+\frac{I^2R}{2}\times\frac{1}{f}$$

$$\frac{(5)^2\times10}{2}\times\frac{1}{7.958\times10^3}+\frac{(5)^2\times10}{2}\times\frac{1}{7.958\times10^3}$$

= 0.0157074 + 0.0157074

= 0.0314149

$$E_{dissipated/cycle} = 0.0314149 = 31.41mJ$$

Quality factor

Now the quality factor of a series RCL circuit at resonance is

$$Q = 2\pi\times\frac{Maximum\ Energy\ Stored}{Energy\ Dissipated\ \ per\ Cycle}$$

Or $Q = 2\pi\times\frac{0.0250}{0.0314} = 2\pi\times0.7958$

Q = 5

Bandwidth of the circuit:-

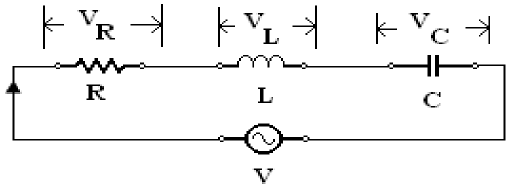
Therefore, the relation between B.W, Quality factor and resonance frequency is

$$Q = \frac{f_r}{B.W} \qquad \text{or}$$

$$B.W = \frac{f_r}{Q} = \frac{7958}{5} = 1590.79 \qquad \text{or}$$

B.W = 1.6 KHZ

Normalised form of the Admittance of the Circuit:-



Now the impedance of the circuit is

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$R + j\left(\omega L - \frac{1}{\omega C}\right)$$

=

$$X = \left(\omega L - \frac{1}{\omega C}\right)$$

Or $Z = R + jX$ where

Now the admittance of the circuit 'Y' is

$$Y = \frac{1}{Z} = \frac{1}{R + jX}$$

Q.70 For the given network function $F(s) = \dfrac{26}{s^2 + s + 26}$, draw its pole-zero plot and determine (i) ω_{max}, the frequency at which $F(j\omega)$ attains its maximum value, (ii) $|F(j\omega_{max})|$, (iii) the half power points, and (iv) the magnitude of the function at half power points. Using this information, draw a neat sketch of the magnitude and the phase responses. **(16)**

Ans:

The given network function F(S) is $F(s) = \dfrac{26}{s^2 + s + 26}$

In factored form, F(S) is

$$F(s) = \frac{26}{(s + 0.5 + j5)(s + 0.5 - j5)} \qquad \text{--------------------- (1)}$$

The amplitude $|H(jW_{\max})|$ is then

$$|H(j4.97)| = \left|\frac{26}{(j4.97 + 0.5 + j5)(j4.97 + 0.5 - j5)}\right|$$

$$= \left|\frac{26}{(0.5 + j9.97)(0.5 - j0.03)}\right|$$

$$= \frac{26}{5} = 5.2$$

$[\because\ (0.5 + j9.97)(0.5 - j0.03)$

$R = \sqrt{(0.5)^2 + (9.97)^2} \quad R = \sqrt{(0.5)^2 + (0.03)^2}$

$= (9.982)(0.50089)$

$= 4.999 \cong 5$

Therefore, $|H(j4.97)| = 5.2$

The point A at which the peaking circle intersects the positive real axis is located at S = 4.5. With the center at A, we draw a circle of radius AB(equal to $5\sqrt{2.23}$ in this case). At the point C, where this new circle intersects the jw axis, we have ω_C. By measurement, we find $\omega_C \simeq 5.95$

Let us check this result, referring to Fig.6, we know that the line segment AB is of length $5\sqrt{2.23}$; it follow that AC is also $5\sqrt{2.23}$ units long. The line segment AO is of length $AO = 5 - 0.5 = 4.5$ units. The pole of the given network function F(S) are at (-0.5+j5) and (-035-j5). The poles of F(S) are shown in Fig.6.

Next we draw the peaking circle with the center at S = -0.5 and the radius equal to 5. At the point where the circle intersects the jw axis. We see that $\omega_{\max} = 4.97$ in Fig.6.

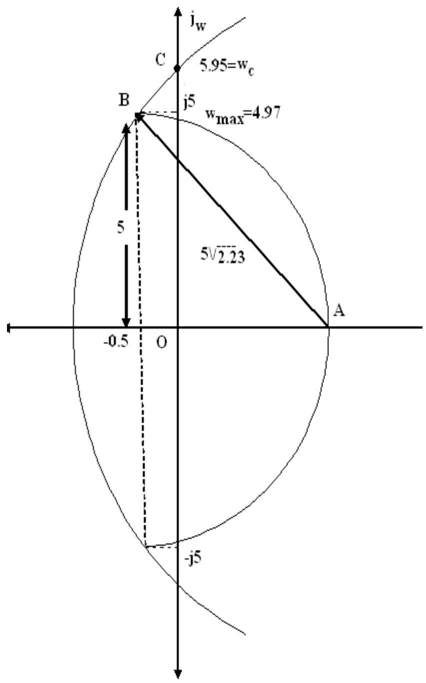


Fig.6

To check this result, the equation

$\omega_{\max}^2 = \beta^2 - \alpha^2$ gives

$\omega_{\max} = \sqrt{5^2 - 0.5^2} = \sqrt{24.75} = 4.97$

$\omega_{\max} = 4.97$

Then W_C is given as

$$W_C = \sqrt{(AC)^2 - (AO)^2} = \sqrt{\left(5\sqrt{2.23}\right)^2 - (4.5)^2} = \sqrt{35.5}$$

Or $W_C = 5.95$

Finally, we obtain $|H(j\omega_C)|$ as

$$|H(j5.95)| = \left|\frac{26}{\sqrt{(26-35.5)^2 + \left(5\sqrt{35.5}\right)^2}}\right|$$

$$= \left|\frac{26}{\sqrt{(-9.5)^2 + 887.5}}\right| = \frac{26}{\sqrt{977.75}}$$

Or $|H(j5.95)| = \dfrac{26}{31.2689} = 0.831$

While is precisely $0.707|H(j\omega_{\max})|$.

Q.71 Determine the h-parameters of the network shown in Fig.Q6. **(16)**

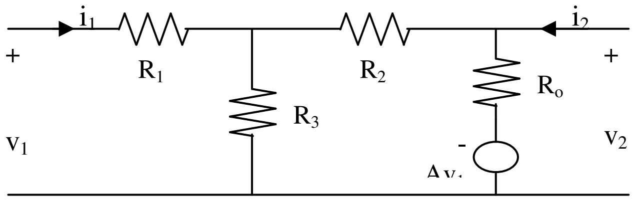


Fig.Q6

Ans:
By converting the voltage source into equivalent current source, then the circuit is shown in Fig.7.1.

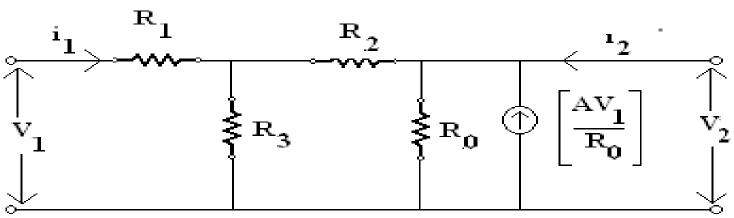


Fig.7.1

The circuit of Fig.7.1 is simplified and as shown in Fig.7.2.

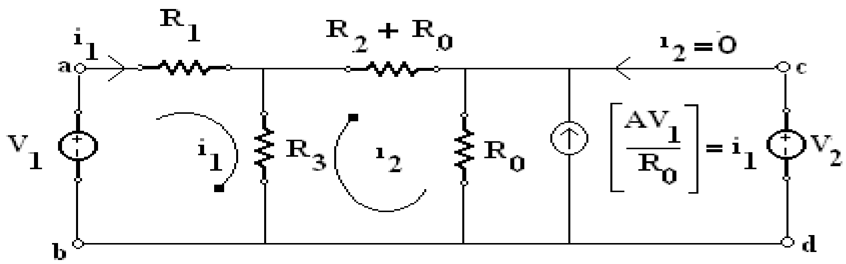


Fig.7.2

<u>Finding of Z-parameters:</u>
With open circuiting the output port c-d and applying a voltage source V_1 at the input port (terminal a-b), the loop equations are given by (ref Fig.7.2)

$V_1 = i_1(R_1 + R_3) - i_2^1 R_3$ ----------------- (1)

However, $i_2 = 0$ output being open circuited

$-i_2^1 = i_1$ ----------------- (2)

By substituting the equation (2) in equation (1), we get

$V_1 = i_1(R_1 + R_3) + i_1 R_3$

Or $V_1 = i_1(R_1 + 2R_3)$

Or $\left.\frac{V_1}{i_1}\right|_{i_2=0} = Z_{11} = (R_1 + 2R_3)$

And from equation (2), we have

$i_2^1 = -i_1 = -\frac{V_1}{(R_1 + 2R_3)}$

Again $V_2 = \left(\frac{AV_1}{R_0} - i_2^1\right)R_3 - (R_2 + R_0)i_2^1$

$$= \left(\frac{AV_1}{R_0} + i_1\right)R_3 + (R_2 + R_0)i_1$$

$$(\because\ i_2^1 = -i_1)$$

Or $V_2 = \left(\frac{AV_1}{R_0} - i_2^1\right)R_3 + R_2 i_1 + R_0 i_1$

Or $V_2 = (R_0 + R_2 + R_3)i_1 + \frac{AV_1}{R_0}.R_3$

$$V_2 = (R_0 + R_2 + R_3)i_1 + i_1.R_3 \qquad \left(\because\ \frac{AV_1}{R_0} = i_1\right)$$

Or $V_2 = (R_0 + R_2 + 2R_3)i_1$

Or $\left.\frac{V_2}{i_1}\right|_{i_2=0} = Z_{21} = (R_0 + R_2 + 2R_3)$

In the next step, the input is opened and V_2 is applied at the output terminals (terminals c-d). The current source becomes useless as with input open i.e. $i_1 = 0$. The network configuration as shown in Fig.7.3.

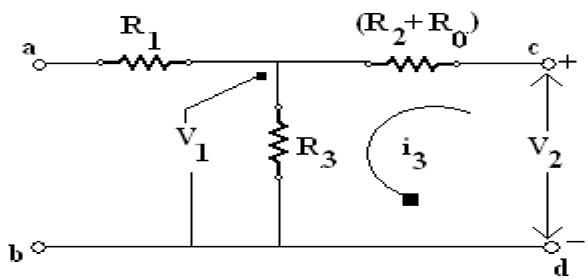


Fig.7.3

Here, $i_2 \equiv i_2^1$ and $V_2 = i_2^1(R_0 + R_2 + R_3)$

Or $V_2 = (R_0 + R_2 + R_3)i_2$

Or $\left.\frac{V_2}{i_2}\right|_{i_1=0} = Z_{22} = (R_0 + R_2 + R_3)$

Also, $V_1 = i_2^1 \times R_3 = R_3 i_2 \qquad (\because\ i_2 = i_2^1)$

Or $\left.\frac{V_1}{i_2}\right|_{i_1=0} = Z_{12} = R_3$

Now $Z_{11} = R_1 + 2R_3$, $Z_{12} = R_3$, $Z_{21} = (R_0 + R_2 + 2R_3)$ & $Z_{22} = (R_0 + R_2 + R_3)$

Now the relationship between Z & h-parameters are:

<u>h-parameters:</u>

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{22}} = \frac{(R_1 + 2R_3)(R_0 + R_2 + R_3) - (R_0 + R_2 + R_3)R_3}{R_0 + R_2 + R_3}$$

$$h_{11} = \frac{R_1R_0 + R_1R_2 + R_1R_3 + R_3R_0 + R_3R_2}{R_0 + R_2 + R_3}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{R_3}{R_0 + R_2 + R_3} = \frac{1}{1 + R_0 + R_2}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} = -\frac{(R_0 + R_2 + 2R_3)}{R_0 + R_2 + R_3}$$

$$h_{22} = \frac{1}{Z_{22}} = \frac{1}{R_0 + R_2 + R_3}$$

Q.72 Determine if the function $F(s) = \dfrac{s^3 + 5s^2 + 9s + 3}{s^3 + 4s^2 + 7s + 9}$ is positive real. **(10)**

Ans:

The given function is $F(s) = \dfrac{s^3 + 5s^2 + 9s + 3}{s^3 + 4s^2 + 7s + 9}$

Now let us proceed with the testing of the function F(s) for positive realness:-

(i) Since all the coefficients in the numerator and denominator are having positive values, hence, for real value of S, Z(s) is real.

(ii) To find whether the poles are on the left half of the S-plane, let us apply the Hurwitz criterion to the denominator using continued fraction method.

Let $P(s) = s^3 + 4s^2 + 7s + 9 = M_2(s) + N_2(s)$

Where $M_2(s) = 4s^2 + 9$ and $N_2(s) = s^3 + 7s$.

Application of continued fraction method is shown below:-

$$\varphi(s) = \frac{N_2(s)}{M_2(s)} = \frac{s^3 + 7s}{4s^2 + 9}$$

$$4s^2+9 \;\big)\; s^3+7s \;\big(\; \frac{s}{4}$$

$$\underline{s^3+9\frac{s}{4}}$$

$$\frac{19s}{4} \;\big)\; 4s^2+9 \;\big(\; \frac{16}{19}s$$

$$\underline{4s^2}$$

$$9 \;\big)\; \frac{19s}{4} \;\big(\; \frac{19s}{4\times 9}$$

$$\underline{\frac{19s}{4}}$$

$$0$$

Since all the quotients are positive in the continued fraction expansion, hence, the polynomial of Z(s) in the denominator is Hurwitz,

(iii) In order to find whether $R_e Z(j\omega) \geq 0$ for all ω, let us adopt slightly more mathematical manipulation.

Let $F(s) = \dfrac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$ where

$M_1(s) = 5s^2 + 3$; $N_1(s) = s^3 + 9s$;

$M_2(s) = 4s^2 + 9$ and $N_2(s) = s^3 + 7s$

Rationalising,

$$F(s) = \frac{M_1 + N_1}{M_2 + N_2} \cdot \frac{M_2 - N_1}{M_2 - N_2}$$

$$= \frac{M_1 M_2 - N_1 N_2}{{M_2}^2 - {N_2}^2} + \frac{N_1 M_2 - M_1 N_1}{{M_2}^2 - {N_2}^2}$$

Here, even part of F(s) is $\dfrac{M_1M_2 - N_1N_2}{M_2^{\ 2} - N_2^{\ 2}}$

$$\therefore \mathrm{Re}al\ \ F(j\omega) = \left.\frac{M_1M_2 - N_1N_2}{M_2^{\ 2} - N_2^{\ 2}}\right|_{s=j\omega} = \frac{D(s)}{M_2^{\ 2} - N_2^{\ 2}}$$

Since, $\left(M_2^{\ 2} - N_2^{\ 2}\right)$ is always positive for $s = j\omega$, $F(j\omega) \geq 0$ provided $D(j\omega) \geq 0$ for any ω.

In this problem,

$D(s) = M_1M_2 - N_1N_2 = \left(5s^2+3\right)\left(4s^2+9\right) - \left(s^3+9s\right)\left(s^3+7s\right)$

Or $D(s) = 20s^4 + 45s^2 + 12s^2 + 27 - s^6 - 7s^4 - 9s^4 - 63s^2$

$= -s^6 + 4s^4 - 6s^2 + 27$

Therefore, $D(j\omega) = -(j\omega)^6 + 4(j\omega)^4 - 6(j\omega)^2 + 27$

Or $D(j\omega) = \omega^6 + 4\omega^4 + 6\omega^2 + 27$

i.e., $D(j\omega) > 0$ for any value of ω. Thus, $Z(j\omega) \geq 0$ for any value of ω.

The above three ((i),(ii), & (iii)) tests certify that the given function F(s) is a PR function.

Q.73 Given that $\mathrm{Re}\,G(j\omega) = \dfrac{\omega^4 + 21\omega^2}{\omega^4 + 17\omega^2 + 16}$, determine a realizable *G(s)*. **(6)**

Ans:

The given real part of the G(s) is

$$\mathrm{Re}\,G(j\omega) = \frac{\omega^4 + 21\omega^2}{\omega^4 + 17\omega^2 + 16} \qquad \text{------------------- (i)}$$

Or $\mathrm{Re}\,G(j\omega) = \dfrac{25\omega^2 - 4\omega^2 + \omega^4}{\omega^4 - 8\omega^2 + 25\omega^2 + 16}$

Or $\mathrm{Re}\,G(j\omega) = \dfrac{25\omega^2 - 4\omega^2 + \omega^4}{\omega^4 - 8\omega^2 - 25j^{-2}\omega^2 + 16}$

Now real & imaginary part of G(jω) is

G(jω) = Real Part of G(jω) + Imaginary Part of G(jω).

$$G(j\omega) = \frac{\left(-25j^2\omega^2 - 4\omega^2 + \omega^4\right) + j\left(20\omega - 5\omega^3 + 5\omega\omega^2\right)}{\left[\left(4-\omega^2\right)^2 - (5j\omega)^2\right]}$$

Or $G(j\omega) = \dfrac{\left(5j\omega - \omega^2\right)\left\{\left(4-\omega^2\right) - 5j\omega\right\}}{\left[\left(4-\omega^2\right) + 5j\omega\right]\left[\left(4-\omega^2\right) - 5j\omega\right]}$

Or $G(j\omega) = \dfrac{\left(5j\omega - \omega^2\right)}{\left[\left(4-\omega^2\right) + 5j\omega\right]}$ ------------------- (ii)

By substituting the value of $s = j\omega$ and $s^2 = j^2\omega^2$

In equation (ii), we get

$$G(j\omega) = \frac{5s - j^2\omega^2}{-j^2\omega^2 + 5s + 4}$$

Or $G(s) = \dfrac{s^2 + 5s}{s^2 + 5s + 4}$ $\quad \left[\because\ s^2 = j^2\omega^2\ \&\ j^2 = -1\right]$

Therefore, the realized G(s) of $\operatorname{Re} G(j\omega)$ is

$$G(s) = \frac{s^2 + 5s}{s^2 + 5s + 4}$$

Q.74 Synthesise the voltage transfer function $T(s) = \dfrac{2Ks}{s^3 + 2s^2 + 2s + 1}$ by any method and obtain the realized value of *K*. **(9)**

Ans:
The given voltage transfer function T(s) is

$$T(s) = \frac{V_2}{V_1} = \frac{2Ks}{s^3 + 2s^2 + 2s + 1}$$

Or $T(s) = \dfrac{V_2}{V_1} = \dfrac{2Ks}{(s+1)(s^2+s+1)}$ --------- (i) $\left[\because\ (s+1)(s^2+s+1) = s^3 + 2s^2 + 2s + 1\right]$

If we split the equation (i) of voltage transfer function into two parts. We obtain

$$T(s) = \frac{V_2}{V_1} = \frac{2Ks}{(s+1)} \cdot \frac{2Ks}{(s^2+s+1)}$$

Or $T(s) = \dfrac{V_2}{V_1} = \dfrac{V_a}{V_1} \cdot \dfrac{V_2}{V_a} = \dfrac{2Ks}{(s+1)} \cdot \dfrac{2Ks}{(s^2+s+1)}$ ------------------- (i)

In equation (i), the value of K must be equal to $\frac{1}{2}$, so that the equation (i) can be simplified in order to get two constant-resistance bridged-T networks i.e.,

$$\frac{V_a}{V_1} = \frac{2Ks}{s+1}$$

Where $K = \frac{1}{2}$, then the above equation becomes

$$\frac{V_a}{V_1} = \frac{s}{s+1} \quad \text{------------------------------- (ii)}$$

$$\frac{V_a}{V_1} = \frac{1}{1 + \left(\frac{1}{s}\right)} \quad \text{------------------------------- (iii)}$$

The equations (ii) and (iii) are compared with the voltage-ratio transfer function of constant-resistance bridged-T circuit is

$$\frac{V_a}{V_1} = \frac{1}{1 + \left(\frac{R}{Z_b}\right)}$$

Where $Z_b = s$, R = 1

Since $Z_a Z_b = 1$, we then obtain $Z_a = \frac{1}{s}$

We see that $Z_b = 1H$ inductor and Z_a is 1F capacitor. Therefore, the final synthesized network for $\frac{V_a}{V_1}$ is shown in Fig 8.1

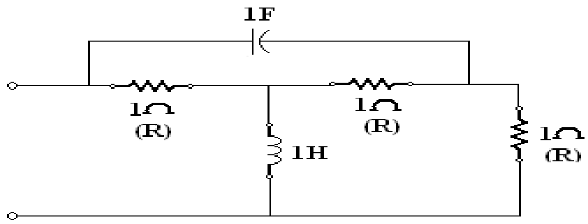


Fig 8.1

<u>Now let us synthesize the $\frac{V_2}{V_a}$</u>:

$\frac{V_2}{V_a}$ is given by

$$\frac{V_2}{V_a} = \frac{2Ks}{\left(s^2 + s + 1\right)}$$

Where $K = \frac{1}{2}$, so that the above equation becomes

$$\frac{V_2}{V_a} = \frac{s}{s^2 + s + 1} = \frac{1}{1 + \left(\frac{s^2 + 1}{s}\right)} \qquad \text{----------------------------- (iv)}$$

So that the equation (iv) is in the form of

$$\frac{V_2}{V_a} = \frac{1}{1 + \left(\frac{Z_a}{R}\right)} \quad \text{where } Z_a = \frac{s^2 + 1}{s} \text{ and R = 1.}$$

Since $Z_a Z_b = 1$, so that $Z_b = \frac{s}{s^2 + 1}$, then the final synthesized network for $\frac{V_2}{V_a}$ is shown in Fig 8.2.

We recognize that Z_a is parallel L-c tank circuit of 1F & 1H and Z_b as a series L-c tank circuit of 1F & 1H. The final synthesized network is shown in Fig.8.2.

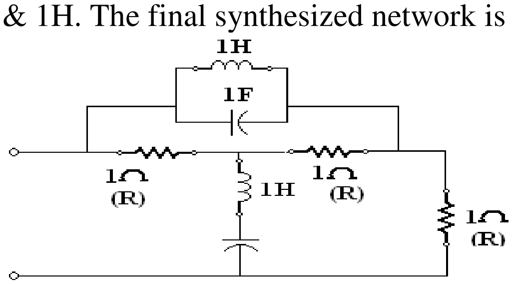


Fig 8.2

The final synthesized network for the given voltage transfer function T(s) is the cascade of $\frac{V_a}{V_1}$ & $\frac{V_2}{V_a}$ is shown in Fig.8.3.

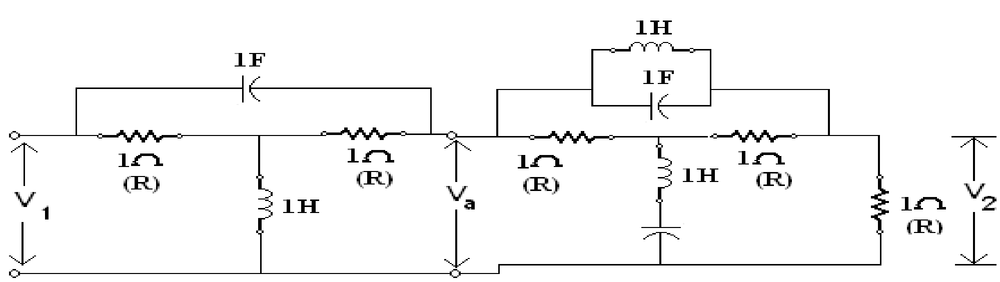


Fig.8.3.

Q.75 Determine the voltage transfer function of the network shown in Fig.Q8. All resistances are 1 ohm, inductors 1H and capacitors 1F. **(7)**

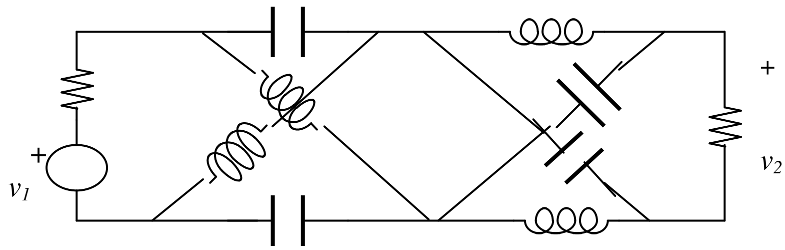


Fig.Q8

Ans:

The given network shown in Fig.8.5, is a two constant resistance two-port networks, connects in tandem. It is called constant resistance, because the impedance looking in at either port is a constant resistance R. When the other port is terminated in the some resistance R as shown in the figure. Hence, if the voltage-ratio transfer function of N_a is $\frac{V_a}{V_1}$ & that of N_b is $\frac{V_2}{V_a}$, then the voltage-ratio transfer function of the total network is

$$\frac{V_2}{V_1} = \frac{V_a}{V_1} \cdot \frac{V_2}{V_a}$$

<u>Finding of voltage transfer $\frac{V_a}{V_1}$ for the first network</u> N_a :-

The constant resistance lattice network is shown in Fig.8.5 is compared with the first network (N_a) shown in Fig.8.6.

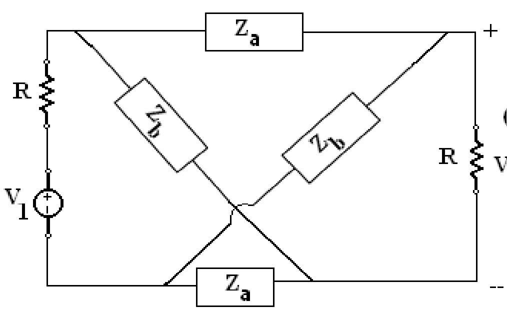


Fig.8.5

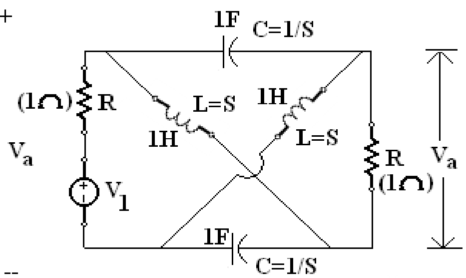


Fig.8.6

Therefore, by comparison of Fig.8.6 & Fig.8.5. We obtain $Z_a = \frac{1}{s}$ and $Z_b = s$; $R = 1\Omega$

Also, the voltage transfer function of a constant resistance lattice network $\frac{V_a}{V_1}$ is given by

$$\frac{V_a}{V_1} = \frac{\frac{1}{2}(Z_b - R)}{Z_b + R}$$

Or $$\frac{V_a}{V_1} = \frac{\frac{1}{2}(s-1)}{s+1}$$

Determination of voltage transfer function $\left(\frac{V_2}{V_a}\right)$ for the second network N_b :-

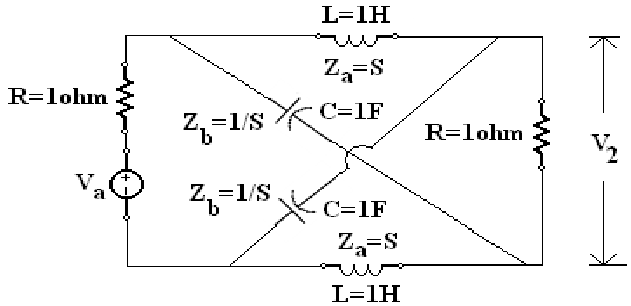


Fig.8.7

By comparing the constant lattice network of Fig.8.5 with the second network N_b of Fig.8.7, we obtain

$Z_a = s$, $Z_b = \frac{1}{s}$ & $R = 1\Omega$

Also the voltage transfer function of a constant resistance lattice network $\frac{V_2}{V_a}$ is given by

$$\frac{V_2}{V_a} = \frac{\frac{1}{2}(Z_b - R)}{(Z_b + R)}$$

Where $Z_b = \frac{1}{s}$, then

$$\frac{V_2}{V_a} = \frac{\frac{1}{2}\left(\frac{1}{s} - 1\right)}{\left(\frac{1}{s} + 1\right)} = \frac{\frac{1}{2}\left(\frac{1-s}{s}\right)}{\left(\frac{1+s}{s}\right)} = \frac{1}{2}\left(\frac{1-s}{1+s}\right)$$

Or $\frac{V_2}{V_a} = \frac{1}{2}\left(\frac{1-s}{1+s}\right)$

Now, the voltage-ratio transfer function of the total network $\frac{V_2}{V_1}$ is

$$\frac{V_2}{V_1} = \frac{V_a}{V_1} \cdot \frac{V_2}{V_a}$$

Where $\frac{V_a}{V_1} = \frac{\frac{1}{2}(s-1)}{(s+1)}$ & $\frac{V_2}{V_a} = \frac{1}{2}\left(\frac{1-s}{1+s}\right)$

Therefore, $\frac{V_2}{V_1}$ is

$$\frac{V_2}{V_1} = \frac{\frac{1}{2}(s-1)}{(s+1)} \cdot \frac{1}{2}\left(\frac{1-s}{1+s}\right) = \frac{1}{4}\frac{(s-1)}{(s+1)}\left(\frac{1-s}{1+s}\right)$$

Or $\frac{V_2}{V_1} = \frac{-s^2 + 2s - 1}{(s+1)^2}$

Hence the voltage transfer function of the Fig.8.5 is

$$\frac{V_2}{V_1}=\frac{\frac{1}{2}(s-1)}{(s+1)}.\frac{1}{2}\left(\frac{1-s}{1+s}\right)$$

Q.76 Realize the impedance $Z(s)=\frac{2(s^2+1)(s^2+9)}{(s^2+4)s}$ in three different ways. **(12)**

Ans:
The given impedance function Z(s) is

$$Z(s)=\frac{2(s^2+1)(s^2+9)}{s(s^2+4)}=\frac{2s^4+20s^2+18}{s^3+4s}$$

A partial fraction expansion of Z(s) gives as

$$\begin{array}{r|l|l} s^3+4s & 2s^4+20s^2+18 & 2s \\ & 2s^4+8s^2 & \\ \hline & 12s^2+18 & \end{array}$$

Therefore, $Z(s)=2s+\frac{12s^2+18}{s(s^2+4)}$

The partial fraction expansion of Z(s) becomes

$$Z(s)=2s+\frac{12s^2+18}{s(s^2+4)}$$

Where $\frac{12s^2+18}{s(s^2+4)}=\frac{A}{s}+\frac{Bs+C}{s^2+4}$

Now, $12s^2+18=A(s^2+4)+Bs^2+Cs$ -------------------------- (1)

By comparing s^2 components on both sides of equation (1),
We get 12 = A + B -------------------------- (i)
By comparing 's' coefficients on both sides of equation (1),
We get C = 0 -------------------------- (ii)
By comparing constants on both sides of equation (1),
We get 18 = A4

Or $A=\frac{18}{4}=\frac{9}{2}$ -------------------------- (iii)

By substituting the value of A from equation (iii) in equation (i), we get

$$12=\frac{9}{2}+B$$

Or $B=12-\frac{9}{2}=\frac{15}{2}$

Therefore the partial fraction expansion of the given functions Z(s) is

$$Z(s)=2s+\frac{\frac{9}{2}}{s}+\frac{\frac{15}{2}s+0}{s^2+4}$$ -------------------------- (2)

We then obtain the synthesized network of Foster Form – I of the given function Z(s) is shown in Fig.9.1

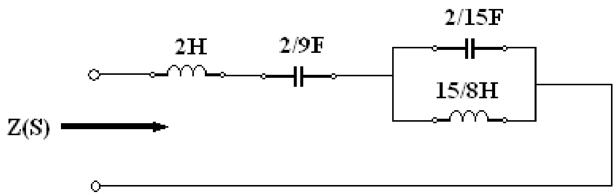


Fig.9.1

Foster-II Form :- In order to find the second Foster Form, we will represent the given impedance function Z(s) into admittance form, so that

$$Y(s) = \frac{s(s^2+4)}{s(s^2+1)(s^2+9)}$$

The partial fraction expansion of Y(s) is

$$Y(s) = \frac{s(s^2+4)}{s(s^2+1)(s^2+9)} = \frac{1}{2}\left[\frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9}\right]$$

Now $s^3 + 4s = (As+B)(s^2+9) + (s+D)(s^2+1)$ ------------------------ (1)

By comparing s^3 coefficients in the above equation (1), we have

1 = A + C ------------------------------------ (i)

By comparing s^2 coefficient on both sides of above equation (1), we have

B + D = 0 ------------------------------------ (ii)

By comparing s coefficient on both sides of above equation (1), we have

9A + C = 4 ------------------------------------ (iii)

By comparing constants on both sides of above equation (1), we have

O = B9 + D ------------------------------------ (iv)

From equation (ii), we have B = -D, by substituting the value of B in equation (iv), we get

9A – A = 0

9(-D) + D = 0

Or -9D + D = 0

Or -8D = 0 or D = 0

By substituting the value of D in equation (iv), we get

B9 + 0 = 0

Or B9 = 0 or B = 0

Equation (i) x 9 $\Rightarrow$ $9\cancel{A} + 9C = 9$

Equation (iiii) $\Rightarrow$ $\cancel{9}A + C = 4$

$-\quad -$

(i) - (iii) 8C = 5

Or $C = \frac{5}{8}$

By substituting the value of C in equation (i), we get

$1 = A + \frac{5}{8}$ or $A = -\frac{5}{8} + 1$

Or $A = \frac{3}{8}$

Therefore,

$$Y(s) = \frac{1}{2}\left[\frac{\frac{3}{8}s}{s^2+1} + \frac{\frac{5}{8}s}{s^2+9}\right]$$

$$= \left[\frac{\frac{3}{16}s}{s^2+1} + \frac{\frac{5}{16}s}{s^2+9}\right]$$

Hence, synthesized network is shown in Fig.9.2

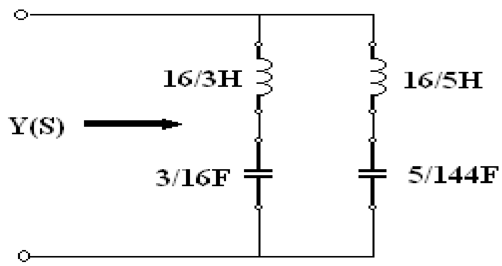


Fig.9.2

<u>CAUER Form – I</u> :- The given impedance function Z(s) is

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{(s^2+4)s} = \frac{2s^4+20s^2+18}{s^3+4s}$$

The continued fraction expansion of Z(s) is

$$s^3+4s\ \big)\ 2s^4+20s^2+18\ \big(\ 2s \leftrightarrow Z_1[2H]$$
$$\underline{2s^4+8s^2}$$
$$12s^2+18\ \big)\ s^3+4s\ \big(\ \frac{1}{12}s \leftrightarrow Y_2\left[\frac{1}{12}F\right]$$
$$\underline{s^3+\frac{3}{2}s}$$
$$\frac{5}{2}s\ \big)\ 12s^2+18\ \big(\ \frac{24}{5}s \leftrightarrow Z_3\left[\frac{24}{5}H\right]$$
$$\underline{12s^2}$$
$$18\ \big)\ \frac{5}{2}s\ \big(\ \frac{5}{36}s \leftrightarrow Y_4\left[\frac{5}{36}F\right]$$
$$\underline{\frac{5}{2}s}$$
$$x$$

Therefore, the final synthesized network [CAUER form – I] is shown in Fig.9.3.

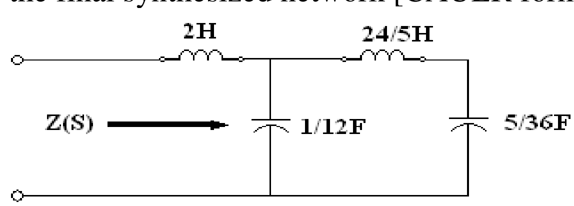


Fig.9.3.

Q.77 Show that the filter described by the transfer function

$$H(s) = \frac{1}{(s^2 + 0.76536s + 1)(s^2 + 1.84776s + 1)}$$ is a low pass filter.

Ans:

The given transfer function H(s) is

$$H(s) = \frac{1}{(s^2 + 0.76536s + 1)(s^2 + 1.84776s + 1)}$$

In low-pass filter design, all the zeros of the system function are at infinity. Therefore, in the given transfer function H(s), the zeros in the numerator are at infinity. Hence the given transfer function H(s) is a Low Pass filter.

Q.78 For the circuit shown in Fig.7. Determine the current i_1, i_2 and i_3. **(8)**

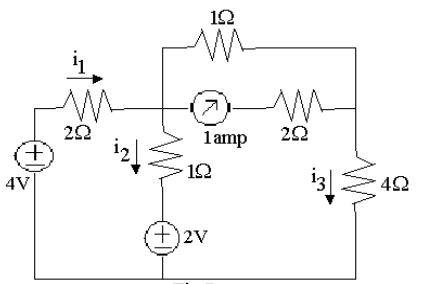


Fig.7

Ans.

The given circuit is redrawn as shown in Fig.7.1.

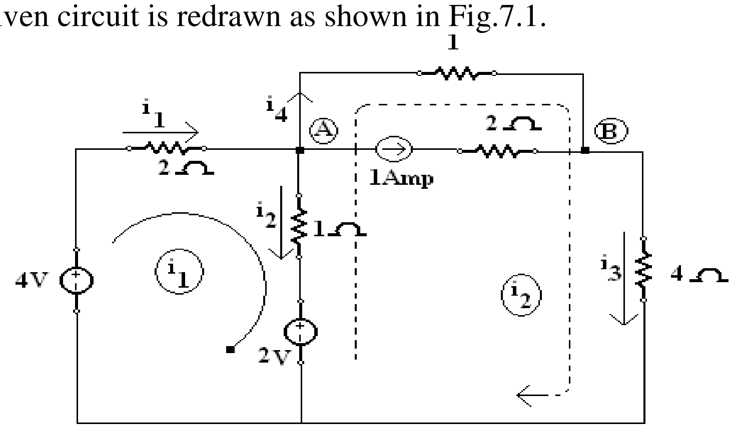


Fig.7.1.

By applying K.C.L at node A, we have

$i_1 = i_2 + i_4 + 1$

Or $i_4 = (i_1 - i_2 - 1)$ ---------------- (1)

By applying K.C.L. at node B, we get

$i_3 = 1 + i_4$ ---------------- (2)

Then, by applying K.V.L. for loop 1, we get

$4 - 2i_1 - i_2 - 2 = 0$

Or $-2i_1 - i_2 + 2 = 0$ ---------------- (3)

Also by applying KVL for loop 2, we get

$2 + i_2 - i_4 - 4i_3 = 0$ ---------------- (4)

From equation (2), we have $i_3 = (1 + i_4)$

By substituting the value of i_3 in equation (4), we get

$2+i_2-i_4-4(1+i_4)=0$

Or $2+i_2-5i_4-4=0$

Or $i_2-5i_4=2$ ---------------- (5)

From equation (1), we have $i_4=(i_1-i_2-1)$

By substituting the value of i_4 in equation (5), we get

$i_2-5(i_1-i_2-1)=2$

Or $6i_2-5i_1=-3$ ---------------- (6)

By multiplying the equation (3), both sides by 6, i.e.,

Equation (3) x 6 $\Rightarrow -12i_1-\cancel{6i_2}=-12$ ---------------- (7)

From eqn (6), we have $-5i_1+\cancel{6i_2}=-3$ ---------------- (8)

Equations (7) & (8) $\Rightarrow -17i_1=-15$

Or $i_1=\dfrac{15}{17}=0.882Amp$

By substituting the value of $i_1=\dfrac{15}{17}Amp$ in equation (6), we get

$6i_2-5i_1=-3$

Or $6i_2-5\left(\dfrac{15}{17}\right)=-3$ or $6i_2-\dfrac{75}{17}=-3$

Or $6i_2-4.4117=-3$

Or $6i_2=4.4117-3$

Or $i_2=0.235Amp$

From equation (1), we have

$i_4=(i_1-i_2-1)$

= 0.882 – 0.235 – 1

= 0.882 – 1.235 = -0.353 Amp

$\therefore\ i_4=-0.353Amp$

From equation (2), we have

$i_3=1+i_4$

Or $i_3=1-0.353=0.647Amp$

$\therefore\ i_3=0.647Amp$

Hence $i_1=0.882Amp$; $i_2=0.235Amp$ & $i_3=0.647Amp$.

Q.79 In the network of the Fig.8, the switch K is open and network reaches a steady state. At t =0, switch K is closed. Find the current in the inductor for t > 0. **(8)**

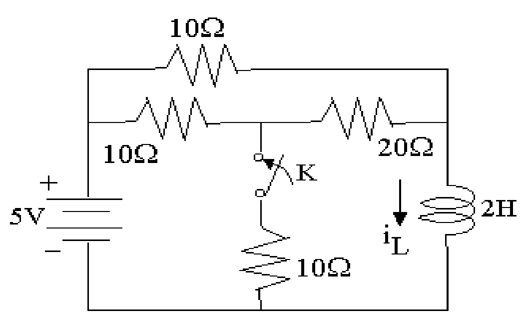


Fig.8

Ans:

At steady state with the switch S is opened.

$$i_L(0^-) = \frac{5}{[(10+20)\|10]} = 0.667\,Amp$$

Now at $t = 0^+$, the switch is closed and using $\Delta - Y$ transformation i.e.,

$$\left[R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{100}{40} = 2.5\Omega \right.$$

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{200}{40} = 5\Omega \quad \text{and}$$

$$\left. R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{200}{40} = 5\Omega \right]$$

After converting Δ (delta) to Y(star) transformation, the circuit becomes as shown in Fig.8.1.

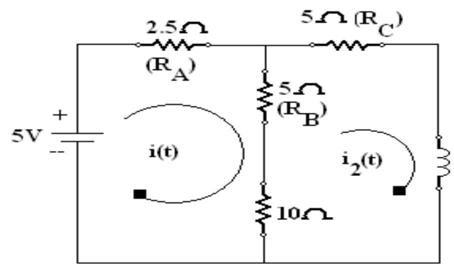


Fig.8.1.

Applying KVL in both the loops in Fig.8.1, we have

$$5 = 2.5i(t) + 15[i(t) - i_L(t)]$$

Or $i(t) = \frac{1}{3.5}[1 + 3i_L(t)]$ ------------------- (1)

And $15[i_L(t) - i(t)] + 5i_L(t) + 2\frac{di_L(t)}{dt} = 0$

Or $2\frac{di_L(t)}{dt} + 20\,i_L(t) = 15\,i(t)$ ------------------- (2)

From equations (1) and (2), we get

$$\frac{di_L(t)}{dt} + \frac{25}{7}i_L(t) = \frac{15}{7}$$

Or $i(t) = \frac{15/7}{25/7} + ke^{-\frac{25}{7}t}$ ------------------- (3)

At $t = 0^+$; $i_L(0^+) = i_L(0^-) = 0.667\,Amp$

From equation (3), we have

Or $0.667 = \frac{15/7}{25/7} + ke^0$

Or $0.667 = 0.599 + k$

Or $k = 0.667$

Therefore, $i(t) = 0.667 + 0.067e^{-3.57t}$ Amp.

Q.80 The network shown in the accompanying Fig.9 is in the steady state with the switch K closed. At t = 0 the switch is opened. Determine the voltage across the switch v_k and $\frac{dv_k}{dt}$ at $t = 0_+$. **(6)**

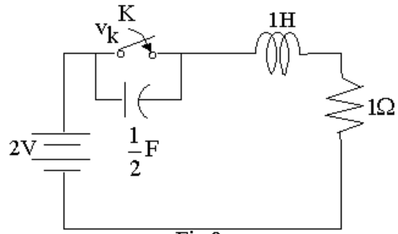


Fig.9

Ans:

When circuit is in the steady state with the switch K is closed, capacitor is short circuited ie., voltage across the capacitor is zero and the inductor is also short circuited. The circuit in the steady state with the switch k is closed is shown in Fig.9.1.

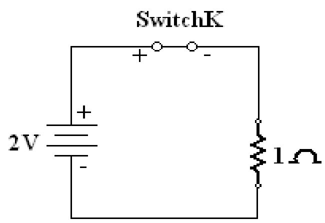


Fig.9.1.

Therefore, $V_K(0^-) = 0$ and the steady state current $i(0^-)$ is $i(0^-) = \frac{V}{R} = \frac{2}{1} = 2Amp$.

And when the switch is opened at t = 0, the capacitor behaves as a short circuit, and the resultant circuit is shown in Fig.9.2.

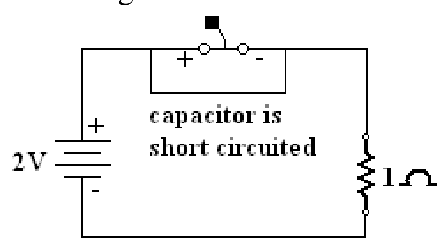


Fig.9.2.

Therefore, V_K at $t = 0^+$ or $V_K(0^+) = 0$

$\therefore\ i(0^+) = i(0^-) = 2A$ [$\because$ capacitor is short circuited and the voltage across the capacitor is zero at $t = 0^+$]

The current through the capacitor is given by

$$i(t) = C\frac{dV_K}{dt} \quad \text{or} \quad \frac{dV_K}{dt} = \frac{1}{C}i(t)$$

Hence $\frac{dV_K}{dt}(0^+) = \frac{1}{C}i(0^+)$

$$= \frac{1}{0.5}\times 2 \qquad \left[\because\ C = \frac{1}{2}F = 0.5F\ \&\ i(0^+) = 2Amp\right]$$

$$= 4\,Volts/\sec$$

Therefore $\frac{dV_K}{dt}(0^+) = 4\,Volts/\sec$

$V_{Th}(s) = 10I(s)$

Where $I(s) = \dfrac{\frac{100}{s}}{s+20} = \dfrac{100}{s(s+20)}$

Then $V_{Th}(s) = 10\left[\dfrac{100}{s(s+20)}\right] = \dfrac{1000}{s(s+20)}$

$\therefore \quad V_{Th}(s) = \dfrac{1000}{s(s+20)}$

$Z_{Th} = s + \left[(s+10)\|10\right] = s + \dfrac{10s+100}{s+20} = \dfrac{s^2+30s+100}{s+20}$

<u>To find</u> $i_1(t)$

Then $I_1(s) = \dfrac{V_{Th}(s)}{Z_{Th}(s)+Z_L} = \dfrac{V_{Th}(s)}{Z_{Th}(s)+10}$

Or $I_1(s) = \dfrac{[(1000)/s(s+20)]}{[(s^2+30s+100)/s+20]+10} = \dfrac{1000}{s(s^2+40s+300)}$

Or $I_1(s) = \dfrac{1000}{s(s+10)(s+30)}$ ---------------- (1)

Using Partial fraction expansion of eqn(1), we get

$$I_1(s) = \frac{1000}{s(s+10)(s+30)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+30}$$

$$A = \left.\frac{1000}{s(s+10)(s+30)}\right|_{s=0} = \frac{1000}{10\times 30} = 3.333$$

$$B = \left.\frac{1000}{s(s+30)}\right|_{s=-10} = \frac{1000}{-10(-10+30)} = \frac{1000}{20\times -10} = -5$$

$$C = \left.\frac{1000}{s(s+10)}\right|_{s=-30} = \frac{1000}{-30(-30+10)} = \frac{1000}{-30(-20)} = 1.666$$

Therefore, $I_1(s) = \dfrac{3.333}{s} - \dfrac{5}{s+10} + \dfrac{1.66}{s+30}$

Hence, the required current $i_1(t)$ is

$i_1(t) = \left[3.33 - 5e^{-10t} + 1.66e^{-30t}\right] Amp$

Q.81 Define Thevenin's theorem. **(4)**

Ans:

Thevenin's Theorem is defined as "any two terminal networks consisting of linear impedances and generators may be replaced by an e.m.f. acting in series with an impedance. The e.m.f is the open circuit voltage at the terminals and the impedance is the impedance viewed at the terminals when all the generators in the network have been replaced by impedances equal to their internal impedances".

Q.82 It is required to find the current $i_1(t)$ in the resistor R_3, by using Thevenin's theorem: The network shown in Fig.10 is in zero state until t = 0 when the switch is closed. **(6)**

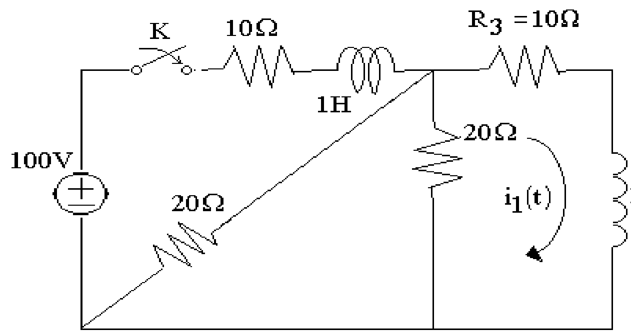


Fig.10

Ans:

In the network shown in Fig.10, the resistance of 20Ω between BC is in parallel with the resistance of 20Ω between BD. Hence the equivalent resistance is

$$\text{Re}\,q = \frac{20\times 20}{20+20} = \frac{400}{40} = 10\Omega$$

Now the equivalent network of Fig.10, after converting into Laplace Transformed Version for t>0 is shown in Fig.10.1.

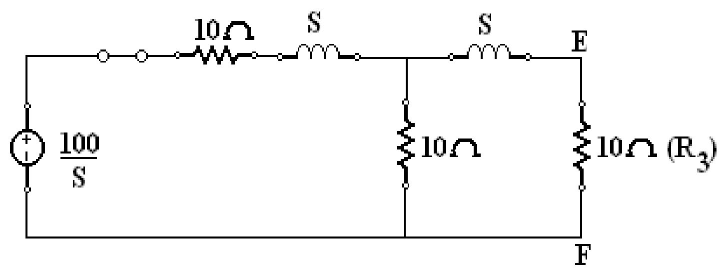


Fig.10.1

The network of Fig.10 is in zero state before the switch is closed i.e., the values of voltages and currents for an excitation which is applied when all initial conditions are zero. i.e.,

$i_{L_1}(0^-) = OA \quad \& \quad i_{L_2}(0^-) = OA$

In order to find $i_1(t)$ in the resister $R_3(10\Omega)$ by using Thevenin's Theorem, first remove the load resistance 10Ω i.e., R_3 and find $V_{TH}(s)$ & $Z_{TH}(s)$. The equivalent circuit of Fig.10.1 is shown in Fig.10.2 for find $V_{TH}(s)$ & $Z_{TH}(s)$.

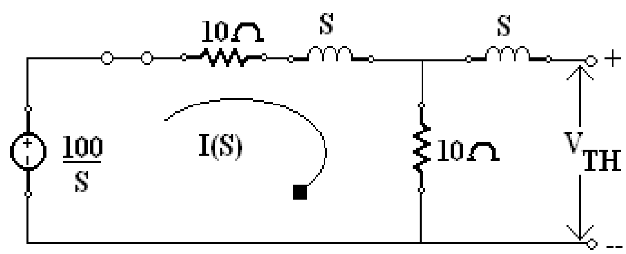


Fig.10.2

Q.83 For the given network in Fig.11, determine the value of R_L that will cause the power in R_L to have a maximum value. What will be the value of power under this condition. **(8)**

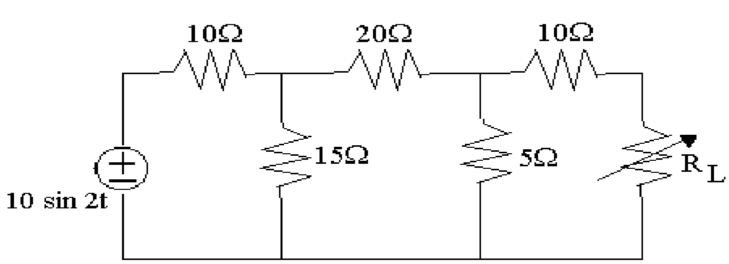


Fig.11

For the given circuit of Fig.11, let us find out the Thevenin's equivalent circuit across AB as shown in Fig.11.1

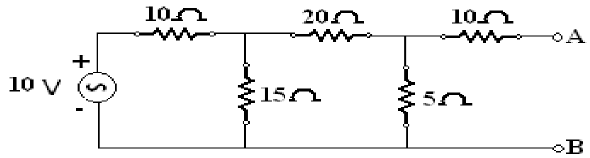


Fig.11.1

The total resistance is

$$R_T = \left[\left(\left\{(10+5)\|20\right\}+15\right)\|10\right]$$

$$= \left[\left\{\left[\frac{15\times 20}{15+20}\right]+15\right\}\|10\right]$$

$$= [8.57+15]\|10$$

$$= (23.57)\|10$$

$$= \frac{235.71}{33.57} = 7.02\Omega$$

Or $R_T = 7.02\Omega$

The total current drawn by the circuit is

$$I_T = \frac{10}{7.02} = 1.424\,Amp$$

The current in the 5Ω resister is

$$I_5 = I_T \times \frac{15}{25+15}$$

$$= 1.424 \times 0.375 = 0.534\,Amp$$

Or $I_5 = 0.534\,Amp$

Thus, Thevenin's Voltage $V_{AB} = V_5 = 5\times I_5$

Or $V_5 = 5\times 0.534$

Or $V_5 = V_{AB} = 2.67\,Volts$

Thevenin's resistance $R_{Th} = R_{AB} = \left[\left\{\left(10\|15\right)+20\right\}\|5\right]+10$

Or $R_{Th} = R_{AB} = \left[\left\{\frac{10\times 15}{10+15}+20\right\}\|5\right]+10$

$$= \left[[6+20]\,\|\,5\right]+10$$

$$= \left[\frac{26\times 5}{26+5}\right]+10 = 14.193\Omega$$

Or $R_{Th} = 14.19\Omega$

The Thevenin's equivalent circuit is shown in Fig.11.2

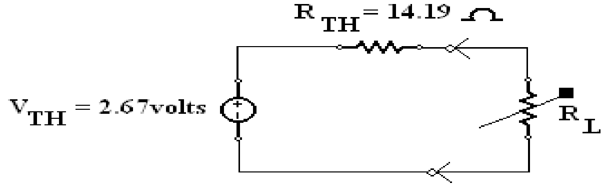


Fig.11.2

According to Maximum Power Transfer Theorem, the maximum power from source to load is transferred, when

$R_S = R_{Th} = R_L = 14.19\Omega$

Current drawn by the load resistance R_L is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{2.67}{14.19 + 14.19} = \frac{2.67}{28.387}$$

$I_L = 0.0940 Amp = 94 mA$

Now, the power delivered to the load R_L is

$P_L = I_L^2 R_L = 0.0940 \times 14.19 = 1.33W$

Q.84 In the network shown in Fig.12 $v_1 = 10 \sin 10^6 t$ and $i_1 = 10 \cos 10^6 t$ and the network is operating in the steady state – For the element values as given, determine the node to datum voltage $v_a(t)$. **(8)**

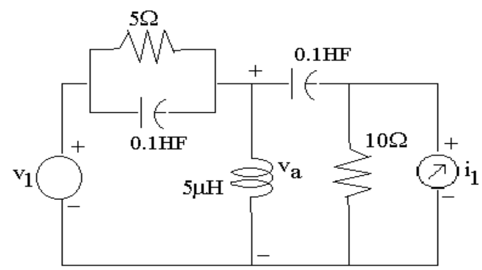


Fig.12

Ans:

Given data is

$V_1 = 10$ & $i_1 = j10$

The Laplace transform equivalent of Fig.12 is shown in Fig.12.1

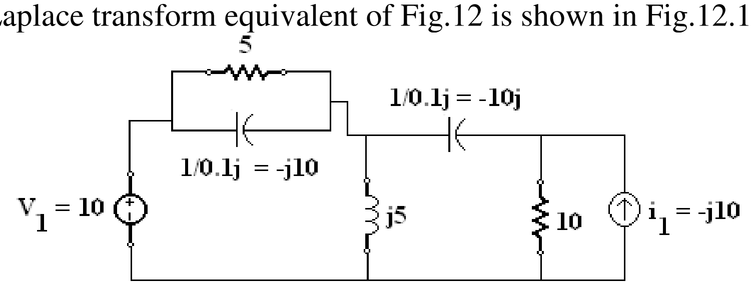


Fig.12.1

The network of Fig.12.1 is redrawn by converting the current source into voltage source which is shown in Fig.12.2

V = -j10 x 10 [∵ V = iR = -j10 x 10 = -j100]

= -j100

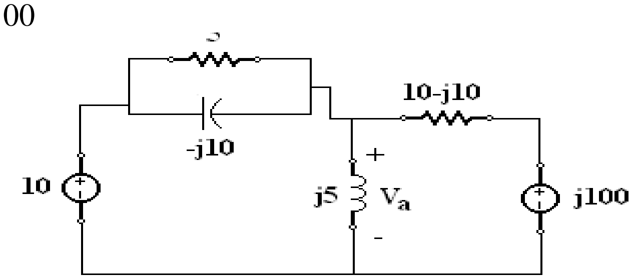


Fig.12.2

Now $$\frac{1}{10 - j10} = \frac{10 + j10}{100 + 100} = \frac{10 + j10}{200} = 0.05 + j0.05$$

Therefore, the Node-to-Datum voltage $V_a(t)$ is determined

$$V_a = \frac{2+j-5+j5}{\left(\frac{1}{5}\right)-j\left(\frac{1}{10}\right)+0.05+j0.05}$$

Or $V_a = \dfrac{-3+j6}{0.2-j0.1+0.05+j0.05}$

Or $V_a = \dfrac{-3+j6}{0.25-j0.05} = \dfrac{6.7e^{j116.6^0}}{0.255e^{-j11.3^0}}$

Or $V_a = 26.3e^{j(116.6+11.3)}$

Or $V_a = 26.3e^{j(127.9)}$

Or $V_a = 26.3\sin\left(10^6 t + 127.9^0\right)$

Or $V_a = 26.3\sin\left(10^6 t + 90^0 + 37.9^0\right)$

Or $V_a = 26.3\cos\left(10^6 t + 37.9^0\right)$

Therefore, the node-to-datum voltage $V_a(t)$ for the given network is

$V_a = 26.3\cos\left(10^6 t + 37.9^0\right)$

Q.85 Determine the amplitude and phase for F(J4) from the pole zero plot in s-plane for the network function $F(s) = \dfrac{s^2+4}{(s+2)\left(s^2+9\right)}$. **(8)**

Ans:

The given network function F(s) is

$$F(s) = \frac{s^2+4}{(s+2)\left(s^2+9\right)} = \frac{s \pm 2j}{(s+2)(s \pm 3j)}$$

Or $F(s) = \dfrac{(s+2j)(s-2j)}{(s+2)(s+3j)(s-3j)}$

Therefore, the network function F(s) has

(i) Two zeros at s = -2j & s = +2j

(ii) Three Poles at s = -2, s = -3j & s = +3j

The Pole-zero diagram of the network function F(s) is shown in Fig.13.1

<u>Finding of Amplitude & Phase for F(j4)</u> :-

Now $F(j4) = \dfrac{(j4)^2+4}{(j4+2)((j4)^2+9)}$

$$= \frac{j^2 16+4}{(j4+2)(j^2 4^2+9)} = \frac{-16+4}{(j4+2)(-16+9)}$$

Or $F(j4) = \dfrac{(-12+j0)}{(2+j4)(-7+j0)}$ ----------------- (1)

Therefore $F(j4) = |M(j4)|.\phi\underline{|(j4)}$

Now by converting the Rectangular form of roots of equation (1) in poles form, we get

-12 + j0	2 + j4	-7 + j0
R$\cos\phi$ = -12	R$\cos\phi$ = 2	R$\cos\phi$ = -7
R$\sin\phi$ = 0	R$\sin\phi$ = 4	R$\sin\phi$ = 0

$R\sqrt{(-12)^2+0^2}$ $\quad$ $R=\sqrt{2^2+4^2}=\sqrt{4+16}$ $\quad$ $R=\sqrt{(-7)^2+0^2}$

R = 12 & $\quad$ $=\sqrt{20}$ $\quad$ $=\sqrt{49}$

$\phi=90^0$ $\quad$ $\phi=Tan^{-1}\left(\frac{4}{2}\right)$ $\quad$ or R = 7

$= 63.43^0$ $\quad$ $\phi=90^0$

$\therefore\ -12+j0$ $\quad$ $\therefore\ 2+j4=\sqrt{20}\underline{|63.43^0}$ $\quad$ $\therefore\ -7+j0^0$

$=12\underline{|90^0}$ $\quad$ $=7\underline{|90^0}$

Therefore the magnitude of the given network function F(s) becomes

$$|M(j4)|=\frac{12}{\sqrt{20}\times 7}=0.3833 \text{ and}$$

The phase of the given network function F(s) becomes

$$\phi(j4)=\frac{90^0}{90^0+63.43^0}$$

Or $\phi(j4)=90^0-90^0-63.43^0$

Or $\phi(j4)=-63.43^0$

The magnitude of zeros and poles & the phases of zeros and poles are shown in Fig.13.1.

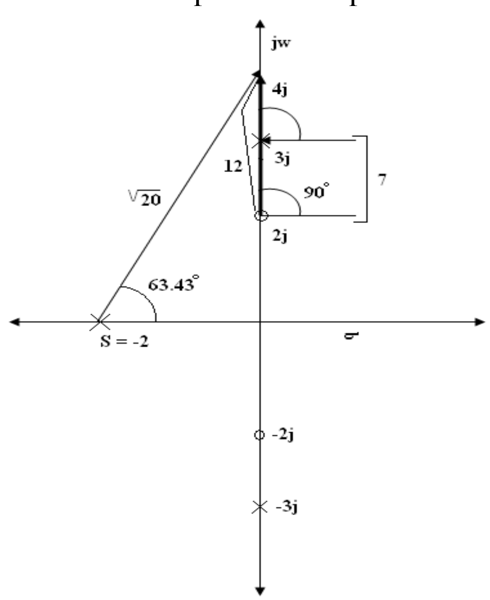


Fig.13.1.

Q.86 A network function consists of two poles at $P_{1,2}=r_i e^{\pm J(\pi-\theta)}=-\sigma_i \pm J\omega_i$ as given in the Fig.13. Show that the square of the amplitude response $M^2(\omega)$ is maximum at $\omega_m^2=r_i^2|\cos 2\theta|$. **(8)**

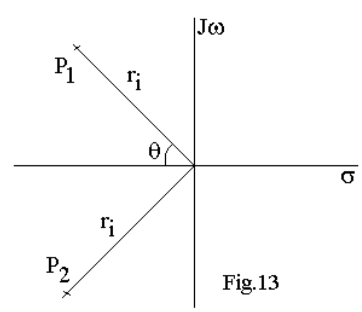


Fig.13

Ans:
Given that the network function consists of two poles at
$P_{1,2} = r_i e^{\pm j(\pi-\theta)} = -\sigma_i + j\omega_i$

------------------ (1)

In terms of σ_i and ω_i in equation (1), H(s) is given by

$$H(s) = \frac{\sigma_i^2 + \omega_i^2}{(s+\sigma_i + j\omega_i)(s+\sigma_i - j\omega_i)}$$

From the pole-zero diagram of H(s) shown in Fig.13.2, we will determine the amplitude response $|H(j\omega)|$.

Let us denote the vectors from the poles to the jw axis as $|M_1|$ & $|M_2|$ as seen in Fig.13.3.

$$|H(j\omega)| = \frac{k}{|M_1||M_2|}.$$

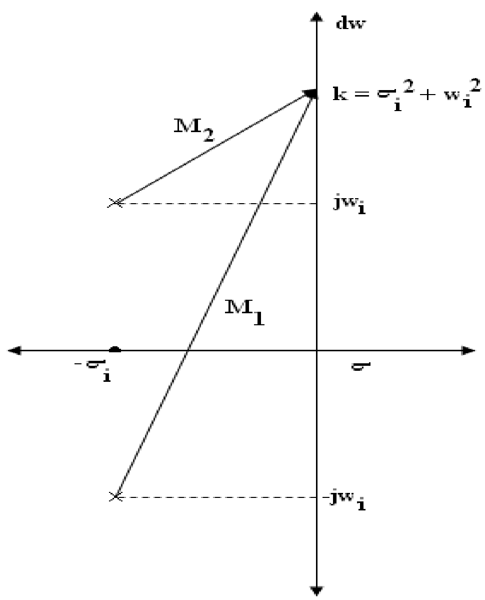


Fig.13.3.

We can then write

Where $k = \sigma_i^2 + \omega_i^2$

$$|M_1| = \left(\sigma_i^2 + (\omega + j\omega_i)^2\right)^{1/2}$$

$$|M_2| = \left(\sigma_i^2 + (\omega - j\omega_i)^2\right)^{1/2}$$

In characterizing the amplitude response, the point $\omega = \omega_{\max}$ at which $|H(j\omega)|$ is maximum is highly significant from both the analysis and design aspects. Since $|H(j\omega)|$ is always positive, the point at which $|H(j\omega)|^2$ is maximum corresponds exactly to the point at which $|H(j\omega)|$ is maximum.

Since $|H(j\omega)|^2$ can be written as

$$|H(j\omega)|^2 = \frac{\left(\sigma_i^2+\omega_i^2\right)^2}{\left[\sigma_i^2+(\omega+j\omega_i)^2\right]\left[\sigma_i^2+(\omega-j\omega_i)^2\right]}$$

$$= \frac{\left(\sigma_i^2+\omega_i^2\right)^2}{\sigma_i^4+2\omega^2\left(\sigma_i^2-\omega_i^2\right)+\left(\sigma_i^2+\omega_i^2\right)^2}$$

We can find $\omega_{\max}$ by taking the derivative of $|H(j\omega)|^2$ write ω^2 and setting the result equal to zero. Thus we have

$$\frac{d|H(j\omega)|^2}{d\omega^2} = -\frac{\left(\sigma_i^2+\omega_i^2\right)^2\left[2\omega^2+2\left(\sigma_i^2-\omega_i^2\right)\right]}{\left[\sigma_i^4+2\omega^2\left(\sigma_i^2-\omega_i^2\right)+\left(\sigma_i^2+\omega_i^2\right)^2\right]^2}$$

From the equation $\frac{d|H(j\omega)|^2}{d\omega^2}=0$, then

We determine $\omega_{\max}^{\ 2} = \left|\omega_i^2-\sigma_i^2\right|$

Now $\sigma_i = r_i\cos(\pi-\theta)$ and

$\omega_i = r_i\sin(\pi-\theta)$

Therefore, $\omega_{\max}^2 = \left|\omega_i^2-\sigma_i^2\right|$

$\omega_{\max}^2 = \left|r_i^2\sin^2(\pi-\theta)-r_i^2\cos^2(\pi-\theta)\right|$

Or $\omega_{\max}^2 = r_i^2\left|\sin^2(\pi-\theta)-\cos^2(\pi-\theta)\right|$

By solving the above equation, we get

$\omega_{\max}^2 = r_i^2|\cos 2\theta|$

Hence, the square of the amplitude response $M^2(\omega)$ is maximum at $\omega_{\max}^2 = r_i^2|\cos 2\theta|$.

Q.87 Following short circuit currents and voltages are obtained experimentally for a two port network

(i) with output short circuited $I_1 = 5\text{mA}$ $I_2 = -0.3\text{mA}$ $V_1 = 25\text{V}$

(ii) with input short circuited $I_1 = -5\text{mA}$ $I_2 = 10\text{mA}$ $V_2 = 30\text{V}$

Determine Y-parameters. **(8)**

Ans:

The given short circuit currents and voltages, when the output is short-circuited i.e. when $V_2 = 0$ are

$I_1 = 5mA$

; $I_2 = -0.3mA$ and $V_1 = 25V\big|_{when\ V_2=0}$

And the given short circuit currents and voltages, when the inupt is short-circuited i.e., when $V_1 = 0$ are

$I_1 = -5mA$; $I_2 = 10mA$ and $V_2 = 30V\big|_{when\ V_1=0}$

Therefore, the Y-parameter equations are

$I_1 = Y_{11}.V_1 + Y_{12}.V_2$ and

$I_2 = Y_{21}.V_1 + Y_{22}.V_2$

Hence $Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0} = \dfrac{5mA}{25volts} = 0.2\times10^{-3}$ mho

$Y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0} = \dfrac{-0.3mA}{25volts} = -0.012\times10^{-3}$ mho

$Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0} = \dfrac{10mA}{30volts} = 0.333\times10^{-3}$ mho

$Y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1=0} = \dfrac{-5mA}{30volts} = -0.166\times10^{-3}$ mho

Therefore the Y-parameters are

$Y_{11} = 0.2\times10^{-3}$ mho; $Y_{22} = 0.333\times10^{-3}$ mho and

$Y_{21} = -0.012\times10^{-3}$ mho; $Y_{12} = -0.166\times10^{-3}$ mho

Q.88 The network of the Fig.14 contains a current controlled current source. For the network find the z-parameters. **(8)**

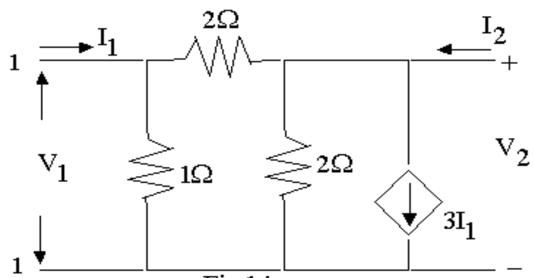


Fig.14

Ans:

First, we convert current source in equivalent voltage source as shown in Fig.14.1.

Then the loop equations for Fig.14.1 are

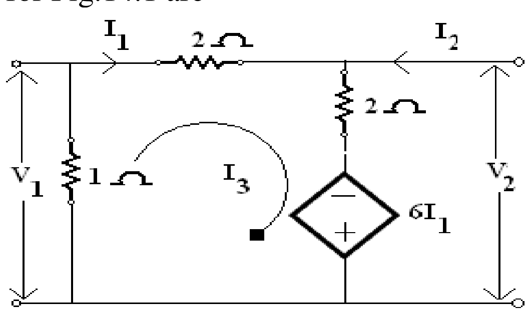


Fig.14.1

$V_1 = 1(I_1 - I_3)$ ------------------- (1)

$6I_1 = 1(I_3 - I_1) + 2I_3 + 2(I_3 + I_2)$

$6I_1 - I_3 + I_1 - 2I_3 - 2I_3 - 2I_2 = 0$

Or

Or $7I_1 - 2I_2 - 5I_3 = 0$ ------------------- (2)

And $V_2 = 2(I_2 + I_3) - 6I_1$

Or $V_2 = 2I_2 + 2I_3 - 6I_1$

Or $V_2 = -6I_1 + 2I_2 + 2I_3$ ------------------- (3)

From equation (2), we have

$7I_1 - 2I_2 = 5I_3$

Or $I_3 = \dfrac{7I_1 - 2I_2}{5} = 1.4I_1 - 0.4I_2$

Or $I_3 = 1.4I_1 - 0.4I_2$ ------------------- (4)

By substituting the value of I_3 from eqn (4), in eqn (1), we get

$V_1 = I_1 - (1.4I_1 - 0.4I_2)$

Or $V_1 = I_1 - 1.4I_1 + 0.4I_2$

Or $V_1 = -0.4I_1 + 0.4I_2$ ------------------- (5)

By substituting the value of I_3 from eqn (4), in eqn (3), we get

$V_2 = -6I_1 + 2I_2 + 2(1.4I_1 - 0.4I_2)$

Or $V_2 = -6I_1 + 2I_2 + 2.8I_1 - 0.8I_2$

Or $V_2 = -3.2I_1 + 1.2I_2$ ------------------- (6)

From equation (5), we have

$V_1 = -0.4I_1 + 0.4I_2$ ------------------- (7)

And from equation (6), we have

$V_2 = -3.2I_1 + 1.2I_2$ ------------------- (8)

Now the Z-parameters are found out by equation (7) & (8)

$Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = -0.4\Omega$ [$\because$ from eqn (7), we have $\dfrac{V_1}{I_1} = -0.4 \Big| when\ I_2 = 0$]

$Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0} = -3.2\Omega$ [$\because$ from eqn (8), we have $\dfrac{V_2}{I_1} = -3.2 \Big| when\ I_2 = 0$]

$Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0} = 0.4\Omega$ [$\because$ from eqn (7), we have $\dfrac{V_1}{I_2} = 0.4 \Big| when\ I_1 = 0$]

$Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0} = 1.2\Omega$ [$\because$ from eqn (8), we have $\dfrac{V_2}{I_2} = 1.2 \Big/ when\ I_1 = 0$].

Q.89. In the network of Fig.15, K is changed from position a to b at t = 0. Solve for i, $di|dt$, and $d^2i|dt^2$ at t = 0 + if R = 1000Ω, L=1H, C=0.1 μF, and V = 100 V. **(8)**

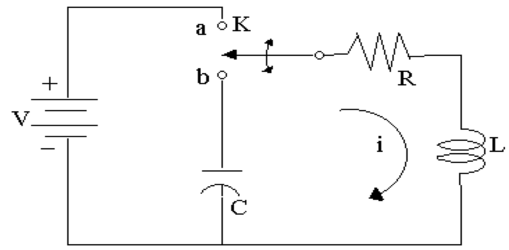


Fig.15

Ans:

At position 'a':

The steady-state value of current $i(0^-)$ is

$$i(0^-) = \frac{100}{1000} = 0.1A$$

At position 'b':

$$i(0^+) = i(0^-) = 0.1A$$

Applying KVL for the network of Fig.15, we have

$$1000\, i(t) + 1\frac{di(t)}{dt} + \frac{1}{0.1\times 10^{-6}}\int_{-\infty}^{t} i(t)dt = 0 \qquad \text{-------------------- (1)}$$

Since initially capacitor is unchanged and at switching instant capacitor behaves as a short circuit i.e., the last term of eqn(1) is equal to zero,Then from eqn(1) we have

$$1000\, i(t) + \frac{di(t)}{dt} = 0$$

$$\text{Or } \frac{di(0^+)}{dt} = -1000\, i(0^+) = -100A/\sec$$

Hence $\frac{di(0^+)}{dt} = -100A/\sec$

On differentiating the eqn (1), we have

$$\frac{d^2 i(t)}{dt^2} = -1000\frac{di(t)}{dt} - \frac{1}{0.1\times 10^{-6}} i(t)$$

Or $\frac{d^2 i(0^+)}{dt^2} = -1000\times(-100) - 10^7 \times (0.1)$

$$\left[\because \frac{di}{dt} = -100 \quad \&\, i(t) = 0.1\right]$$

Or $\frac{d^2 i(0^+)}{dt^2} = (-10^5 + 10^6)$

Or $\frac{d^2 i(0^+)}{dt^2} = -9\times 10^5 A/\sec^2$.

Q.90 Given $z(s) = \frac{s^2 + Xs}{s^2 + 5s + 4}$ what are the restrictions on 'X'. For z(s) to be a positive real function and find 'X' for $\mathrm{Re}[z(J\omega)]$ to have second order zero at $\omega = 0$. **(8)**

Ans:

The given function $z(s) = \frac{s^2 + Xs}{s^2 + 5s + 4}$ is in the form of biquadratic equation as

$$z(s)=\frac{s^2+a_1s+a_0}{s^2+b_1s+b_0}$$

Where $a_1 = X$; $a_0 = 0$ &

$b_1 = 5$; $b_0 = 4$.

For Z(s) to be a positive real function, the following condition is to be satisfied i.e.,

$$a_1b_1 \geq \left(\sqrt{a_0}-\sqrt{b_0}\right)^2$$

i.e. $(X).(5) = X5 \geq \left(\sqrt{0}-\sqrt{4}\right)^2$

or $X5 \geq 4$

in order to have Z(s) to be a positive real function the value X must be equal to 0.8 or more than 0.8.

or (0.8)(5) = 4 or 4 = 4.

Hence, for Z(s) to be a positive real function the value X must $\geq$ 0.8

<u>Finding of value X for Re{Z(jω)}:</u>

The given function Z(s) is given by

$$z(s)=\frac{s^2+Xs}{s^2+5s+4}$$

Now

$$Z(j\omega)=\frac{(j\omega)^2+X(j\omega)}{(j\omega)^2+5(j\omega)+4}=\frac{-\omega^2+j\omega}{-\omega^2+5j\omega+4}$$

Or

$$Z(j\omega)=\frac{\left(Xj\omega-\omega^2\right)}{\left(4-\omega^2\right)+5j\omega}\times\frac{\left(4-\omega^2\right)-5j\omega}{\left(4-\omega^2\right)-5j\omega}$$

$$=\frac{\left(Xj\omega-\omega^2\right)\left(4-\omega^2\right)-5j\omega}{\left(4-\omega^2\right)-\left(5j\omega\right)^2}$$

$$=\frac{4Xj\omega-\omega^2Xj\omega+5X\omega^2-4\omega^2+\omega^4+5\omega^2j\omega}{16+\omega^4-8\omega^2+25\omega^2}$$

Or

$$Zj\omega=\frac{(4Xj\omega-w^2Xj\omega+5\omega^2j\omega)+(\omega^4-4\omega^2+5X\omega^2)}{\omega^4+17\omega^2+16}$$

Hence

$$\text{Re}\{Z(j\omega)\}=\frac{\omega^4-4\omega^2+5X\omega^2}{\omega^4+17\omega^2+16} \qquad \text{---------------------} \ (1)$$

To have second order zero at ω = 0, the eqn (1) becomes equal to zero.

i.e., $5X\omega^2-4\omega^2=0$ [$\because$ second order zeros has been chosen from eqn (1)]

or $5X\cancel{\omega^2}=4\cancel{\omega^2}$

or $5X=4$

or $X=\frac{4}{5}=0.8$

Hence, the value of X must be 0.8, for Z(s) to be a positive real fuction.

Q.91 List out the properties of LC immittance function and then realize the network having the driving point impedance function $z(s)=\dfrac{2s^5+12s^3+16s}{s^4+4s^2+3}$ by continued fraction method. **(8)**

Ans:

1. $Z_{LC}(s)$ or $Y_{LC}(s)$ is the ratio of odd to even (or) even to odd polynomials.
2. The poles and zeros are simple and lie on the jω axis.
3. The poles and zeros interlace on the jω axis.
4. The highest powers of numerator and denominator must differ by unity; the lowest powers also differ by unity.
5. There must be either a zero or a pole at the origin and infinity.

Realization of the network having the driving point impedance function $z(s)=\dfrac{2s^5+12s^3+16s}{s^4+4s^2+3}$ by continued fraction method:

The given driving point impedance function Z(s) is

$$z(s)=\frac{2s^5+12s^3+16s}{s^4+4s^2+3}$$

By taking continued fraction expansion, [CAUER-I] we get

$$s^4+4s^2+3\;\Big)\;2s^5+12s^3+16s\;\Big(\;2s \leftarrow L_1$$
$$\underline{2s^5+8s^3+16s}$$
$$4s^3+10s\;\Big)\;s^4+4s^2+3\;\Big(\;\frac{s}{4}\leftarrow C_2$$
$$\underline{s^4+\frac{5}{2}s^2}$$
$$\frac{3s^2}{2}+3\;\Big)\;4s^3+10s\;\Big(\;\frac{8}{3}s\leftarrow L_3$$
$$\underline{4s^3+8s}$$
$$2s\;\Big)\;\frac{3s^2}{2}+3\;\Big(\;\frac{3}{4}s\leftarrow C_4$$
$$\underline{\frac{3s^2}{2}}$$
$$3\;\Big)\;2s\;\Big(\;\frac{2}{3}s\leftarrow L_5$$
$$\underline{2s}$$
$$0$$

Hence Z(s) = $2s+\cfrac{1}{\cfrac{s}{4}+\cfrac{1}{\cfrac{8}{3}s+\cfrac{1}{\cfrac{3}{4}s+\cfrac{1}{\cfrac{2}{3}s}}}}$

The resulting network for the given driving point impedance function Z(s) is shown in Fig.16.

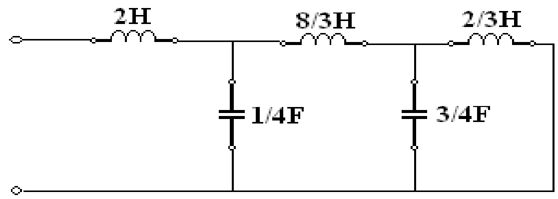


Fig.16

Q.92 For the network function $Y(s)=\dfrac{2(s+1)(s+3)}{(s+2)(s+4)}$ synthesize in one Foster and one Cauer form.

(8)

Ans:

The given network function Y(s) is

$$Y(s)=\frac{2(s+1)(s+3)}{(s+2)(s+4)}=2-\frac{1}{s+2}-\frac{3}{s+4}$$

Thus the residues of Y(s) are real but negative and poles and zeros alternative on negative real axis. Therefore, in order to remove negative sign, we take $\frac{Y(s)}{s}$ & at last, then multiply S.

So, $\dfrac{Y(s)}{s}=\dfrac{2(s+1)(s+3)}{s(s+2)(s+4)}$

By taking the partial fractions for the above equation, we get

$$\frac{Y(s)}{s}=\frac{2(s+1)(s+3)}{s(s+2)(s+4)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+4}$$

$$A=\left.\frac{2(s+1)(s+3)}{(s+2)(s+4)}\right|_{s=0}=\frac{6}{8}=\frac{3}{4}$$

$$B=\left.\frac{2(s+1)(s+3)}{s(s+4)}\right|_{s=-2}=\frac{2\times(-1)\times1}{(-2)(-2+4)}=\frac{1}{2}$$

$$C=\left.\frac{2(s+1)(s+3)}{s(s+2)}\right|_{s=-4}=\frac{2\times(-3)\times(-1)}{(-4)(-2)}=\frac{3}{4}$$

Therefore, $\dfrac{Y(s)}{s}=\dfrac{3}{4s}+\dfrac{1}{2(s+2)}+\dfrac{3}{4(s+4)}$

By multiplying the above equation both sides by s, we get

$$\frac{Y(s)}{s}=\frac{3}{4}+\frac{s}{2(s+2)}+\frac{3s}{4(s+4)}$$

Therefore, the first term is of Resistance (R) of value $\frac{3}{4}\Omega$. The second term is the parallel combination of Resistance (R) of value $\frac{1}{2}\Omega$ and inductance of value $\frac{1}{4}H$. The third term is the Parallel Combination Resistance (R) of value $\frac{3}{4}\Omega$ and inductance of value $\frac{3}{16}H$.

Hence, the synthesized network for the given function Y(s) is shown in Fig.17.

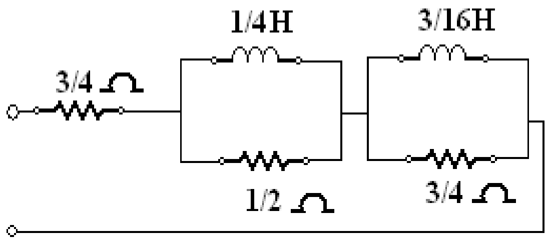


Fig.17.

Synthesizing of the network function Y(s) = $\frac{2(s+1)(s+3)}{(s+2)(s+4)}$ in CAUER FORM – I:

The given network function is

$$Y(s)=\frac{2(s+1)(s+3)}{(s+2)(s+4)}=\frac{2(s^2+3s+s+3)}{(s^2+4s+2s+8)}=\frac{2(s^2+4s+3)}{s^2+6s+8}$$

Or $Y(s)=\frac{2s^2+8s+6}{s^2+6s+8}$

Now $Z(s)=\frac{1}{Y(s)}=\frac{s^2+6s+8}{2s^2+8s+6}$

Now taking the continued fraction expansion of Z(s), we have

$$2s^2+8s+6\,\big)\,s^2+6s+8\,\big(\,\tfrac{1}{2}\leftarrow Z_1$$
$$\underline{s^2+4s+4}$$
$$2s+4\,\big)\,2s^2+8s+6\,\big(\,s\leftarrow Y_2$$
$$\underline{2s^2+4s}$$
$$4s+6\,\big)\,2s+4\,\big(\,\tfrac{1}{2}\leftarrow Z_3$$
$$\underline{2s+3}$$
$$1\,\big)\,4s+6\,\big(\,4s\leftarrow Y_4$$
$$\underline{4s}$$
$$6\,\big)\,1\,\big(\,\tfrac{1}{6}\leftarrow Z_5$$
$$\underline{1}$$
$$0$$

The synthesized CAUER Form-I network for the given fuction is shown in Fig.18.

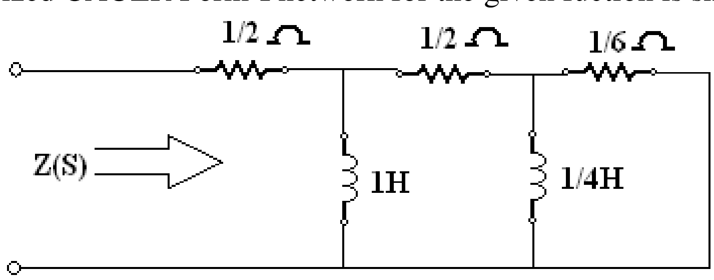


Fig.18

Q.93 The voltage ratio transfer function of a constant-resistance bridged-T network is given by $\frac{v_2}{v_1}=\frac{s^2+1}{s^2+2s+1}$ synthesize the network that terminated in a 1Ω resistor. **(8)**

Ans: The given voltage ration transfer function $\frac{V_2}{V_1}$ is

$$\frac{V_2}{V_1} = \frac{s^2+1}{s^2+2s+1} = \frac{1}{\frac{s^2+1+2s}{s^2+1}}$$

$$\frac{V_2}{V_1} = \frac{1}{1+\left[\frac{2s}{s^2+1}\right]} \qquad \text{---------------- (1)}$$

But the voltage ration transfer function of a constant-resistance bridged-T network is given by

$$\frac{V_2}{V_1} = \frac{R}{R+Z_a} = \frac{Z_b}{Z_b+R}$$

Where $R = R_{in} = R_L = 1\Omega$, so that

$[\because R = R_{in} = R_L = 1\Omega]$

$$\frac{V_2}{V_1} = \frac{1}{1+Z_a} = \frac{Z_b}{1+Z_b} \qquad \text{--------------- (2)}$$

Or $$\frac{V_2}{V_1} = \frac{1}{1+Z_a} = \frac{1}{1+\left(\frac{1}{Z_b}\right)} \qquad \text{--------------- (3)}$$

By comparing eqns (1) and (3), we get

$$Z_a = \frac{2s}{s^2+1} \quad \& \quad Z_b = \frac{s^2+1}{2s}$$

Hence, we recognize Z_a as a parallel L-C tank circuit and Z_b as a series L-C tank circuit.
Therefore, the final synthsized network for the given voltage ratio transfer function of a constant-resistance bridged-T network is shown in fig.18.

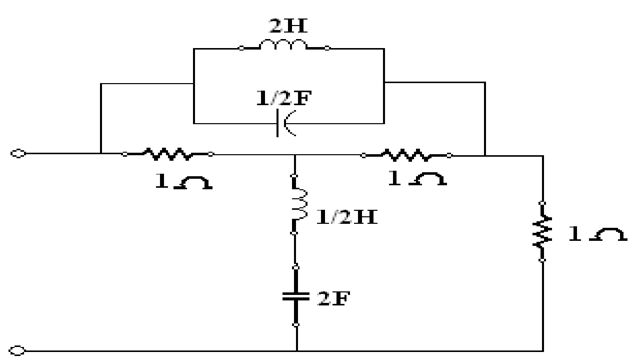


fig.18.

Q.94 Find the poles of system functions for low-pass filter with n =3 and n = 4 Butterworth characteristics. (Do not use the tables) **(8)**

Ans: The general form of cascaded Low Pass Filter Transfer function $\left|H(\omega)\right|^2 = \dfrac{A_0^2}{1+f(\omega)^2}$ if we select $f(\omega)^2$ as $\left(\dfrac{\omega}{\omega_0}\right)^n$, the cascaded transfer function takes the form of

$$\left|H(\omega)\right|^2 = \frac{A_0^2}{1+\left(\dfrac{\omega}{\omega_0}\right)^n} = \frac{A_0^2}{B_n^2(\omega)}$$

Where $B_n^2(\omega) = 1+\left(\dfrac{\omega}{\omega_0}\right)^{2n}$ is called the Butterworth Polynomial, n being a positive integer indicating the order of filter cascade.

Therefore, the Butterworth polynomial is given by

$$B_n^2(\omega) = 1+\left(\frac{\omega}{\omega_0}\right)^{2n}$$

Normalising for $\omega_0 = 1$ *rad/sec*

$\therefore\ B_n^2(\omega) = 1+\omega^{2n}$

However, $B_n^2(\omega) = \left|B_n(j\omega)\right|^2 = \left|B_n(s).B_n(-s)\right|_{s=j\omega}$

$$= \left|1+(-s^2)^n\right|_{s^2=-\omega^2}$$

The zeros (2n numbers) of [$B_n(s).B_n(-s)$] are obtained by solving $1+(-1)^n s^{2n} = 0$

Case 1: [when n is even]

The equation $1+(-1)^n s^{2n} = 0$ reduces to $s^{2n} = -1$ which can also be written in form of $s^{2n} = -1 = e^{j(2i-1)\pi}$,

Obviously, the 2n roots are given by $P_i = e^{j\frac{2i-1}{2n}.\pi}$, where i = 1, 2. ...., 2n

Or $P_i = \cos\left(\dfrac{2i-1}{2n}.\pi\right) + j\sin\left(\dfrac{2i-1}{2n}.\pi\right)$

Case 2 :- [when n is odd]

The equation $1+(-1)^n s^{2n} = 0$ reduces to $s^{2n} = 1$, which can be further written as $s^{2n} = 1 = e^{j2i\pi}$

The 2n roots are then given by

$P_i = e^{j\frac{i}{n}\pi}$, where i = 0, ..., 1 ...., (2n – 1)

i.e. $P_i = \cos\left(\dfrac{i}{n}\pi\right) + j\sin\left(\dfrac{i}{n}\pi\right)$

In general, whether n is ODD (or) EVEN,

$P_i = e^{j[(2i+n-1)/2n]\pi}$

The Butterworth Polynomial can then be evaluated from the Transfer Formation

$$T(\omega) = \left|H(\omega)\right|^2 = \frac{A_0^2}{B_n^2(\omega)}$$

Finding of Poles of System functions for n=3 Butterworth Polynomial:

In general, the Butterworth Polynomial for n-odd is given by

$$B_n^2(s) = \left[\prod_{i=1}^{(n-1)/2} \left(s^2 + 2\cos\theta_i s + 1\right)\right](s+1)$$

By substituting the value of n=3 in the above equation, we get

$$B_n^2(s) = (s+1)(s^2 + 2\cos\theta_1 s + 1)$$

Or $B_n^2(s) = (s+1)\left(s + e^{j\theta_1}\right)\left(s + e^{-j\theta_1}\right)$ ----------------- (1)

Where $\theta_1 = \dfrac{\pi}{n}$ for n = 3 giving $\theta_1 = \dfrac{\pi}{3}$

By substituting the value of θ_1 in equation (1), we get

$$B_n^2(s) = (s+1)\left[s^2 + 2\cos^{(\pi/3)} s + 1\right]$$

Or $B_n^2(s) = (s+1)(s^2 + s + 1) = s^3 + 2s^2 + 2s + 1$

Therefore, the poles of system function for low-pass filter with n=3 Butterworth characteristics is

$$T(\omega = \left|H(\omega)\right|^2 = \frac{A_0^2}{\left|B_n(s).B_n(-s)\right|}$$

$$= \left[\frac{A_0^2}{(1+2s+2s^2+2s^3).(1-2s+2s^2-2s^3)}\right]$$

$$= B_n(s).B_n(-s)$$

We then have

$$B_n(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Or $B_n(s) = \dfrac{1}{(s+1)\left(s + \dfrac{1}{2} + j\dfrac{\sqrt{3}}{2}\right)\left(s + \dfrac{1}{2} - j\dfrac{\sqrt{3}}{2}\right)}$

Finding of Poles of System functions with = n = 4 using Butterworth Polynomial:

In general, the Butterworth Polynomial (for n = even) is given by

$$B_n^2(s) = \prod_{i=1}^{n/2} \left(s^2 + 2\cos\theta_i s + 1\right)$$

For $\theta_i = \left[\dfrac{2i-1}{2n}.\pi\right]$

Similarly for n = 4, the Butterwork Polynomial is given by

$$B_n(s) = \frac{1}{\left(s + e^{j5/8\pi}\right)\left(s + e^{j7/8\pi}\right)\left(s + e^{j9/8\pi}\right)\left(s + e^{j11/8\pi}\right)}$$

$$B_n(s) = \frac{1}{\left(s^2 + 0.76536s + 1\right)\left(s^2 + 1.84775s + 1\right)}$$

Q.95 Determine the loop currents, I_1, I_2, I_3 and I_4 using mesh (loop) analysis for the network shown in Fig.6. **(8)**

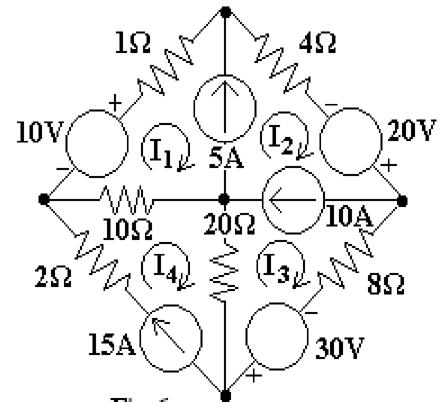


Fig.6

Ans:

The branches AE, DE and BC consists of current sources, shown in Fig.2.1. Here we have to apply supermesh analysis.

The combined supermesh equation is

$$10(I_1 - I_4) + I_1 - 10 + 4I_2 - 20 + 8I_3 - 30 + 20(I_3 - I_4) = 0$$

Or $11I_1 + 4I_2 + 28I_3 - 30I_4 = 60$ ---------------- (1)

In branch AE, by applying KCL, we have

$I_2 - I_1 = 5A$ ----------------- (2)

In branch BC, $I_4 = 15A$ ----------------- (3)

In branch DE, $I_2 - I_3 = 10A$ ----------------- (4)

From equation (2), we have $I_2 = (I_1 + 5)$ ----------------- (5)

From equation (4), we have $I_3 = (I_2 - 10)$ ---------------- (6)

By substituting the values of I_4, I_2 and I_3 from equations (3), (5) and (6) respectively, in equations (1), we get

$$11I_1 + 4I_2 + 28I_3 - 30I_4 = 60$$

$$11I_1 + 4(I_1 + 5) + 28(I_2 - 10) - 30(15) = 60$$

Or $15I_1 + 20 + 28I_2 - 280 - 450 = 60$

Or $\cancel{15I_1} + 28I_2 = 770$ ------------------ (7)

[15 x eqn (2)] $\cancel{-15I_1} + 15I_2 = 75$ ------------------ (8)

(7) + (8) $\longrightarrow$ $43I_2 = 845$

Hence $43I_2 = 845$

Or $I_2 = \frac{845}{43} = 19.65A$

From eqn (2), we have

$I_2 - I_1 = 5$

$19.65 - I_1 = 5$ ($\because$ $I_2 = 19.65A$)

$-I_1 = 5 - 19.65$

$-I_1 = -14.65$

Or $I_1 = 14.65A$

From eqn (6), we have

$I_3 = I_2 - 10$

$I_3 = 19.65 - 10$ ($\because$ $I_2 = 19.65A$)

Or $I_3 = 9.65A$

Hence $I_1 = 14.65A$, $I_2 = 19.65A$, $I_3 = 9.65A$ and $I_4 = 15A$

Q.96 Find the power delivered by the 5A current source (in Fig.7) using nodal analysis. **(8)**

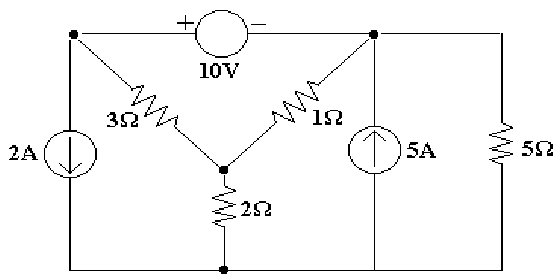


Fig.7

Ans:

Assume the voltages V_1, V_2 and V_3 at nodes 1, 2, and 3 respectively. Here, the 10V source is common between nodes 1 and 2. So, by applying the Supernode technique, the combined equation at node 1 and 2 is

$$\frac{V_1 - V_3}{3} + 2 + \frac{V_2 - V_3}{1} - 5 + \frac{V_2}{5} = 0$$

Or $0.34V_1 + 1.2V_2 - 1.34V_3 = 3$ -------------------- (1)

At node 3,

$$\frac{V_3 - V_1}{3} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$$

Or $-0.34V_1 - V_2 + 1.83V_3 = 0$ -------------------- (2)

Also $V_1 - V_2 = 10$ -------------------- (3)

By multiplying the eqn (1) with 1.366, we have

$0.464V_1 + 1.639V_2 - 1.83V_3 = 4.098$ -------------------- (4)

By solving equations (2) and (4), we have summation of (2) and (4) gives i.e., ((2)+(4) is

$-0.34V_1 - V_2 + 1.83V_3 = 0$

$0.464V_1 + 1.639V_2 - 1.83V_3 = 4.098$

$0.124V_1 + 0.639V_2 = 4.098$ -------------------- (5)

From equation (3), we have

$V_1 - V_2 = 10$

Or $V_1 = V_2 + 10$ -------------------- (6)

By substituting the value of V_1 from eqn (6) in eqn (5), we have

$0.124(V_2 + 10) + 0.639V_2 = 4.098$

Or $0.124V_2 + 1.24 + 0639V_2 = 4.098$

Or $0.763V_2 = 4.098 - 1.24$

Or $0.763V_2 = 2.858$

Or $V_2 = \dfrac{2.858}{0.763} = 3.74V$

From eqn (3), we have

$V_1 - V_2 = 10$

$V_1 = V_2 + 10$

$V_1 = 3.74 + 10$

Or $V_1 = 13.74V$

By substituting the values of V_1 & V_2 in eqn (1), we have

$0.34(13.74) + 1.2(3.74) - 1.34V_3 = 3$

Or $9.165 - 1.34V_3 = 3$

Or $V_3 = 46V$

Hence the Power delivered by the source 5A is

$P_{5A} = V_2 \times 5$

= 3.74 x 5 = 18.7 W

Q.97 The capacitor in the circuit of Fig.8 is initially charged to 200V. Find the transient current after the switch is closed at t=0. **(8)**

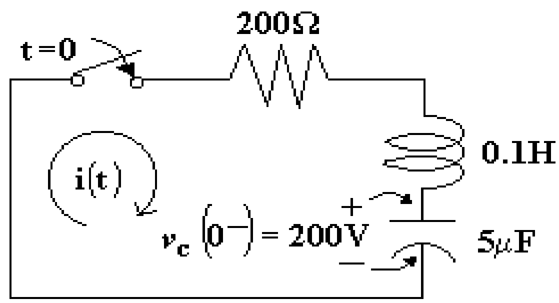


Fig.8

Ans:

The differential equation for the current i(t) is

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{c}\int_0^t i(t)dt + V_c(0^+) = 0$$

And the corresponding Laplace Transformed equation is

$$RI(s) + L\left[SI(s) - i(o^+)\right] + \frac{1}{cs}I(s) + \frac{V_C(o^+)}{s} = 0$$

The given parameters are

$$C = 5\mu F = 5\times10^{-6}F = \frac{1}{5\times10^{-6}S}$$

$L = 0.1H = 0.1S$

$R = 200\Omega = 200$ and

$V_C(o^+) = +200V$ (with the Polarity given)

The initial current $i(o^+) = 0$, because initially inductor behaves as a open circuit. The equivalent Laplace Transformed network representation is shown in Fig.3.2.

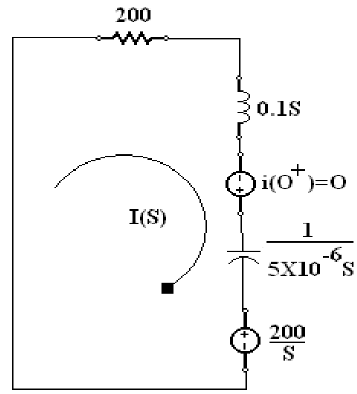


Fig.3.2

The Laplace Transformed equation for the Fig.3.2, then becomes

$$200I(s)+0.1sI(s)+\frac{1}{5\times10^{-6}s}I(s)+\frac{200}{s}=0$$

$$I(s)\left[200+0.1s+\frac{1}{5\times10^{-6}s}\right]=-\frac{200}{s}$$

Or $I(s)\left[200s+0.1s^2+\frac{1}{5\times10^{-6}}\right]=-200$

Or $I(s)\left[0.1s^2+200s+\frac{1}{5\times10^{-6}s}\right]=-200$

Or $I(s)\left[s^2+2000s+2000000\right]=-200$

Or $I(s)=-\left[\frac{200}{s^2+2000s+2000000}\right]$

$$I(s)=-\frac{200}{(s+1000+1000j)(s+1000-1000j)}$$

$$=-200\left[\frac{K_1}{(s+1000+1000j)}+\frac{K_2}{(s+1000-1000j)}\right]$$

$$I(s)=-200\left[\frac{-\frac{1}{2000j}}{(s+1000+1000j)}+\frac{\frac{1}{2000j}}{(s+1000-1000j)}\right]$$

Now $i(t)=L[I(s)]$

$$I(t)=-200\left[-\frac{1}{2000j}e^{-1000t\left[e^{-j1000t}-e^{-j1000t}\right]}\right]$$

Q.98 Determine the r.m.s. value of current, voltage drops across R and L, and power loss when 100 V (r.m.s.), 50 Hz is applied across the series combination of R=6 Ω and $L=\frac{8}{314}$, H. Represent the current and voltages on a phasor diagram. **(8)**

Ans:

The equivalent circuit is shown in Fig.3.3 and the given data is source voltage

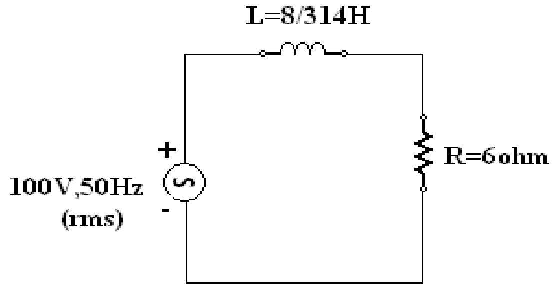


Fig.3.3

$V_c = 100\angle 0^0$ (r.m.s)

Frequency, $f = 50HZ$

Resistance $R = 6\Omega$ and

Inductance $L = \dfrac{8}{314H}$

In Rectangular Form, the total impedance

$Z_T = R + jX_L$

Where $X_L = 2\pi f L$

$= 2\pi \times 50 \times \dfrac{8}{314}$ $\quad [\because\ f = 50HZ \ \&\ L = \dfrac{8}{314}H\ ]$

$X_L = 8\Omega$

Therefore, $Z_T = R + jX_L$

$Z_T = (6 + j8)\Omega$

R.M.S. Value of Current $I = \dfrac{V_S}{Z_T} = \dfrac{100\angle 0^0}{(6 + j8)}$

Conversion of rectangular form of (6 + j8) into Polar form

$R\cos\phi = 6$ ----------------- (1)

$R\sin\phi = 8$ ----------------- (2)

Squaring and adding the above two equations, we get

$R^2 = 6^2 + 8^2$ or $R = \sqrt{6^2 + 8^2}$ or $R = \sqrt{36 + 64} = 10$

From equations (1) & (2), $\tan\phi = \dfrac{8}{6}$ or

$\phi = \tan^{-1}\left(\dfrac{8}{6}\right) = 53.13^0$

Hence the polar form is $10\angle 53.13^0$

Therefore, the current $I = \dfrac{V_S}{V_T} = \dfrac{100\angle 0^0}{10\angle 53.13^0}$

Or $I = 10\angle -53.13^0$

Hence the phase angle between voltage and current is

$\theta = 53.13^0$

The r.m.s. voltage across the resistance is

$V_{r.m.s.}(V_R) = I.R$

$= 10 \times 6 = 60V$

$V_{r.m.s.}(V_R) = 60V$

The r.m.s. voltage across the inductive reactance is

$V_L = I.X_L$

$= 10 \times 8 = 80V$

The phase diagram for the problem is shown in Fig.3.4.

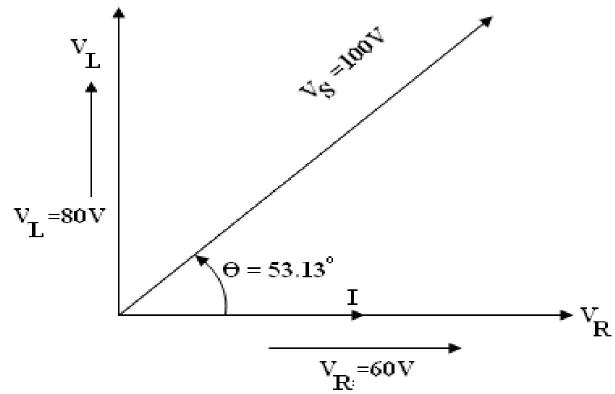


Fig.3.4

Q.99 Using Kirchhoff's laws to the network shown in Fig.9, determine the values of v_6 and i_5. Verify that the network satisfies Tellegen's theorem. **(8)**

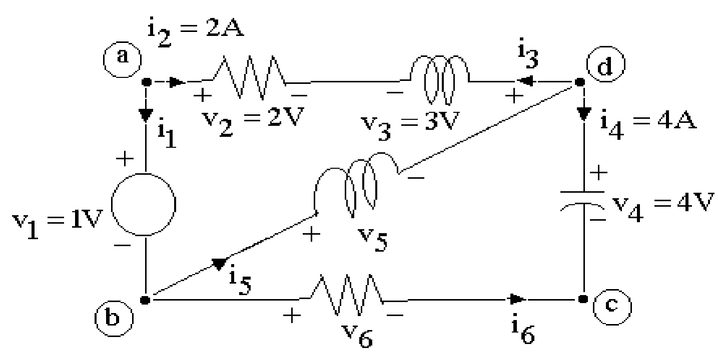


Fig.9

Ans:

For the loop b.a.d, by applying KVL, we have

$V_5 = -V_1 + V_2 - V_3$

$V_5 = -1 + 2 - 3 = -4 + 2 = -2V$

$\therefore\ V_5 = -2V$

For the loop bdc, by applying KVL, we have

$V_6 = V_5 + V_4$

$= -2 + 4 = 2V$

$\therefore\ V_6 = 2V$

By applying KCL at node (a), we have

$i_1 = -i_2 = -2A$ &

$i_3 = -i_2 = -2A$

Now, by applying KCL at node (c), we have

$i_6 = -i_4 = -4A$

$\therefore\ i_6 = -4A$

Therefore, by applying KCL at node (b), we get

$i_5 = i_1 - i_6$

$= -2-(-4) = -2+4 = 2A$

$\therefore \quad i_5 = 2A$

Hence the voltage $V_6 = 2v$ and the current $i_5 = 2A$

Also KCL at node (d) becomes

$i_5 = i_3 + i_4$

= -2 + 4 = 2A

Now, applying Tellegen's Theorem for voltages & currents, then

$$\sum_{K=1}^{6} = V_K i_K = (V_1 i_2) + (V_2 i_2) + (V_3 i_3) + (V_4 i_4) + (V_5 i_5) + (V_5 i_6)$$

= (1x-2) + (2x2) +(3x-2) + (4x4) + (-2x2) + (2x-4)

= 0

$$\sum_{K=1}^{6} = 0$$ Hence Proved.

Q.100 State Reciprocity Theorem for a linear, bilateral, passive network. Verify reciprocity for the network shown in Fig.10. **(8)**

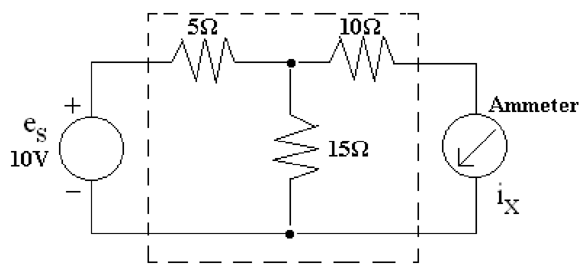


Fig.10

Ans:

This theorem states that in any linear network containing bilateral linear impedances and generators, the ratio of voltage V introduced in one mesh to the current I in any second mesh is the same as the ratio obtained if the positions of V and I are interchanged, other emf being removed.

Verification of Reciprocity Theorem for the network shown in Fig.4.2.:

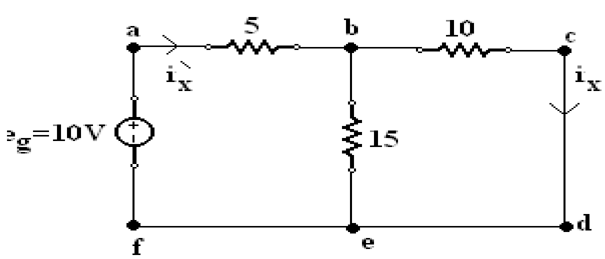


Fig.4.2

The given network can be drawn by short circuiting the Ammeter is shown in Fig.4.3.

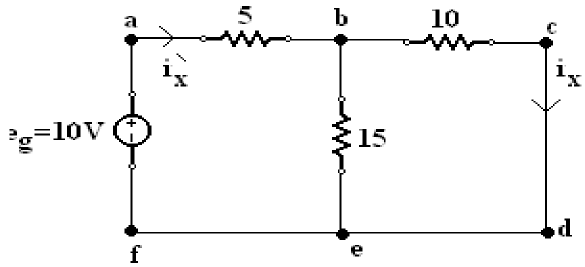


Fig.4.3

Suppose that a voltage source of 10v in branch of causes a current i_x' given out by 10v source is

$$i_x' = \frac{10}{5+(10\|15)} = \frac{10}{5+6} = \frac{10}{11} = 0.909A$$

By current division

$$i_x = i_x'.\frac{15}{15+10} = 0.909 \times \frac{15}{25} = 0.5454A$$

Therefore, the current i_x when the voltage source of 10v is placed between *af* is

$$i_x = 0.5454A$$

Now, the voltage source of 10v is removed from branch *af* and connected in the branch *cd* as shown in Fig.4.4.

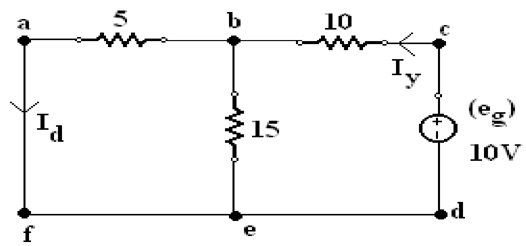


Fig.4.4

Let the current I_j flow in branch *af* due to the source 10v acting in the branch *cd*. Now, we have to find the value of I_j.

From Fig.4.4, we observe that 5Ω is in parallel with 15Ω resistance. Their equivalent resistance is $\frac{5\times5}{5+15} = \frac{75}{20} = 3.75\Omega$. Therefore, the current given out by the 10v source is

$$I_y = \frac{10}{10+(5\|15)} = \frac{10}{10+3.75} = 0.7272A$$

$$\therefore \quad I_y = 0.7272A$$

Hence the current $I_j = I_y \times \frac{15}{15+5} = 0.7272 \times \frac{15}{20} = 0.5454A$

Therefore $I_j = 0.5454A$

So the Reciprocity Theorem is verified for

$$i_x = i_y = 0.5454A$$

Q.101 Find

(i) the r.m.s. value of the square-wave shown in Fig.11.

(ii) the average power for the circuit having $z_{in} = 1.05 - j0.67, \Omega$ when the driving current is $40 - j3$, A. **(8)**

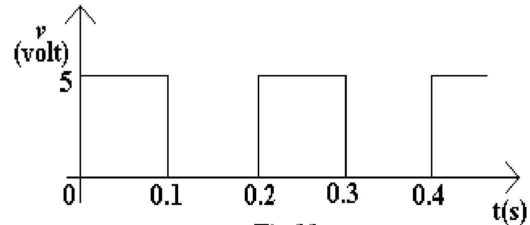


Fig.11

Ans:

(i) The square wave for the Fig.11 is given by

V = 5 for 0 < t < 0.1

= 0 for 0.1 < t < 0.2 and

The period is 0.2 seconds

The r.m.s. value of the square wave shown in Fig.5.1 is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 dt}$$

$$= \left[\sqrt{\frac{1}{T}\int_0^{0.1} 5^2 dt + \int_1^{0.2} (0)^2 dt}\right]$$

$$= \sqrt{\frac{1}{0.2}.25\int_0^{0.1} tdt} = \sqrt{\frac{25}{0.2}\left[\frac{t^2}{2}\right]_0^{0.1}}$$

$$= \sqrt{\frac{25}{0.2}\frac{(0.1)^2}{2}} = \sqrt{0.625} = 0.7906$$

Hence $V_{rms} = 0.796V$

(ii) <u>Finding of Average Power:</u>

Given data

$Z_{in} = 1.05 - j0.67\Omega$ &

The driving current $I = 40 - j3A$

The Average Power in the circuit is the Power dissipated in the resistive part only i.e.,

$$P_{average} = \frac{I_m^2}{2}.R$$

Where $I_m = 40$ and

$R = 1.05\Omega$

Therefore, $P_{av} = \frac{(40)^2}{2}.1.05 = 840W$

$\therefore \quad P_{av} = 840W$

Q.102 The voltage across an impedance is 80+j60 Volt, and the current though it is 3+j4 Amp. Determine the impedance and identify its element values, assuming frequency to be 50Hz. From the phasor diagram, identify the lag or lead of current w.r.t. voltage. **(8)**

Ans:

Given that the voltage across an impedance V_Z is $V_Z = 80 + j60$ volts and the current through the impdance I_Z is

$I_Z = 3 + j4$ Amp and

Frequency (f) = 50 Hz and the

Equivalent circuit shown in Fig.5.2

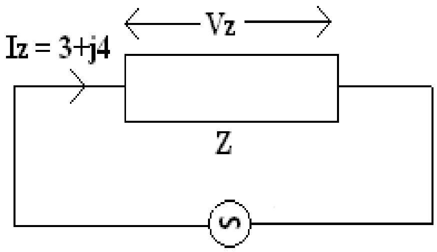


Fig.5.2

The impedance (Z) for the circuit of Fig.5.2 is

$$Z = \frac{V_Z}{I_Z} = \frac{80 + j60}{3 + j4}$$

Converting of 80+j60 into Polar Form	Converting of 3+j4 into Polar Form
$R\cos\phi = 80$ ------------------ (1) $R\sin\phi = 60$ ----------------- (2) Squaring & adding the above equations, we get $R^2 = 80^2 + 60^2$ or $R = \sqrt{80^2 + 60^2} = \sqrt{6400 + 3600}$ $= \sqrt{10{,}000} = 100$ From eqn (1) & (2), $Tan\phi = \frac{60}{80}$ or $\phi = Tan^{-1}\left(\frac{60}{80}\right) = Tan^{-1}(0.75)$ Or $\phi = 36.869^0$ or $\phi = 36.87^0$ Now the Polar form is $100\angle 36.87^0$	$R\cos\phi = 3$ --------------------- (1) $R\sin\phi = 4$ --------------------- (2) Squaring & adding the above equations, we get $R^2 = 3^2 + 4^2$ or $R = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ From eqn (1) & (2) $Tan\phi = \frac{4}{3}$ or $\phi = Tan^{-1}\left(\frac{4}{3}\right)$ or $\phi = 53.13^0$ Now the Polar form is $5\angle 53.13^0$

Therefore the impedance (Z) becomes

$$Z = \frac{V_Z}{I_Z} = \frac{80 + j60}{3 + j4} = \frac{100\angle 36.87^0}{5\angle 53.13^0} = 20\angle 36.87^0 - 53.13^0$$

Or the impedance $Z = 20\angle -16.26^0$

Converting of $20\angle -16.26^0$ into Rectangular Form

$$a + jb = R(\cos\phi + j\sin\phi)$$

Where $R = 20$ & $\underline{|\phi} = -16.26^0$

$R\cos\phi = 20\cos(-16.26^0) = 0.96 \times 20 = 19.2$

$R\sin\phi = 20\sin(-16.26^0) = 20 \times -0.2799$

$= -5.598$

$a + jb = 19.2 - j5.598$

Therefore $Z = 20\underline{|-16.26^0} = 19.2 - j5.598$

Hence the given circuit has a resistance of 19.2Ω in series with capacitive reactance of 5.598. The phase angle between the voltage and current is $\theta = 16.26^0$. Here, the current leads the voltage by 16.26^0. The resultant phasor diagram is shown in Fig.5.3.

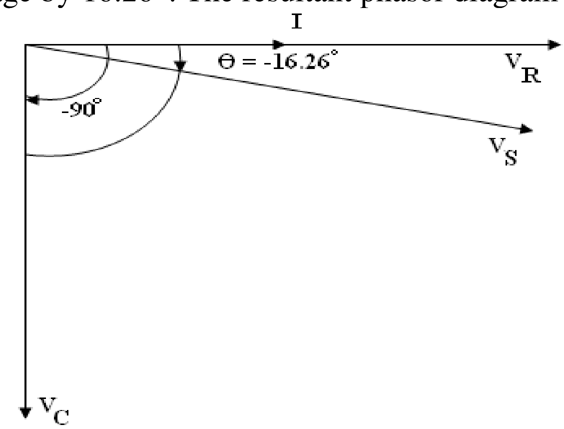


Fig.5.3

Q.103 Consider the function $F(s) = \dfrac{s^2 + 1.03}{s^2 + 1.23}$. Plot its poles and zeroes. Sketch the amplitude and phase for F(s) for $1 \le \omega \le 10$. **(8)**

Ans:

The given function F(s) is

$F(s) = \dfrac{s^2 + 1.03}{s^2 + 1.23}$ and F(s) can be factorised as

$$F(s) = \frac{s^2 + 1.03}{s^2 + 1.23} = \frac{s^2 + (1.05)^2}{s^2 + (1.09)^2} = \frac{(s + j1.015)(s - j1.015)}{(s + j1.109)(s - j1.109)} \quad \text{-----------------} \ (1)$$

For the steady-state, s = jω

Hence, $$F(j\omega) = \frac{(j\omega + j1.015)(j\omega - j1.015)}{(j\omega + j1.109)(j\omega - j1.109)} \quad \text{-----------------} \ (2)$$

F(s) has two complex conjugate zeros, a_1 and a_1^* at $s = \pm 1.015$ and are shown in Fig.6.1. Likewise, F(s) has two complex conjugate poles b_1 and b_1^* at $s = \pm 1.109$ as shown in Fig.6.1.

At ω = 1.015, the phasors from zero a_1 to the frequency ω = 1.015 is of zero magnitude, as is evident from equation (2) and from Fig.6.1.

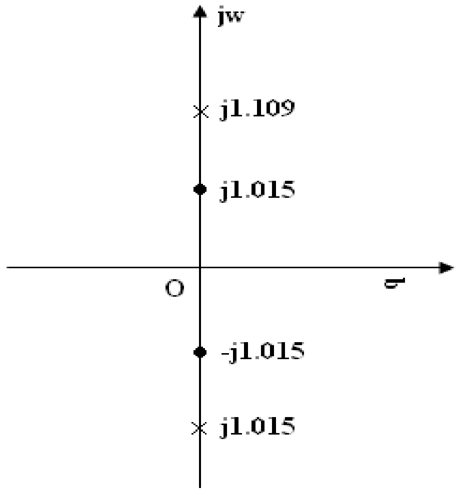


Fig.6.1.

Therefore, at zero on jω-axis, the amplitude response is zero.

At ω = 1.109, phasor from the pole b_1 to the frequency ω = 1.109 is of zero magnitude, as a result of which amplitude response is infinite at pole b_1.

When ω < 1.015, it is apparent from the pole-zero diagram that the phase is zero. However, when ω < 1.015 and ω < 1.109, the phasor from the zero at ω = 1.015 will point upward while the phasor from the other poles and zeros are oriented in the same direction as far as ω<1.105. Thus, Amplitude Response is shown in fig.6.2.

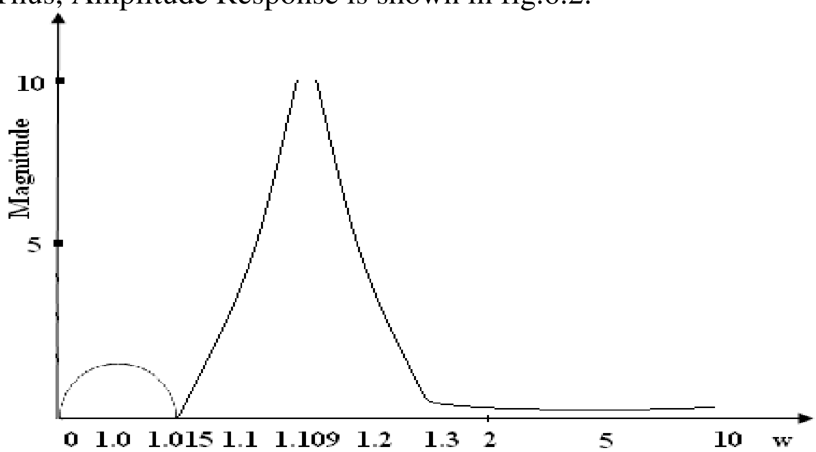


fig.6.2.

Phase Response:

We see that a zero on the jω-axis, the phase response has a step discontinuity of $+180^0$ for increasing frequency. Likewise at a pole on the jω-axis, the phase response is discontinuous by -180^0 as shown in Fig.6.3.

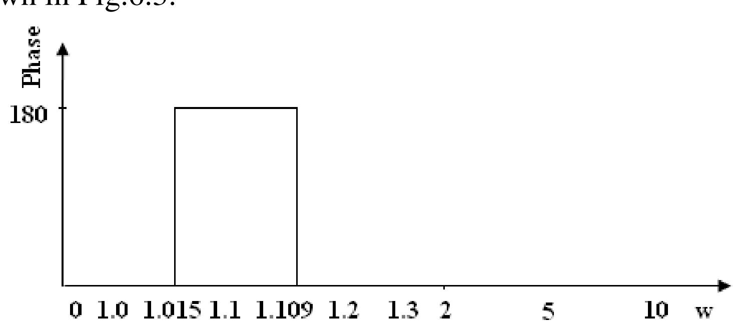


Fig.6.3.

Amplitude and Phase Response Plot for the given network function for the frequency range $10 \le \omega \le 10$ is given in Fig.6.2 & fig.6.3 respectively.

Q.104 Determine whether the function $F(s) = \dfrac{s^3 + 2s^2 + 3s + 1}{s^3 + s^2 + 2s + 1}$ is positive real or not. **(8)**

Ans:

The given function F(s) is

$$F(s) = \frac{s^3 + 2s^2 + 3s + 1}{s^3 + s^2 + 2s + 1}$$

Let $F(s) = \dfrac{P(s)}{Q(s)} = \dfrac{s^3 + 2s^2 + 3s + 1}{s^3 + s^2 + 2s + 1}$

<u>Condition (1):</u>

For F(s) to be Positive Real, P(s) and Q(s) should be Hurwitz Polynomials.

First, checking of whether Q(s) = $s^3 + s^2 + 2s + 1$ is Hurwitz or not we have

$$\varphi(s) = \frac{n(s)}{m(s)} = \frac{s^3 + 2s}{s^2 + 1}$$

$$\begin{array}{l}
s^2 + 1 \,\big)\, s^3 + 2s \,\big(\, s \\
\qquad \underline{-s^3 \pm s} \\
\qquad 0 \qquad s \,\big)\, s^2 + 1 \,\big(\, s \\
\qquad\qquad\quad \underline{-s^2} \\
\qquad\qquad\qquad 1 \,\big)\, s \,\big(\, s \\
\qquad\qquad\qquad\quad \underline{-s} \\
\qquad\qquad\qquad\quad 0
\end{array}$$

Hence $\alpha_1 = 1$, $\alpha_2 = 1$ & $\alpha_3 = 1$ are all positive and real. Therefore, Q(s) is Hurwitz.

Next, checking of whether $P(s) = s^3 + 2s^2 + 3s + 1$ is Hurwitz or not, we have

$$\varphi(s) = \frac{n(s)}{m(s)} = \frac{s^3 + 3s}{2s^2 + 1}$$

$$\begin{array}{l}
2s^2 + 1 \,\big)\, s^3 + 3s \,\big(\, \tfrac{1}{2}s \\
\qquad\quad \underline{-s^3 - \tfrac{1}{2}s} \\
\qquad\qquad \tfrac{5}{2}s \,\big)\, 2s^2 + 1 \,\big(\, \tfrac{4}{5}s \\
\qquad\qquad\qquad \underline{-2s^2} \\
\qquad\qquad\qquad\quad 1 \,\big)\, \tfrac{5}{2}s \,\big(\, \tfrac{5}{2}s \\
\qquad\qquad\qquad\qquad \underline{-\tfrac{5}{2}s} \\
\qquad\qquad\qquad\qquad 0
\end{array}$$

Hence $\alpha_1 = \dfrac{1}{2}$, $\alpha_2 = \dfrac{4}{5}$, and $\alpha_3 = \dfrac{5}{2}$ are all Positive and Rea. Therefore, P(s) is Hurwitz Polynomial.

<u>Condition (2):</u>

Since F(s) does not have poles on the jω-axis, then the function F(s) is Positive Real Funciton.

<u>Condition (3):</u>

The third condition requires that $M_1M_2 - N_1N_2\big|_{s-j\omega} \geq 0$ for all ω.

For $P(s) \to s^3 + 2s^2 + 3s + 1$

Where $M_1 = 2s^2 + 1$ & $N_1 = s^3 + 3s$

For $Q(s) \to s^3 + s^2 + 2s + 1$

Where $M_2 = s^2 + 1$ & $N_2 = s^3 + 2s$

$$A(\omega^2) = M_1M_2 - N_1N_2 = \left(2s^2+1\right)\left(s^2+1\right) - \left(s^3+3s\right)\left(s^3+2s\right)\Big|_{s=j\omega}$$

$$= 2s^4 + 2s^2 + s^2 + 1 - s^6 + 2s^4 + 3s^4 + 6s^2\Big|_{s=j\omega}$$

$$= 2s^4 + 3s^2 + 1 - s^6 + 5s^4 + 6s^2\Big|_{s=j\omega}$$

$$= 2(j\omega)^4 + 3(j\omega)^2 + 1 - (j\omega)^6 + 5(j\omega)^4 + 6(j\omega)^2$$

$$= 2\omega^4 - 3\omega^2 + 1 + \omega^6 + 5\omega^4 - 6\omega^2$$

$$A(\omega^2) = 2\omega^4 - 9\omega^2 + 1 + \omega^6 + 5\omega^4$$

For all the roots of $A(\omega^2)$, from $0 \to \alpha$, $A(\omega^2) \geq 0$, so that the function F(s) is Positive Real Function.

Q.105 Given the Z parameters of a two-port network, determine its Y parameters. **(8)**

Ans:

The Z-parameters of a two-port network are given by

$V_1 = Z_{11}I_1 + Z_{12}I_2$ ------------- (1) and

$V_2 = Z_{21}I_1 + Z_{22}I_2$ ------------- (2)

Where Z_{11}, Z_{12}, Z_{21}, Z_{22} are called Z-Parameters or impedance (z) parameters.

These parameters can be represented by matrix form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{------------- (3)}$$

From equation (3), the current I_1 and I_2 in matrix form as

$$I_1 = \begin{bmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{bmatrix} \Big/ \Delta_Z \quad \text{------------- (4)}$$

$$I_2 = \begin{bmatrix} Z_{11} & V_1 \\ Z_{22} & V_2 \end{bmatrix} \Big/ \Delta_Z \quad \text{------------- (5)}$$

Where Δ_Z is the determinant of Z matrix given by

$$\Delta_Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \quad \text{------------- (6)}$$

From equations (4) and (5), we can write

$$I_1 = \frac{Z_{22}}{\Delta_Z}V_1 - \frac{Z_{12}}{\Delta_Z}V_2 \quad \text{------------- (7)}$$

$$I_2 = \frac{-Z_{21}}{\Delta_Z}V_1 + \frac{Z_{11}}{\Delta_Z}V_2 \quad \text{------------- (8)}$$

The Y-parameters of a two-port network are given as

$I_1 = Y_{11}V_1 + Y_{12}V_2$ ------------- (9)

$I_2 = Y_{21}V_1 + Y_{22}V_2$ ------------- (10)

Comparing equation (7) with equation (9), we have

$Y_{11} = \dfrac{Z_{22}}{\Delta_Z}$; $Y_{12} = \dfrac{-Z_{12}}{\Delta_Z}$ and

Comparing equation (8) with equation (10), we have

$Y_{21} = \dfrac{-Z_{21}}{\Delta_Z}$ and $Y_{22} = \dfrac{Z_{11}}{\Delta_Z}$

Q.106 Find the y-parameters for the two-port network of Fig.12. **(8)**

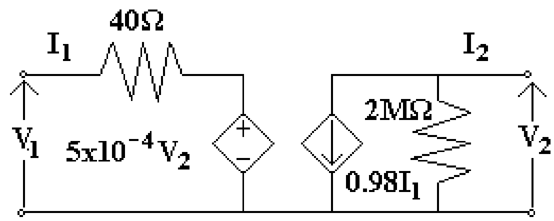


Fig.12

Ans:

The h-mode ac equivalent circuit of transistor amplifier is shown in Fig.7.2.

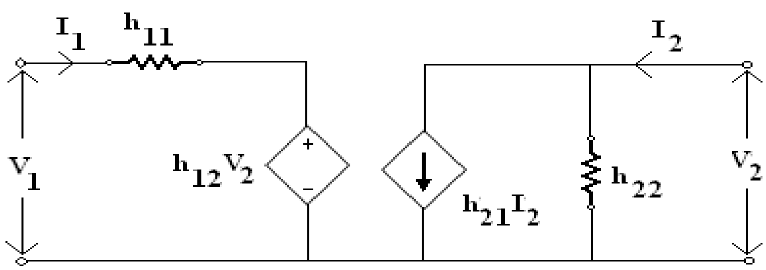


Fig.7.2.

By comparing the given two port network shown in Fig.12 with the h-mode a-c equivalent circuit of transistor amplifier is shown in Fig.7.2, we have

$h_{11} = 40\Omega$

$h_{12} = 5\times10^{-4}$

$h_{21} = 0.98$ and

$h_{22} = 2M\Omega$

Now, the Y-parameters can be written in terms of h-parameters as

$$Y_{11} = \frac{1}{h_{11}} = \frac{1}{40} = 0.025\,\text{mho}$$

$$Y_{12} = -\frac{h_{12}}{h_{11}} = -\frac{5\times10^{-4}}{40} = -0.125\times10^{-4}$$

$$Y_{21} = \frac{h_{21}}{h_{11}} = \frac{0.98}{40} = 0.025$$

$$Y_{22} = \frac{\Delta h}{h_{11}} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} = \frac{40\times2\times10^{6} - 5\times10^{-4}0.98}{40}$$

$$\frac{80\times10^{6}-4.9\times10^{-4}}{40}=\frac{8\times10^{6}-0.00049}{40}$$

$$Y_{22}=2\times10^{6}\Omega$$

Q.107 Synthesise a one-port L-C network whose driving-point impedance is $Z(s)=\dfrac{6s^3+2s}{12s^4+8s^2+1}$ **(8)**

Ans:

Synthesizing the given driving point impedance function Z(s) by CAUER-2 Network. First reorient the function as shown below to get the CAUER-2 network i.e.,

$$Z(s)=\frac{2s+6s^3}{1+8s^2+12s^4}$$

Since $Z(s)\to 0$ with zero, the first element C_1 is absent and with $s\to\alpha$, $Z(s)\to 0$; then the last element is a capacitor.

CAUER-2 Network is obtained by continued fraction method on inverting and dividing as shown below:-

$$2s+6s^3\,\big)\,1+8s^2+12s^4\,\big(\,\frac{1}{2s}$$
$$\underline{1+3s^2}$$
$$5s^2+12s^4\,\big)\,2s+6s^3\,\big(\,\frac{2}{5s}$$
$$\underline{2s+\frac{24}{5}s^3}$$
$$\frac{6}{5}s^3\,\big)\,5s^2+12s^4\,\big(\,\frac{25}{6s}$$
$$\underline{5s^2}$$
$$12s^4\,\big)\,\frac{6}{5}s^3\,\big(\,\frac{1}{10s}$$
$$\underline{\frac{6}{5}s^3}$$
$$0$$

Therefore $$Z(s)=\cfrac{1}{Y_2(s)+\cfrac{1}{Z_3(s)+\cfrac{1}{Y_4(s)+\cfrac{1}{Z_5(s)}}}}$$

Or $$Z(s)=\cfrac{1}{L_2(s)+\cfrac{1}{C_3(s)+\cfrac{1}{L_4(s)+\cfrac{1}{C_5(s)}}}}$$

Hence, the first element is absent i.e., $C_1=0$ and the first element would now be shunt inductor.

Therefore,

$$Y_2(s)=\frac{1}{2s} \qquad \text{giving} \qquad L_2=2H$$

$Z_3(s) = \frac{2}{5s}$ giving $C_3 = \frac{5}{2}F$

$Y_{34}(s) = \frac{25}{6s}$ giving $L_4 = \frac{6}{25}H$

$Z_5(s) = \frac{1}{10s}$ giving $C_5 = 10F$

Hence the synthesized CAUER-2 network for the driving-point impedance function is given in Fig.8.1.

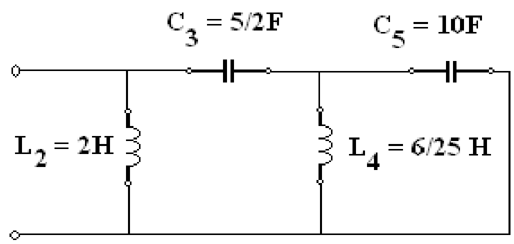


Fig.8.1

Q.108 Determine the condition for a lattice terminated in R as shown in Fig.13 to be a constant-resistance network. **(8)**

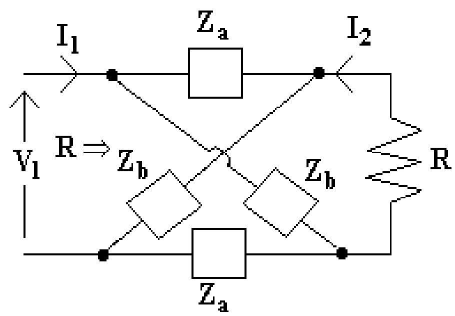


Fig.13

Ans:

The given lattice network is redrawn as a bridge network by removing the terminated resistance R as shown in Fig.8.3

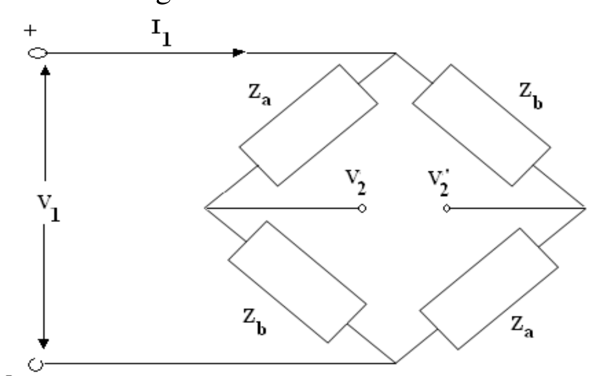


Fig.8.3

In order to find the condition for Constant-Resistance Network for the Fig.8.3, first determine the Z-parameters (Z_{11} & Z_{21}) for the network shown in Fig.8.3

We know that $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

When $I_2 = 0$; $V_1 = I_2\left[(Z_a + Z_b)\|(Z_a + Z_b)\right]$

Or $V = I_1 \dfrac{(Z_a + Z_b)(Z_a + Z_b)}{Z_a + Z_b + Z_a + Z_b}$ ------------------------ (1)

Therefore $Z_{11} = \dfrac{V}{I_1} = \dfrac{(Z_a + Z_b)(Z_a + Z_b)}{Z_a + Z_b + Z_a + Z_b}$

Or $Z_{11} = \dfrac{Z_a + Z_b}{2}$ ------------------------- (2)

Next find

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

When $I_2 = 0$, V_2 is the voltage across 2 – 2'

$$V_2 = V_1\left[\frac{Z_b}{Z_a + Z_b} - \frac{Z_a}{Z_a + Z_b}\right] \qquad \text{------------------------ (3)}$$

By substituting the value of V_1 from eqn (1), in eqn (3), we get

$$V_2 = \left[\frac{I_1(Z_a + Z_b)(Z_a + Z_b)}{Z_a + Z_b + Z_a + Z_b}\right]\left[\frac{Z_b(Z_a + Z_b) - Z_a(Z_a + Z_b)}{(Z_a + Z_b)(Z_a + Z_b)}\right]$$

Or $\dfrac{V_2}{I_1} = \dfrac{Z_b(Z_a + Z_b) - Z_a(Z_a + Z_b)}{Z_a + Z_b + Z_a + Z_b} = \dfrac{Z_a + Z_b(Z_b - Z_a)}{Z_a + Z_b(1+1)}$

Or $Z_{21} = \dfrac{V_2}{I_1} = \dfrac{Z_b - Z_a}{2}$ ----------------------- (4)

Now the Input Impedance (Z_{in}) in terms of the Z-Parameters is given by

$$Z_{in} = \frac{V_1}{I_1} = \left[Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_L}\right]$$

Or $Z_{in} = \dfrac{(Z_{11}Z_{22} - Z_{12}Z_{21}) + Z_{11}Z_L}{Z_{22}Z_L}$ ----------------------- (5)

From the given Fig.8.2, the load impedance is equivalent to 'R' or $Z_L = R$

By substituting the valued of Z_L in equation (5), we get

$$Z_{in} = \frac{(Z_{11}Z_{22} - Z_{12}Z_{21}) + Z_{11}Z_R}{Z_{22}Z_R} \qquad \text{----------------------- (6)}$$

For a symmetrical Two-Port network, the impedance $Z_{22} = Z_{11}$ and for a reciprocal two-port network, the impedance $Z_{12} = Z_{21}$, then we have

$$Z_{in} = \frac{Z_{11}R + Z_{11}{}^2 + Z_{21}{}^2}{Z_{11} + R} \qquad \text{----------------------- (7)}$$

For the given constant-resistance Lattice network for the Fig.8.2, the input impedance is equal to R i.e., $Z_{in} = R$

By substituting the value of Z_{in} in equation (7), we have

$$R = \frac{Z_{11}R + Z_{11}{}^2 + Z_{21}{}^2}{Z_{11} + R} \qquad \text{----------------------- (8)}$$

In order for $Z_{in} = R$, the following condition must be hold as

$Z_{11}{}^2 - Z_{21}{}^2 = R^2$ from eqn (8) ----------------------- (9)

For the lattice network, it is found from equation (2)

$\left(Z_{11} = \dfrac{Z_a + Z_b}{2}\right)$ and $\left(Z_{21} = \dfrac{Z_b - Z_a}{2}\right)$ from equation (4).

By substituting the values of Z_{11} and Z_{21} in equation (9), we have

$$\left(\frac{Z_a + Z_b}{2}\right)^2 - \left(\frac{Z_a - Z_b}{2}\right)^2 = R^2$$

Or $\dfrac{1}{4}\left[(Z_a + Z_b)^2 - (Z_a + Z_b)^2\right] = R^2$

Or $\dfrac{1}{4}\left[Z_a{}^2 + Z_b{}^2 + 2Z_aZ_b - Z_a{}^2 - Z_b{}^2 + 2Z_aZ_b\right] = R^2$

$\dfrac{1}{4}\left[4Z_aZ_b\right] = R^2$

Or $Z_aZ_b = R^2$ --------------------- (10)

In order for a Lattice network of Fig.13 to be a constant-resistance network the equation (10) must be hold.

Q.109 Find the y-parameters of the circuit of Fig.14 in terms of s. Identify the poles of $y_{ij}(s)$. Verify whether the residues of poles satisfy the general property of L-C two-port networks. **(8)**

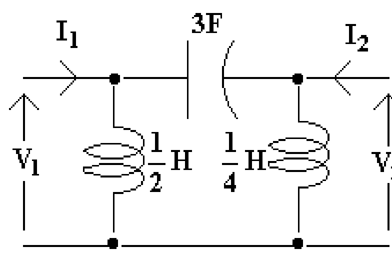


Fig.14

Ans:

The Laplace Transformed of the given circuit is shown in Fig.9.2.

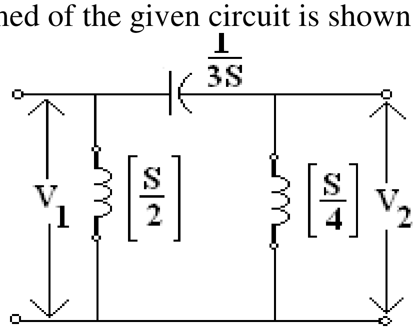


Fig.9.2

The Y-Parameters for the equivqalent Π network for the Fig.9.2 is

$$Y_{11} = (Y_A + Y_B) = \left(\frac{s}{2} + \frac{1}{3s}\right) = \left(\frac{3s^2 + 4}{6s}\right)$$

$$Y_{21} = -Y_B = -\frac{1}{3s}$$

$$Y_{12} = -Y_B = -\frac{1}{3s}$$

$$Y_{22} = (Y_B + Y_C) = \left(\frac{s}{4} + \frac{1}{3s}\right) = \left(\frac{3s^2+4}{12s}\right)$$

Therefore $Y_{11} = \dfrac{3s^2+2}{6s}$, $Y_{12} = Y_{21} = -\dfrac{1}{3s}$ & $Y_{22} = \dfrac{3s^2+4}{12s}$

Finding of $Y_{ij}(s)$:-

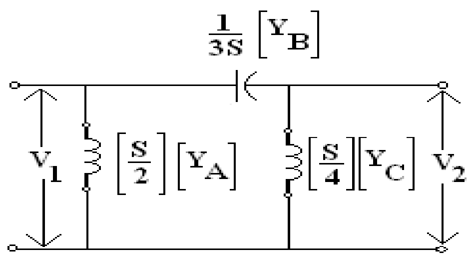


Fig.9.3

The Admittance is

$$Y_{ij}(s) = \frac{1}{Z_{ij}(s)}$$

Now to find the impedance for the network of Fig.9.3 i.e.

$$Z_{ij}(s) = \frac{V_1(s)}{I_1(s)} = \left[\frac{s}{2} \Big\| \left(\frac{1}{3s} + \frac{s}{4}\right)\right]$$

$$= \left[\frac{s}{2} \Big\| \left(\frac{4+3s^2}{12s}\right)\right]$$

$$= \left[\frac{\frac{s}{2} \times \left(\frac{4+3s^2}{12s}\right)}{\frac{s}{2} + \frac{4+3s^2}{12s}}\right] = \left[\frac{\frac{4s}{24s} + \frac{3s^3}{24s}}{\frac{6s^2+4+3s^2}{12s}}\right]$$

Hence

$$Z_{ij}(s) = \left(\frac{4s+3s^3}{24s}\right)\left(\frac{12s}{9s^2+4}\right) = \frac{3s^3+4s}{2(9s^2+4)}$$

Or $Z_{ij}(s) = \dfrac{3s^3+4s}{18s^2+8}$

Therefore $Y_{ij}(s) = \dfrac{1}{Z_{ij}(s)} = \dfrac{18s^2+8}{3s^3+4s} = \dfrac{2s^2+1}{s(3s^2+4)}$

Hence $Y_{ij}(s) = \dfrac{2s^2+1}{s(3s^2+4)}$

Verification of residues of Poles satisfying the general property of L-C two-port network:-

$$Y_{ij}(s) = \frac{2s^2+1}{s(3s^2+4)}$$

(1) Y_{ij}(s) is the ratio of even to odd polynomial

i.e. $$\frac{2s^2+1\ (even)}{s\left(3s^2+4\right)\ (odd)}$$

(2) The poles & zerosof $Y_{ij}(s)$ are simple & lie on the jω axis only

(3) The poles and zeros alternate on the jω axis

i.e. s = 0 ⟶ Pole

$s=\pm\frac{1}{2}=\pm 0.5$ ⟶ Zero

$s=\pm\frac{4}{3}=\pm 1.33$ ⟶ Pole

They alternate with one another

(4) The highest power of numerator i.e., $2s^2$ & denominator i.e. ($3s^3$) is differ by unit; and the lowest power of numerator i.e.e, 1 and the lowest power of denominator i.e. s is differ by unity.

(5) There is a pole at zero & pole at infinity.

Therefore, the above (5) conditions are satisfied. Hence $Y_{ij}(s)$ is L-C Admittance function.

Q.110 A third-order Butterworth polynomial approximation is desired for designing a low-pass filter. Determine H(s) and plot its poles. Assume unity d-c gain constant. **(8)**

Ans:

The general form of cascaded Low Pass Filter Transfer Function

$|H(\omega)|^2=\frac{A_0{}^2}{1+f(\omega)^2}$ if we select $f(\omega)^2$ as $\left(\frac{\omega^2}{\omega_0{}^2}\right)^n$, the cascaded Transfer Function takes takes the form of

$$|H(\omega)|^2=\frac{A_0{}^2}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}=\frac{A_0{}^2}{B_n{}^2(\omega)}$$

Where $B_n{}^2(\omega)=1+\left(\frac{\omega}{\omega_0}\right)^{2n}$ is called the Butterworth Polynomial, n being a Positive integer indicating the order of the filter.

Therefore, the Butterworth Polynomial is given by

$$B_n{}^2(\omega)=1+\left(\frac{\omega}{\omega_0}\right)^{2n}$$

Normalising for $\omega_0=1$ rad/sec,

$B_n^2(\omega)=|B_n(j\omega)|^2=[B_n(s)B_n(-s)]$ are obtained by solving $1+(-1)^n s^{2n}=0$

<u>When n is odd</u>:- (i.e., n = 3)

The equation $1+(-1)^n s^{2n}=0$ reduces to $s^{2n}=1$, which can be further written as

$s^{2n}=1=e^{j2i\pi}$

The 2n roots are then given by

$P_i = e^{j\frac{i}{n}\pi}$, i = 0, ...1, ... (2n – 1)

i.e., $P_i = \cos\left(\frac{i}{n}\pi\right) + j\sin\left(\frac{i}{n}\pi\right)$

In general, P_i can be written as [if n is odd or even]

$P_i = e^{j[(2i+n-1)/2n]\pi}$

The Butterworth Polynomial can then be evaluated from the Transfer Formation as

$$T(\omega) = |H(\omega)|^2 = \frac{A_0^2}{B_n^2(\omega)} \qquad \text{-------------------- (1)}$$

The denominator Polynomial can have factored form of representation as follows :-

$B_n^2(s) = (s+1)(s+e^{j\theta_1})(s-e^{j\theta_1})$ for n = 3

$\quad = (s+1)(s^2 + 2\cos\theta_1 s + 1)$

Where $\theta_1 = \frac{\pi}{n}$ for n = 3 giving $\theta_1 = \frac{\pi}{3}$.

$B_n^2(s) = (s+1)(s^2 + 2\cos\frac{\pi}{3} + 1)$

$B_n^2(s) = (s+1)(s^2 + s + 1) = s^3 + 2s^2 + 2s + 1$ -------------------- (2)

By substituting the value of $B_n^2(s)$ from eqn (2) in eqn (1), we get

$$T(\omega) = |H(\omega)|^2 = \frac{A_0^2}{s^3 + 2s^2 + 2s + 1}$$

Factoring $|H(\omega)|^2 = \frac{1}{1+2s+2s^2+s^3} \cdot \frac{1}{1-2s+2s^2-s^3}$ [$\because A_0 = 1$, since d-c gain constant is unity]

= H(s) . H(-s)

We then have

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$H(s) = \frac{1}{(s+1)\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}$$

The Pole diagram of H(s) for n = 3 Butterworth filter is shown in Fig.9.4.

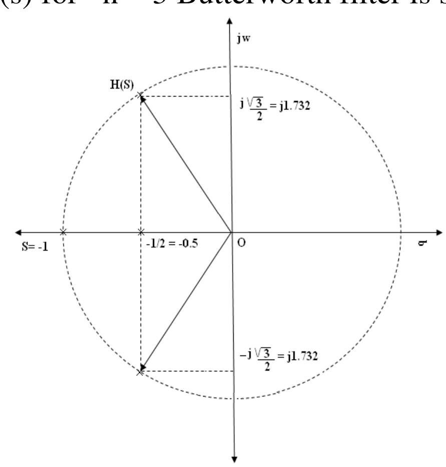


Fig.9.4

Q.111 Find the power dissipated in the 4Ω resistor in the circuit shown in Fig.7, using loop analysis. **(8)**

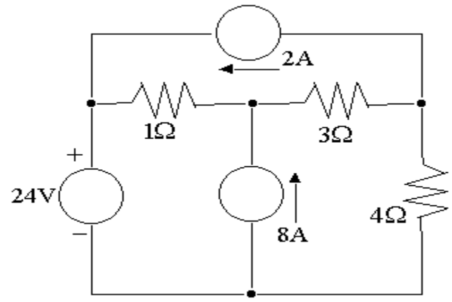


Fig.7

Ans:

KVL to supermesh (excluding branches having only sources) taking loop currents I_1, I_2 and I_3:

$-24 + 4I_3 + 3(I_3 - I_2) + 1(I_1 - I_2) = 0 \Rightarrow I_1 - 4I_2 + 7I_3 = 24$ ---------- (1)

For the branches having sources $\Rightarrow$ $I_2 = -2$,A, $I_3 - I_1 = 8$, A ---------- (2)

$\therefore$ from (1) and (2) $\Rightarrow$ $I_3 - 8 + 8 + 7I_3 = 24 \Rightarrow I_3 = 3$, A

$\therefore$ Power dissipated in the 4Ω resistor = $I_3^2 R = 9\times 4 = 36, W$

Q.112 Find v_x in the network of Fig.8, if the current through $(2+j3)$ element is zero. **(8)**

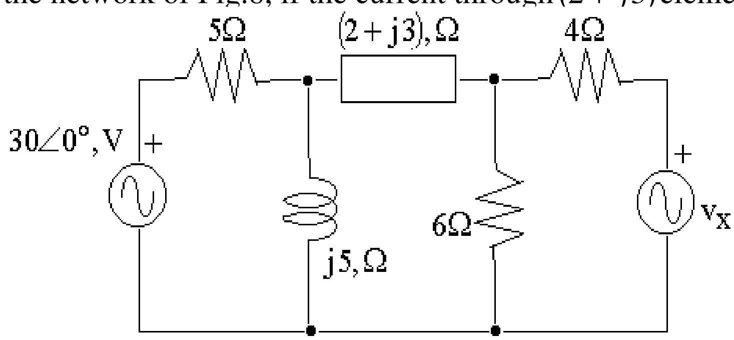


Fig.8

Ans:

No current through $(2+j3) \Rightarrow v_2 = v_3$

Also, $v_4 = v_x$, $v_1 = 30\angle 0^0$, v.

KCL (node v_2) $\Rightarrow i_1 = i_2 \Rightarrow \frac{v_1 - v_2}{\cancel{5}} = \frac{v_2}{j\cancel{5}} \Rightarrow v_2.(1-j) = 30\angle 0^0 \Rightarrow v_2 = \frac{30\angle 0^0}{\sqrt{2}\angle -45^0} = \frac{30}{\sqrt{2}}\angle 45^0$,

v.

KCL (node v_3)$\Rightarrow i_3 = i_4 \Rightarrow \frac{v_4 - v_3}{4} = \frac{v_3}{6} \Rightarrow v_x = \frac{5}{3}v_3 = \frac{5}{3}v_2 = \frac{50}{\sqrt{2}}\angle 45^0, v = 35.36\angle 45^0$, v.

Q.113 Derive the expression for transient current i(t) for a series R-L-C circuit with d-c excitation of V, volts, assuming zero initial conditions. What will i(t) if $R = 200\Omega$, L=0.1H, $\omega_o = 100\sqrt{10}$ rad/s and $\frac{di(0^+)}{dt} = 2000 A/s$? **(4+4)**

Ans:

KVL to loop of v and series R-L-C with i(t) $\Rightarrow V = L\frac{di}{dt} + Ri + \frac{1}{C}\int idt$

Differentiation $\Rightarrow O = L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} \underset{\rightarrow}{L} O = \left(s^2L + Rs + \frac{1}{C}\right)I(s)$.

$\therefore$ Roots of $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha \pm \beta$ (say).

Where $\alpha = \frac{R}{2L}, \omega_0^2 = \frac{1}{LC}, \, and \;\; \beta = \sqrt{\alpha^2 - \omega_0^2}$.

$\therefore$ Transient current

$i(t) = Ae^{s_1 t} + Be^{s_2 t}, A$ -------- (1) (A, B $\longrightarrow$ constants, derived usint inital conditions)

$\alpha = \frac{R}{2L} = \frac{200}{2 \times 0.1} = 1000; \alpha^2 = 10^6; \omega_0^2 = 10^5; \frac{DI}{DT}(o^+) = 2000, A/s = As_1e^{s_1 o} + Bs_2e^{s_2 o}$ ------ (2)

$\therefore \;\; \beta = \sqrt{\alpha^2 - \omega_0^2} = \sqrt{10^6 - 10^5} = 948.68; s_1 = -\alpha + \beta = -51.32; s_2 = -\alpha - \beta = -1948.68$ ---- (3)

From (2) and (3) $\Rightarrow A = 1.054 = -B.$

$\therefore \;\; i(t) = 1.054\left(e^{-51.32t} - e^{-1948.68}\right), A$

Q.114 Find the transform current I(s) drawn by the source shown in Fig.9 when switch K is closed at time t = 0. Assume zero initial conditions.

(8)

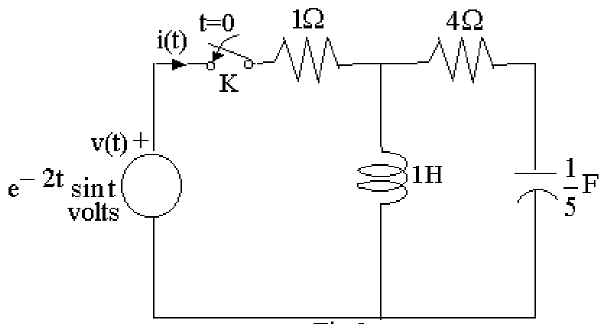
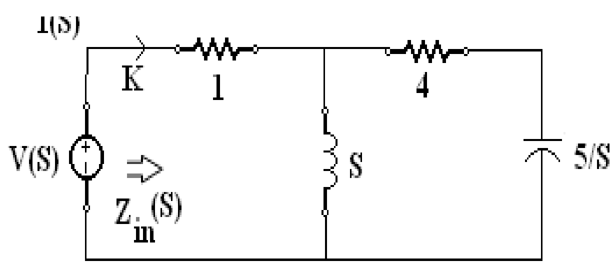


Fig.9

Ans:



$$Z_{in}(s) = 1 + \frac{s\left(4 + \frac{5}{s}\right)}{s + 4 + \frac{5}{s}} = 1 + \frac{4s^2 + 5s}{s^2 + 4s + 5}$$

$$=\left(\frac{5s^2+9s+5}{s^2+4s+5}\right)$$

$$V(s)=\frac{1}{(s+2)^2+1}=\frac{1}{s^2+4s+5}$$

$$\therefore\ I(s)=\frac{V(s)}{Z_{in}(s)}=\frac{1}{5s^2+9s+5}=\frac{1}{5}\frac{1}{\left(s+\frac{9}{10}\right)^2+1-\frac{81}{100}}=\frac{1}{5}\frac{1}{\frac{\sqrt{19}}{10}}\left[\frac{\frac{\sqrt{19}}{10}}{(s+0.9)^2+\left(\frac{\sqrt{19}}{10}\right)^2}\right]$$

$$\therefore\ i(t)\underset{\Rightarrow}{L^{-1}}\frac{2}{\sqrt{19}}.e^{-0.9t}.\sin\left(\frac{\sqrt{19}}{10}t\right)=0.4588e^{-0.9t}.\sin 0.4359t, A.$$

Q.115 State Thevenin theorem. Obtain the Thevenin equivalent of the network across the terminal AB as shown in Fig.10 (all element values are in Ω). **(4+4)**

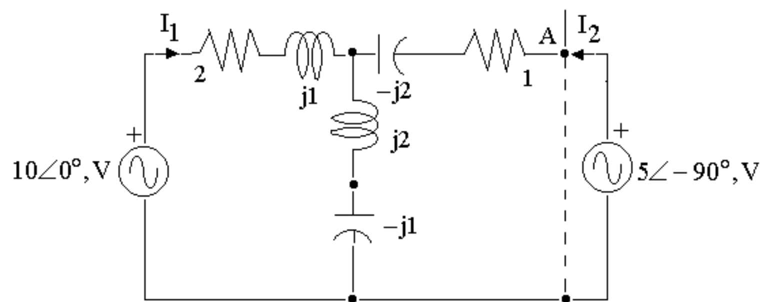
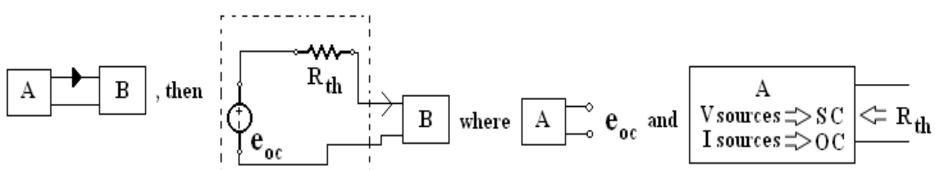


Fig.10

Ans:

"Given any linear active circuit, it can be replaced by a voltage source e_{oc} (= voltage at the output with load disconnected) with a series resistance R_{Th} (= equivalent resistance of network as seen from the O.C. output end, when all voltage sources are replaced by shrt-circuits and al current sources by open-circuits."



$$Z_{Th}=1-j2+\frac{(j2-j1)(j1+2)}{j2-j1+j1+2}=1-j2+\frac{j2-1}{2(j+1)}=1-j2+\frac{(j2-1)(-j+1)}{2(\sqrt{2})}=1-j2+\frac{1+j3}{4}$$

$$=\frac{5-j5}{4},\Omega=1.25-j1.25=1.25(1-j)$$

$$E_{oc}=10\underline{|0^0}\,\frac{j2-j1}{j2-j1+2+j1}=\frac{5j(1-j)}{(1+j)(1-j)}=2.5+j2.5$$

$$\therefore\ I_2=\frac{V_2}{Z_{Th}}=\frac{E_{oc}-5\underline{|-90^0}}{1.25(1-j)}=\frac{2.5+j2.5+j5}{1.25(1-j)}=\frac{2.5(1+j3)}{1.25(1-j)}=2\frac{(1+j3)(1+j)}{1+1}$$

$= 1-3+j4 = -2+j4, A$.

$= 4.472\underline{|-63^0.43}, A$.

Q.116 For the transform current function $I(s) = \dfrac{4s(s+2)}{(s+1)(s+3)}$, draw its pole-zero plot. Compute the inverse laplace transform. **(8)**

Ans:

Two poles $\Longrightarrow$ s = -1, -3. $\{ I(s) = K\dfrac{N(s)}{D(s)} \}$

$\therefore\ i(t) = Ae^{-t} + Be^{-3t}, A$ ---------------- (1)

Magnitude and phase contributions;

$\therefore\ A = 4\dfrac{1.e^{j\pi}}{2.e^{j0}} = -2$.

Pole at $s=-1 \Rightarrow M_{Z_1P_1}=1, M_{Z_2P_1}=1, M_{P_2P_1}=3; \phi_{Z_1P_1}=\pi, \phi_{Z_2P_1}=0=\phi_{P_2P_1}$ $B = 4\dfrac{3.e^{j3\pi}}{2.e^{j0}} = -6$.

Pole at $s=-3 \Rightarrow M_{Z_1P_2}=3, M_{Z_2P_2}=1, M_{P_1P_2}=2; \phi_{Z_1P_2}=\pi,= \phi_{Z_2P_2}=\phi_{P_1P_2}$

$\therefore\ i(t) = -2e^{-t} - 6e^{-3t} = -2e^{-t}(1+3e^{-2t}), A$

By partial fractions, $I(s) = \dfrac{A}{s+1} + \dfrac{B}{s+3} \Rightarrow A = \dfrac{4(-1)(-1+2)}{(-1+3)} = -2; B = \dfrac{4(-3)(-1)}{(-3+1)} = -6$

$\therefore\ i(t) = -2e^{-t} - 6e^{-3t}, A$.

Q.117 Derive the condition for maximum power transfer to the load $(R_\ell + jX_\ell)$ from a voltage source v_S having source impedance $(R_S + jX_S)$. Calculate this power if a $50\angle 0°$ voltage source having source impedance of 15+j20, Ω drives the impedance-matched load. **(5+3)**

Ans:

Load power = $|I_L|^2 Z_L = |I_L|^2 R_L$.

Or $P_L = \dfrac{{V_S}^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}$

For maximum power to load, $\left.\dfrac{dP_L}{dR_L}\right|_{X_L=fixed} = 0$ and $\left.\dfrac{dP_L}{dX_L}\right|_{R_L=fixed} = 0$ ----------------- (1)

i.e. ${V_S}^2\left[(R_S+R_L)^2.1 - R_L.2.(R_S+R_L).1\right] = 0 \Rightarrow (R_S - R_L)^2 = 0, or, R_S = R_L$ ---------------- (2)

And $R_L {V_S}^2[-1.2.(X_S+X_L).1] = 0 \Rightarrow X_L = -X_S$ ----------------- (3)

Combining (2) and (3) $\Rightarrow Z_L = Z_S^* \Rightarrow$ condition for maximum power transfer.

$U_S = 50\angle 0^0 V; Z_S = 15 + j20, \Omega; Z_L = Z_S^* = 15 - J20, \Omega$ (for matched impedance)

$\therefore\ I_L = \dfrac{U_S}{Z_S + Z_L} = \dfrac{50\angle 0^0}{30} = 1.66\angle 0^0, A.$

$\therefore\ P_L = |I_L|^2.R_L = (1.66)^2 15 = 41.33, W$

Q.118 For the circuit of Fig.11, determine e(t). Assume zero initial conditions. **(8)**

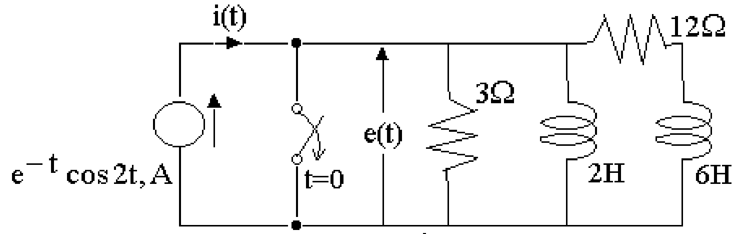


Fig.11

Ans:

$$Y(s)=\frac{1}{3}+\frac{1}{2s}+\frac{1}{6(s+2)}=\frac{2s(s+2)+3(s+2)+s}{6s(s+2)}$$

$$=\frac{2s^2+8s+6}{6s(s+2)}=\frac{s^2+4s+3}{3s(s+2)}=\frac{(s+1)(s+3)}{3s(s+2)}$$

$$I(s)=\frac{(s+1)}{(s+1)^2+4}=\frac{(s+1)}{s^2+2s+5}$$

$$\therefore\quad E(s)=\frac{I(s)}{Y(s)}=\frac{3s(s+2)}{(s+3)(s^2+2s+5)}$$

$$E(s)=\frac{A}{s+3}+\frac{Bs+C}{s^2+2s+5}\Rightarrow A(s^2+2s+5)+(Bs+C)(s+3)=3s(s+2)$$

$$A=\left.\frac{3s(s+2)}{s^2+2s+5}\right|_{s=-3}=\frac{3(-3)(-3+2)}{9-6+5}=\frac{9}{8};\ B=3-\frac{9}{8}=\frac{15}{8};\quad \left[\begin{array}{l} s^2: A+B=3 \\ s^1: 2A+3B+C=6 \\ s^0: 5A+3C=0 \end{array}\right.$$

$$C=-\frac{5}{3}A=-\frac{5}{3}\times\frac{9}{8}=-\frac{15}{8}.$$

$$\therefore\quad E(s)=\frac{9}{8}\left(\frac{1}{s+3}\right)+\frac{15}{8}\left(\frac{s-1}{s^2+2s+5}\right)=1.125\left(\frac{1}{s+3}\right)+1.875\left[\frac{(s+1)-2}{(s+1)^2+2^2}\right]$$

$$\therefore\ E(s)\underset{\Rightarrow}{L^{-1}}e(t)=1.125e^{-3t}+1.875e^{-t}\left[\cos 2t-\sin 2t\right]$$

$$e(t)=1.125e^{-3t}+1.875\sqrt{2}e^{-t}\left[\cos 2t\cos 45^0-\sin 2t\sin 45^0\right]$$

$$==1.125e^{-3t}+1.875\sqrt{2}e^{-t}\cos\left(2t+45^0\right).$$

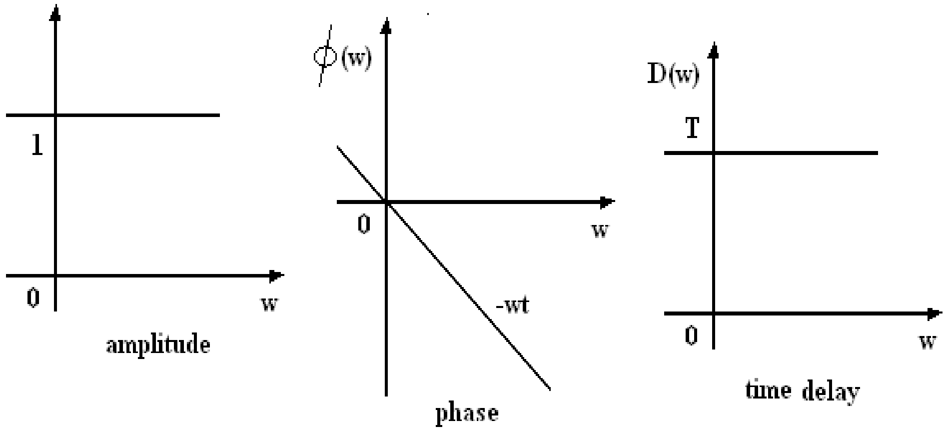
Q.119 Consider the transfer function of pure delay $H(s)=e^{-sT}$, where T = delay w.r.t. the excitation. Sketch the amplitude and phase responses and the delay characteristics. **(8)**

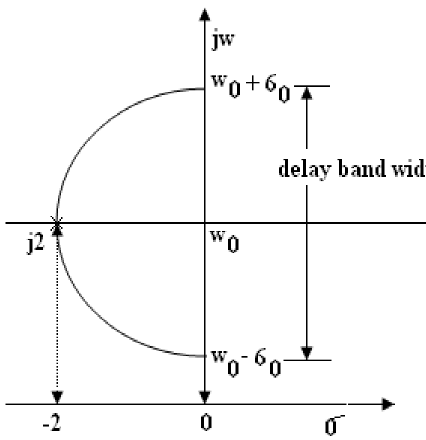
Ans:

$H(s)=e^{-sT}\Rightarrow H(j\omega)=e^{-j\omega T}\Rightarrow$ Amplitude $\left|H(j\omega)\right|=1$

Time delay $T\overset{\Delta}{=}-\frac{d}{d\omega}\phi(\omega)=T$. Phase = $\phi(\omega)=-\omega T$

Delay Bandwith = $26_0=2\times 2=4$ units.





Q.120 The peaking circle for a single-tuned circuit is shown in Fig.12. State the conditions on α and β for ω_{max} to exist. Determine ω_{max}, circuit Q and half-power points for $\alpha = 3, \beta = 5, A : (2,0)$. What is the condition for a high-Q circuit? **(2+5+1=8)**

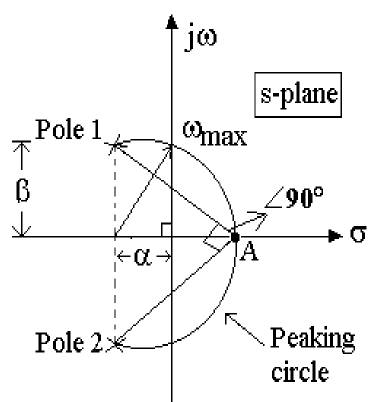
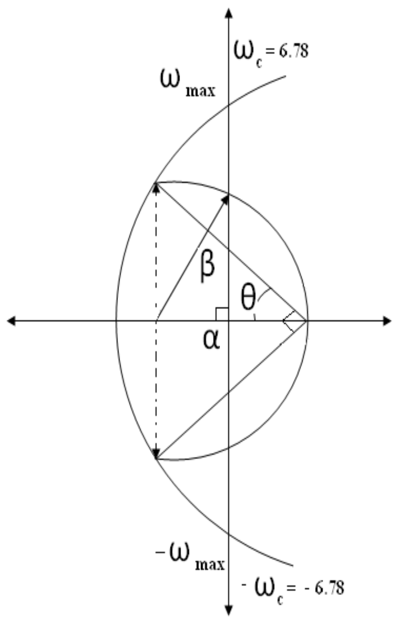


Fig.12

Ans:



Conditions for ω_{max} to exit;

(i) $\alpha = \beta \Rightarrow$ peaking circle cuts jω axis at $\omega = 0 \Rightarrow \omega_{max} = 0$.

(ii) $\alpha << \beta \Rightarrow \omega_{max} \approx \beta$.

(iii) $\alpha > \beta \Rightarrow \omega_{max}$ undetermined.

$\therefore$ For ω_{max} to be real and +ve, $\alpha < \beta$. ($\omega_{max} = 0$ for $\alpha = \beta$).

$\alpha = 3, \beta = 5 \Rightarrow \omega_{max} = \pm 4$ (on jω-axis); circuit-$Q \stackrel{\Delta}{=} (2\varsigma)^{-1} = \dfrac{1}{2\cos\theta}$

$$\cos\theta = \frac{3}{\sqrt{34}} = 0.5145 \Rightarrow \theta = 59^0 \Rightarrow Q = \frac{\sqrt{34}}{2\times 3} = 0.97.$$

From the graph $\Rightarrow \omega_{C1} = 6.78$ and $\omega_{C2} = -6.78$ rad/s.

(half power points)

For a high-Q circuit, $\varsigma^2 << 1$ and $\omega_{max} \approx \omega_0$ is the condition.

Q.121 Determine the y-parameters of the network of Fig.13. **(8)**

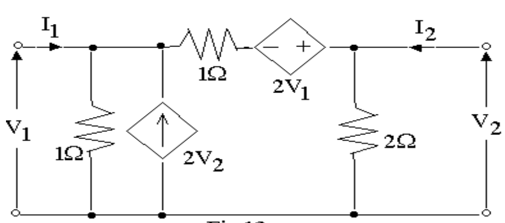


Fig.13

Ans:

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

KCL(a) $\Rightarrow I_1 = -2V_2 - I_C = -2V_2 - V_2 = -3V_2$

KCL(b) $\Rightarrow I_2 = I_c + I_d = V_2 + \dfrac{V_2}{2} = \dfrac{3}{2}V_2$

$$I_a = \frac{V_1}{1} = V_1$$

$$I_2 = -3V_1$$
$$I_b = -I_2$$
$$KVL \Rightarrow V_1 = 1(-I_2) - 2V_1$$
$$I_2 = -3V_1$$
$$I_1 = I_a + I_b = -I_2 + V_1 = 4V_1.$$
$$\therefore \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -3, S;$$
$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{2}, S;$$
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 4, S;$$
$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -3, S.$$

Q.122 Find the voltage transfer function, current transfer function, input and transfer impedances for the network of Fig.14. **(8)**

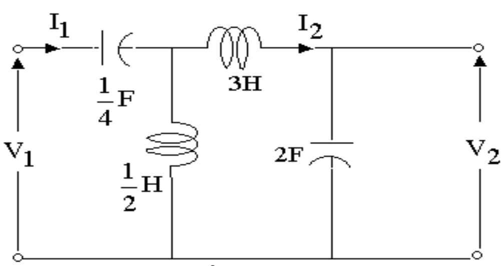


Fig.14

Ans:

$$\frac{I_2(s)}{V_2(s)} = \frac{2s}{1} = 2s$$
$$V_a(s) = 3s$$
$$\frac{V_a(s) - V_2(s)}{3s} = I_2(s)$$
$$\therefore \; V_a(s) = 3s(2s) + 1 = 6s^2 + 1$$
$$I_1(s) = \frac{V_1(s) - V_a(s)}{\frac{4}{s}} = \frac{s}{4}\left[V_1(s) - (6s^2 + 1)\right]$$
$$\Rightarrow I_1(s) = \frac{V_a(s)}{\left(\frac{1}{\frac{2}{s}}\right)} + I_2(s) = \frac{2}{s}\left(6s^2 + 1\right) + 2s = \frac{14s^2 + 2}{s} = 14s + \frac{2}{s}$$
$$V_1(s) = \frac{A}{s}.I_1(s) + V_a(s) = \frac{4}{s}\left(14s + \frac{2}{s}\right) + 6s^2 + 1 = 6s^2 + 1 + 56 + \frac{8}{s^2}$$
$$V_1(s) = \frac{6s^4 + 57s^2 + 8}{s^2}.$$
$$\therefore \quad Z_{in}(s) = \frac{I_1(s)}{V_1(s)} = \frac{6s^4 + 57s^2 + 8}{\cancel{s}^2\left(\frac{14s^2 + 2}{\cancel{s}}\right)} = \left(\frac{6s^4 + 57s^2 + 8}{14s^3 + 2s}\right);$$

$$\frac{V_2(s)}{V_1(s)} = \frac{1.s^2}{6s^4 + 57s^2 + 8};$$

$$\frac{I_2(s)}{I_1(s)} = \frac{2s}{\dfrac{14s^2+2}{s}} = \frac{s^2}{7s^2+1};$$

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{1.s^2}{14s^2+2}.$$

Q.123 From the given pole-zero configuration of Fig.15, determine the four possible L-C network configurations. **(8)**

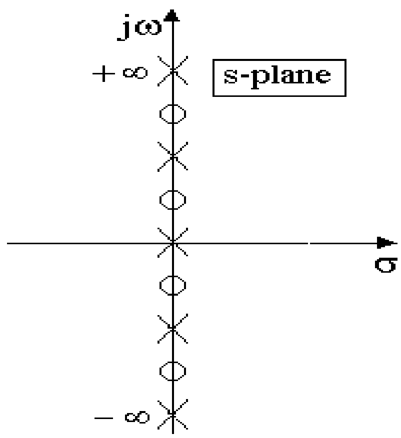


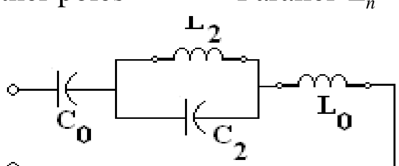
Fig.15

Ans:

(i) Pole at origin ω = 0 ⟹ first element is C_0.

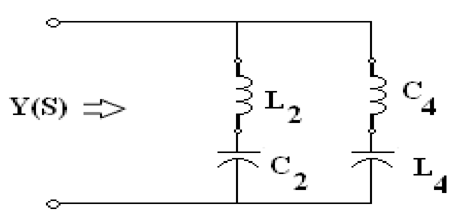
Pole at infinity ω = ∞ ⟹ last element is L_0.

Other poles ⟹ Parallel $L_n - C_n \Rightarrow$ I Foster Form.



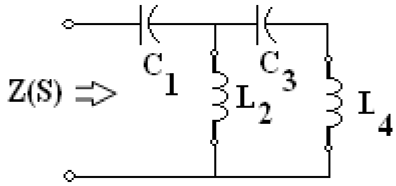
(ii) $Y(s) = \dfrac{1}{Z(s)} \Rightarrow$ No zero at origin ω = 0 ⟹ L_o absent.

Zeros at origin ω = 0 and infinity. ω = ∞ ⟹ Zeros $L_n - C_n \Rightarrow$ II Foster Form.



(iii) Pole at ∞ = ω ⟹ series inductor.
Pole at origin ω = 0 ⟹ last element is C.

(iv) Pole at origin ω = 0 ⟹ first element ⟹ series C.
Pole at infinity ω = ∞ ⟹ last element ⟹ inductor ⟹ II Cauer Form.



Q.124 Synthesise an L-C network with 1-Ω termination given the transfer impedance function:
$Z_{21}(s) = \dfrac{2}{s^3 + 3s^2 + 4s + 2}$. **(8)**

Ans:

$Z_{21}(s) = K.\dfrac{N(s)}{D(s)}$; KN(s) = 2 (even)(all zeros at ∞).

$\therefore \quad z_{21} = \dfrac{2}{s^3 + 4s}$ and $z_{22} = \dfrac{3s^2 + 2}{s^3 + 4s} \Rightarrow$ (Both have same poles)

$$y_{22} \Rightarrow \; 3s^2 + 2 \,\Big)\, s^3 + 4s \,\Big(\, \frac{1}{3}s \Rightarrow y \Rightarrow \text{ shunt C}$$

$$s^3 + \frac{2}{3}s$$

$$\overline{\qquad\qquad}$$

$$\frac{10}{3}s \,\Big)\, 3s^2 + 2 \,\Big(\, \frac{9}{10}s \Rightarrow Z \Rightarrow \text{ series L}$$

$$3s^2$$

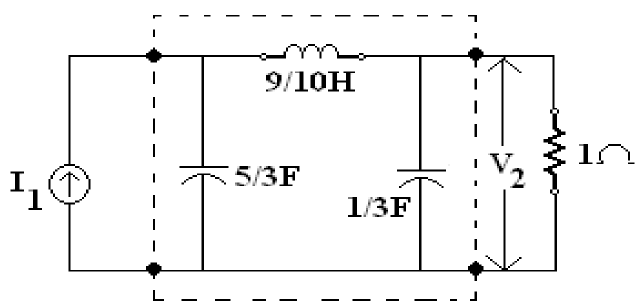
$$\overline{\qquad\qquad}$$

$$2 \,\Big)\, \frac{10}{3}s \,\Big(\, \frac{5}{3}s \Rightarrow y \Rightarrow \text{ shunt C}$$

$$\frac{10}{3}s$$

$$\overline{\qquad\qquad}$$

$$0$$



Q.125 Determine the range of constant 'K' for the polynomial to be Hurwitz.
$P(s) = S^3 + 3s^2 + 2s + K$ **(8)**

Ans:

$P(s) = S^3 + 3s^2 + 2s + K$

S^3	1	2
S^2	3	K
S^1	$\dfrac{6-K}{3}$	0
S	K	

For P(s) to be Hurwitz K > 0

$\frac{6-K}{3} > 0$ or $K < 6$

Range by 'K' $0 < K < 6$.

Q.126 Synthesize the admittance function $Y(s) = \frac{(s+2)(s+4)}{(s+1)(s+5)}$ in the form shown in Fig. 8 below.

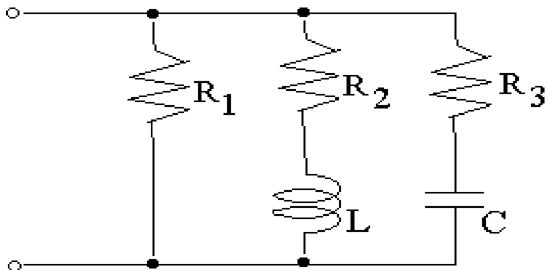


Fig.8

(14)

Ans:

The partial fraction expansion for Y(S) is

$$Y(S) = \frac{S^2+4S+2S+8}{S^2+5S+S+5} = \frac{S^2+6S+8}{S^2+6S+5} \quad \text{OR}$$

$$Y(S) = 1 + \frac{3}{(S^2+6S+5)} = 1 + \frac{3}{(S+1)(S+5)} = 1 + \frac{A_1}{S+1} + \frac{A_2}{S+5} \qquad \text{--------------- (1)}$$

$$A_2 = \left.\frac{3}{(S+1)}\right|_{S=-5} = \frac{3}{(-5+1)} = -\frac{3}{4}$$

$$A_1 = \left.\frac{3}{(S+5)}\right|_{S=-1} = \frac{3}{(-1+5)} = \frac{3}{4}$$

Therefore, the partial fraction expansion for Y(S) is

$$Y(S) = 1 + \frac{3/4}{S+1} + \frac{-3/4}{S+5} \qquad \text{----------------- (2)}$$

Since one of the residues in equation (2) is negative. This partial fraction expansion cannot be used for synthesis. An alternative method would be used to expand $\frac{Y(S)}{S}$ and then multiply the whole expansion by S. Hence

$$\frac{Y(S)}{S} = \frac{(S+2)(S+4)}{S(S+1)(S+5)} = \frac{A_1}{S} + \frac{A_2}{S+1} + \frac{A_3}{S+5}$$

$$A_1 = \left.\frac{(S+2)(S+4)}{S(S+1)(S+5)}\right|_{S=0} = \frac{(2)(4)}{(1)(5)} = \frac{8}{5}$$

Where

$$A_2 = \left.\frac{(S+2)(S+4)}{S(S+5)}\right|_{S=-1} = \frac{(-1+2)(-1+4)}{(-1)(-1+5)} = \frac{(1)(3)}{-4} = -\frac{3}{4}$$

and

$$A_3 = \left.\frac{(S+2)(S+4)}{S(S+1)}\right|_{S=-5} = \frac{(-5+2)(-5+4)}{(-5)(-5+1)} = \frac{(-3)(-1)}{(-5)(-4)} = \frac{3}{20}$$

Therefore, the partial fraction expansion for $\frac{Y(S)}{S}$ is

$$\frac{Y(S)}{S} = \frac{\frac{8}{5}}{S} - \frac{\frac{3}{4}}{S+1} + \frac{\frac{3}{20}}{S+5} \qquad \text{----------------- (3)}$$

By multiplying the equation(3) with 'S', we obtain

$$Y(S) = \frac{\frac{8}{5}(S)}{S} - \frac{\frac{3}{4}S}{S+1} + \frac{\frac{3}{20}S}{S+5}$$

OR

$$Y(S) = \frac{8}{5} - \frac{\frac{3}{4}S}{S+1} + \frac{\frac{3}{20}S}{S+5} \qquad \text{----------------- (4)}$$

If we observe in equation(4), Y(S) also has a negative term. If we divide the denominator of this negative term into the numerator, we can rid ourselves of any terms with negative signs.

Hence $$Y(S) = \frac{8}{5} - \left(\frac{3}{4} - \frac{\frac{3}{4}}{S+1}\right) + \frac{\frac{3}{20}S}{S+5}$$

$$= \frac{8}{5} - \frac{3}{4} + \frac{\frac{3}{4}}{S+1} + \frac{\frac{3}{20}S}{S+5}$$

$$= \left(\frac{32-15}{20}\right) + \frac{\frac{3}{4}}{S+1} + \frac{\frac{3}{20}S}{S+5}$$

$$Y(S) = \underbrace{\frac{17}{20}}_{R_1} + \underbrace{\frac{\frac{3}{4}}{S+1}}_{(R_2\,\&\,L)} + \underbrace{\frac{\frac{3}{20}S}{S+5}}_{(R_3\,\&\,C)} \qquad \text{----------------- (5)}$$

From equation (5),

(i) The first term of value $\frac{20}{17}S$ is the Resistance (R)

(ii) The second term is the series combination of Resistance $\left(\frac{4}{3}\Omega\right)$ and inductance of value $\frac{4}{3}$ Henrys

(iii) The third term is the series combination of Resistance of value $\frac{20}{3}\Omega$ and capacitance of value $\frac{3}{100}$ Farads. The final synthesized network for the admittance function Y(S) is shown in fig.7.1.

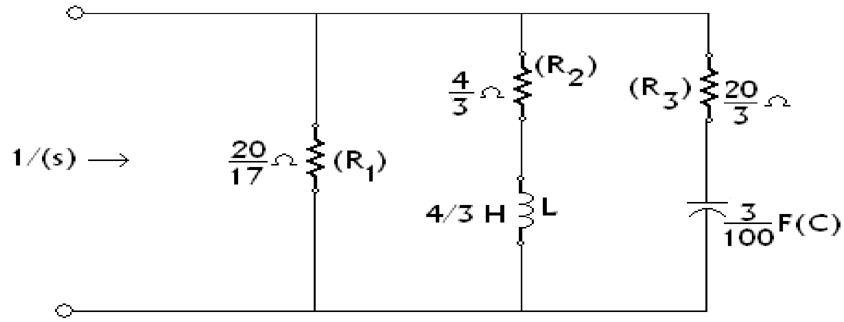


fig.7.1

Q.127 Synthesize an RC ladder and an RL ladder network to realize the function $F(s)=\frac{s^2+4s+3}{s^2+8s+12}$ as an impedance or an admittance. **(14)**

Ans:
The given function is

$$F(S)=\frac{S^2+4S+3}{S^2+8S+12}=\frac{(S+1)(S+3)}{(S+2)(S+6)}$$

RC Impedance Function: The partial fraction expansion of Z(S) will result in negative residues at poles S = -2 and S = -6. Therefore, we expand $\frac{F(S)}{S}$ by Partial fraction and then multiply by S. Hence

$$\frac{F(S)}{S}=\frac{(S+1)(S+3)}{S(S+2)(S+6)}=\frac{A_1}{S}+\frac{B_1}{S+2}+\frac{C_1}{S+6}$$

Where

$$A_1=\left.\frac{(S+1)(S+3)}{(S+2)(S+6)}\right|_{S=0}=\frac{(1)(3)}{(2)(6)}=\frac{3}{12}=\frac{1}{4}$$

$$B_1=\left.\frac{(S+1)(S+3)}{S(S+6)}\right|_{S=2}=\frac{(-2+1)(-2+3)}{(-2)(-2+6)}=\frac{(-1)(1)}{(-2)(4)}=\frac{1}{8}$$

And

$$C_1=\left.\frac{(S+1)(S+3)}{S(S+2)}\right|_{S=-6}=\frac{(-6+1)(-6+3)}{(-6)(-6+2)}=\frac{(-5)(-3)}{(-6)(-4)}=\frac{15}{24}=\frac{5}{8}$$

Therefore, $\dfrac{F(S)}{S} = \dfrac{1}{4S} + \dfrac{1}{8(S+2)} + \dfrac{5}{8(S+6)}$ ---------------- (1)

Now, from equation (1), none of the residues are negative. Hence multiplying the equation (1) by S, we have

$F(S) = \dfrac{1}{4} + \dfrac{S}{8(S+2)} + \dfrac{5S}{8(S+6)}$ ---------------- (2)

$$F(S) = \underbrace{\frac{1}{4}}_{R} + \underbrace{\frac{1}{\dfrac{1}{1/8} + \dfrac{1}{S/16}}}_{(R \parallel C)} + \underbrace{\frac{1}{\dfrac{1}{5/8} + \dfrac{1}{5S/48}}}_{(R \parallel C)}$$

The resultant function F(S) is a RC Ladder impedance function.

(i) The first term is the Resistance of value $\frac{1}{4}\Omega$

(ii) The second term is the Parallel combination of resistance of value $\frac{1}{8}\Omega$ and capacitance of value $\frac{1}{16}F$.

(iii) The third term is the parallel combination of resistance of value $\frac{5}{8}\Omega$ and capacitance of value $\frac{5}{48}$ Farads.

The resultant RC Impedance Ladder network is shown in Fig.8.1

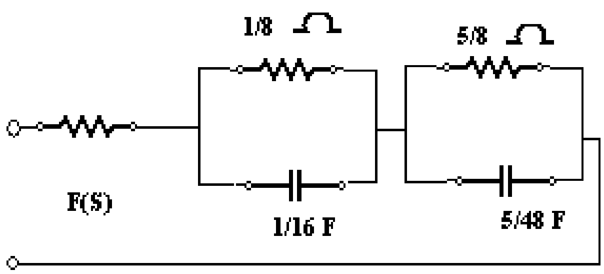


Fig.8.1

RL Admittance Function: RL admittance function is obtained by repeated removal of poles at S = 0 which corresponds to arranging numerator and denominator of F(S) in ascending powers of S and then find continued fraction expansion.

Therefore, $F(S) = \dfrac{S^2 + 4S + 3}{S^2 + 8S + 12}$

$12+8S+S^2 \,\big)\, 3+4S+S^2 \,\big(\, \frac{1}{4} \rightarrow Z$ (Resistance)

$3+2S+\frac{S^2}{4}$

$-\quad -\quad -$

$2S+\frac{3}{4}S^2 \,\big)\, 12+8S+S^2 \,\big(\, \frac{6}{S} \rightarrow Y$ (Inductance)

$12+\frac{18}{4}S$

$-\quad -$

$\frac{7}{2}S+S^2 \,\big)\, 2S+\frac{3}{4}S^2 \,\big(\, \frac{4}{7} \rightarrow Z$ (Resistance)

$2S+\frac{4}{7}S^2$

$-\quad -$

$$\frac{5}{28}S^2 \Big) \; \frac{7}{2}S + S^2 \; \Big(\frac{196}{105} \rightarrow Y \quad \text{(Inductance)}$$

$$\frac{7}{2}S$$

$$-$$

$$S^2 \Big) \; \frac{5}{28}S^2 \; \Big(\frac{5}{28} \rightarrow Z \quad \text{(Resistance)}$$

$$\frac{5}{28}S^2$$

$$-$$

$$0$$

Therefore, the resultant RL admittance Ladder network for the function F(S) is shown in fig.8.2

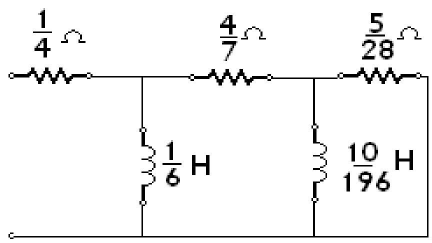


fig.8.2